# FOUR-JET PRODUCTION IN $k_{\mathrm{T}}$-FACTORIZATION: SINGLE AND DOUBLE PARTON SCATTERING*

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We present a preliminary study of both Single and Double Parton Scattering contributions to the inclusive 4-jet production in the $k_{\mathrm{T}}$-factorization framework at Leading Order and $E_{\mathrm{CM}} = 7$ TeV. We compare our results to collinear results in the literature and to the ATLAS and CMS data at 8 and 7 TeV, respectively. We also discuss the importance of double-parton scattering for relatively soft cuts on the jet transverse momenta and find out that symmetric cuts do not suit quite well to $k_{\mathrm{T}}$-factorization predictions because of a kinematic effect suppressing the double parton scattering contribution.

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## 1. Introduction

We employ fully gauge-invariant amplitudes with initial state off-shell particle to assess both the Single Parton Scattering and Double Parton Scattering contributions to four-jet production. This allows us to expand the analysis of Ref. [1] and assess the differences between the collinear approach and the high-energy factorization (HEF) (or $k_{\mathrm{T}}$-factorization).

## 2. Calculations and comparison to experimental data

### *2.1. Single-parton scattering production of four jets*

The collinear factorization formula for the calculation of the inclusive partonic 4-jet cross section reads

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$$\sigma^B_{4\text{-jets}} = \sum_{i,j} \int \frac{\mathrm{d}x_1}{x_1} \frac{\mathrm{d}x_2}{x_2} \, x_1 f_i(x_1, \mu_\mathrm{F}) \, x_2 f_j(x_2, \mu_\mathrm{F}) \times \frac{1}{2\hat{s}} \prod_{l=1}^{4} \frac{\mathrm{d}^3 k_l}{(2\pi)^3 2E_l} \Theta_{4\text{-jet}} \, (2\pi)^4 \, \delta\left( x_1 P_1 + x_2 P_2 - \sum_{l=1}^{4} k_l \right) \times \overline{|\mathcal{M}(i, j \to 4\,\text{part.})|^2} \,. \tag{1}$$

Here, $f_i(x_{1,2}, \mu_\mathrm{F})$ are the collinear PDFs for the $i^\mathrm{th}$ parton, carrying $x_{1,2}$ momentum fractions of the proton and evaluated at the factorization scale $\mu_\mathrm{F}$; the index $l$ runs over the four partons in the final state, $P$ is the total initial state partonic momentum, associated to the center-of-mass energy $\hat{s} = P^2 = (P_i + P_j)^2 = 2\, P_i \cdot P_j$; the $\Theta$ function takes into account the kinematic cuts applied, and $\mathcal{M}$ is the partonic on-shell matrix element, which includes symmetrization effects due to identity of particles in the final state.

The analogous formula to (1) for HEF is

$$\sigma^B_{4\text{-jets}} = \sum_{i,j} \int \frac{\mathrm{d}x_1}{x_1} \frac{\mathrm{d}x_2}{x_2} \, \mathrm{d}^2 k_{\mathrm{T}1} \mathrm{d}^2 k_{\mathrm{T}2} \, \mathcal{F}_i(x_1, k_{\mathrm{T}1}, \mu_\mathrm{F}) \, \mathcal{F}_j(x_2, k_{\mathrm{T}2}, \mu_\mathrm{F}) \times \frac{1}{2\hat{s}} \prod_{l=1}^{4} \frac{\mathrm{d}^3 k_l}{(2\pi)^3 2E_l} \Theta_{4\text{-jet}} \, (2\pi)^4 \times \delta\left( x_1 P_1 + x_2 P_2 + \vec{k}_{\mathrm{T}1} + \vec{k}_{\mathrm{T}2} - \sum_{l=1}^{4} k_l \right) \overline{|\mathcal{M}(i^*, j^* \to 4\,\text{part.})|^2} \,. \tag{2}$$

Here, $\mathcal{F}_i(x_k, k_{\mathrm{T}k}, \mu_\mathrm{F})$ is a transverse momentum dependent (TMD) parton distribution function for a given type of parton. Similarly, as in the collinear factorization case, $x_k$ is the longitudinal momentum fraction and $\mu_\mathrm{F}$ is a factorization scale. The new degree of freedom is introduced via the transverse $k_{\mathrm{T}k}$, which is perpendicular to the collision axis. The formula is valid when the $x$s are not too large and not too small and, in order to construct a full set of TMD parton densities, we apply the Kimber–Martin–Ryskin (KMR) prescription [2, 3], which, at the end of the day, amounts to applying the Sudakov form factor onto the PDFs.

$\mathcal{M}(i^*, j^* \to 4\,\text{part.})$ is the gauge invariant matrix element for $2 \to 4$ particle scattering with two initial off-shell legs. We rely on the numerical Dyson–Schwinger recursion in the AVHLIB[1] for its computation. If complete calculation of 4-jet production in $k_\mathrm{T}$-factorization was still missing in

[1] Available for download at `https://bitbucket.org/hameren/avhlib`

the literature, it was mainly because computing gauge-invariant amplitudes with off-shell legs is definitely non-trivial. Techniques to compute such amplitudes in gauge invariant ways are by now analytically and numerically well-established [4–6].

We use a running $\alpha_{\rm s}$ provided with the MSTW2008 PDF sets and set both the renormalization and factorization scales equal to half the transverse energy, which is the sum of the final state transverse momenta, $\mu_{\rm F} = \mu_{\rm R} = \frac{\hat{H}_{\rm T}}{2} = \frac{1}{2}\sum_{l=1}^{4} k_{\rm T}^{l}$, working in the $n_{\rm f} = 5$ flavour scheme.

There are 19 different channels contributing to the cross section at the parton-level, of which the dominant ones, contributing together to $\sim 93\%$ of the total cross section, are

$$\begin{aligned} &gg \to gggg\,, \quad gg \to q\bar{q}gg\,, \quad qg \to qggg\,, \quad q\bar{q} \to q\bar{q}gg\,, \\ &qq \to qqgg\,, \quad qq' \to qq'gg\,. \end{aligned} \tag{3}$$

### 2.2. Double-parton scattering production of four jets

The SPD contribution is expected to dominate for high momentum transfer, because, as it is intuitively clear as well, it is highly unlikely that two partons from one proton and two from the other one are energetic enough for two hard scatterings to take place, as the well-known behaviour of the PDFs for large momentum fractions suggests. However, if the cuts on the transverse momenta of the final state are lowered, a window opens to observe significant double-parton scattering effects, as often stated in the literature on the subject and recently analysed for 4-jet production in the framework of collinear factorization [1]. Here, we perform the same analysis in HEF, with the goal to assess the difference in the predictions.

First of all, let us present the standard formula for the computation of DPS cross section for a four-parton final state,

$$\frac{d\sigma^{B}_{4\text{-jet,DPS}}}{d\xi_1 d\xi_2} = \frac{m}{\sigma_{\rm eff}} \sum_{i_1,j_1,k_1,l_1;i_2,j_2,k_2,l_2} \frac{d\sigma^{B}(i_1 j_1 \to k_1 l_1)}{d\xi_1}\,\frac{d\sigma^{B}(i_2 j_2 \to k_2 l_2)}{d\xi_2}\,, \tag{4}$$

where the $\sigma(ab \to cd)$ cross sections are obtained by restricting formulas (1) and (2) to a single channel and the symmetry factor $m$ is $1/2$ if the two hard scatterings are identical, in order to avoid double counting, and is otherwise 1, whereas $\xi_1$ and $\xi_2$ are for generic kinematical variables for the first and second scattering, respectively.

The effective cross section $\sigma_{\rm eff}$ can be loosely interpreted as a measure of the transverse correlation of the two partons inside the hadrons. In this paper, we stick to the widely used value $\sigma_{\rm eff} = 15$ mb.

We also have to use an Ansatz for DPDFs, which for collinear factorization is

$$D_{1,2}(x_1, x_2, \mu) = f_1(x_1, \mu)\, f_2(x_2, \mu)\, \theta(1 - x_1 - x_2)\,, \tag{5}$$

where $D_{1,2}(x_1, x_2, \mu)$ is the DPDF and $f_i(x_i, \mu)$ are the ordinary PDFs. The subscripts 1 and 2 distinguish the two generic partons in the same proton. Of course, this Ansatz can be automatically generalised to the case when parton transverse momenta are included by simply including the dependence on the transverse momentum.

Coming to DPS contributions, in principle, we must include all the possible 45 channels which can be obtained by coupling in all possible distinct ways the 8 channels for the $2 \to 2$ SPS process, *i.e.*

$$\begin{aligned}
\#1 &= gg \to gg\,, & \#5 &= q\bar{q} \to q'\bar{q}'\,,\\
\#2 &= gg \to q\bar{q}\,, & \#6 &= q\bar{q} \to gg\,,\\
\#3 &= qg \to qg\,, & \#7 &= qq \to qq\,,\\
\#4 &= q\bar{q} \to q\bar{q}\,, & \#8 &= qq' \to qq'\,.
\end{aligned}$$

We find that the pairs $(1,1)$, $(1,2)$, $(1,3)$, $(1,7)$, $(1,8)$, $(3,3)$ $(3,7)$, $(3,8)$ together account for more than 95% of the total cross section. This was tested for all the sets of cuts considered in this paper.

### *2.3. Comparison to the collinear approach and to the ATLAS data with hard central kinematic cuts*

Our HEF calculation was first tested against the 8 TeV data recently reported by the ATLAS Collaboration [7]. The kinematic cuts are $p_{\mathrm{T}} >$ 100 GeV for the leading jet and $p_{\mathrm{T}} > 64$ GeV for the first three subleading jets; in addition, $|\eta| < 2.8$ is the pseudorapidity cut and $\Delta R > 0.65$ is the constraint on the jet cone radius parameter.

We employ the running NLO $\alpha_{\mathrm{s}}$ coming with the MSTW2008 sets. For such hard central cuts, not much difference is expected between the two approaches and, indeed, we find none. Also, DPS effects are irrelevant with this kinematics and this is confirmed by our analysis, which is presented in much more detail in [8].

The collinear factorization performs slightly better for intermediate values and HEF does a better job for the last bins, except for the $4^{\mathrm{th}}$ jet. All in all, both approaches are consistent with the data in this kinematic region.

### *2.4. Comparison to CMS data with softer cuts*

As discussed in Ref. [1], so far the only experimental analysis of four-jet production relevant for the DPS studies was realized by the CMS Collaboration [9]. The cuts used in this analysis are $p_{\mathrm{T}} > 50$ GeV for the first and second jets, $p_{\rm T} > 20$ GeV for the third and fourth jets, $|\eta| < 4.7$ and the jet cone radius parameter $\Delta R > 0.5$. In the rest of this section, we present our results for such cuts.

As for the total cross section for the four-jet production, the experimental and theoretical LO results are:

$$
\begin{aligned}
\text{CMS Collaboration}: &\quad \sigma_{\rm tot} = 330 \pm 5\,(\text{stat.}) \pm 45\,(\text{syst.})\ \text{nb}\,,\\
\text{LO collinear factorization}: &\quad \sigma_{\rm SPS} = 697\ \text{nb}\,, \qquad \sigma_{\rm DPS} = 125\ \text{nb}\,,\\
&\quad \sigma_{\rm tot} = 822\ \text{nb}\,,\\
\text{LO HEF } k_{\rm T}\text{-factorization}: &\quad \sigma_{\rm SPS} = 548\ \text{nb}\,, \qquad \sigma_{\rm DPS} = 33\ \text{nb}\,,\\
&\quad \sigma_{\rm tot} = 581\ \text{nb}\,. \qquad (6)
\end{aligned}
$$

It is apparent that the LO results need refinements from NLO contributions, much more than it does in the case of the ATLAS hard cuts, as we are, of course, not that deep into the perturbative region. For this reason, in the following, we will always perform comparisons only to data normalised to the total (SPS+DPS) cross sections. We find that this is better than introducing fixed K-factors, whose phase-space dependence is never really under control. What is immediately apparent in the HEF total cross section is the dramatic damping of the DPS contribution with respect to the collinear case. The effect of the damping is of kinematical nature and can be understood by an analogy of a similar effect first observed in NLO jet photoproduction at HERA [10]. The point is that the emission of gluon radiation, which is taken into account via the TMDs in our approach and via the real contribution in a collinear NLO calculation, alters the exact momentum balance of the final state two-jet system, so that a lot of events are not taken into account for the higher transverse momentum just above the cut. In Fig. 1, we compare the predictions in HEF to the CMS data for the 1$^{\rm st}$ and 2$^{\rm nd}$

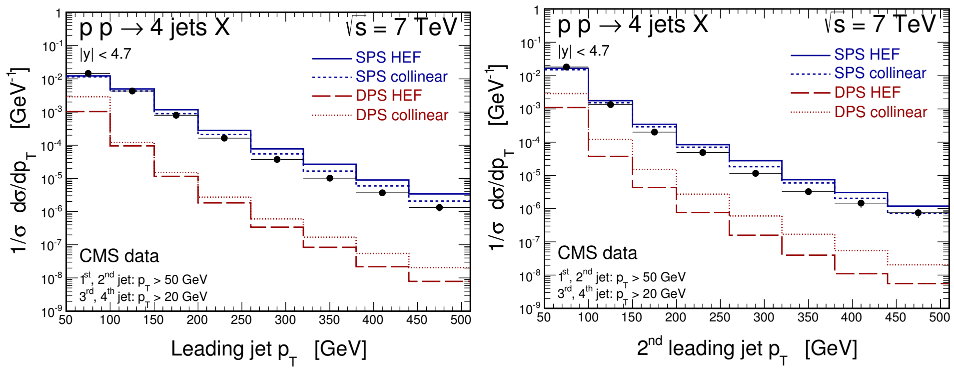


Fig. 1. Comparison of the LO collinear and HEF predictions to the CMS data for the 1$^{\rm st}$ and 2$^{\rm nd}$ leading jets.

leading jets transverse momenta spectra. Here, both the SPS and DPS contributions are normalized to the total cross section, *i.e.* the sum of the SPS and DPS contributions. In all cases, the renormalized transverse momentum distributions agree quite well with the CMS data.

### 2.5. HEF predictions for a possible set of asymmetric cuts

Moving from the previous considerations, we present our results for four-jet production by employing also another set of cuts, which are asymmetric with respect to the final state transverse momenta. Specifically, we require $p_{\rm T} > 35$ GeV for the leading jet, $p_{\rm T} > 20$ GeV for all the other jets, and we stick to $|\eta| < 4.7$, $\Delta R > 0.65$ for rapidity and jet size parameter. An experimental analysis with such cuts is not available: while the CMS Collaboration did perform the analysis for soft enough cuts as to allow for significant DPS contributions to show up, they did not impose such asymmetry, as discussed above, whereas both analyses presented by the ATLAS Collaboration employ too hard cuts for multi-parton interactions to be any significant at all [7]. Of course, it would be desirable to have such an analysis in the future.

The theoretical total cross sections for these cuts for four-jet production are:

$$\begin{aligned} \text{LO collinear factorization:} \quad & \sigma_{\rm SPS} = 1969 \text{ nb}\,, \qquad \sigma_{\rm DPS} = 514 \text{ nb}\,, \\ \text{LO HEF } k_{\rm T}\text{-factorization:} \quad & \sigma_{\rm SPS} = 150 \text{ nb}\,, \qquad \sigma_{\rm DPS} = 297 \text{ nb}\,. \end{aligned} \tag{7}$$

When comparing to (6), it is apparent that now the drop in the total cross section for DPS when moving from LO collinear to HEF approach is considerably smaller, as argued.

In Fig. 2, we show our predictions for the normalized transverse momentum distributions with the new set of cuts.

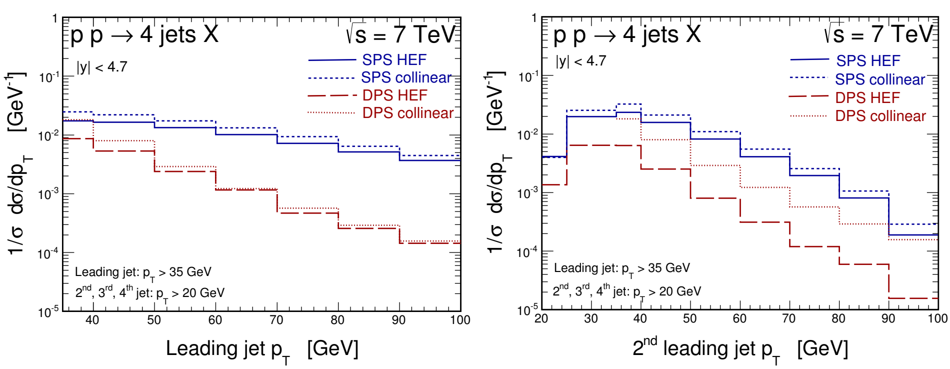


Fig. 2. LO collinear and HEF predictions for the 1st and 2nd leading jets with the asymmetric cuts.

## 3. Conclusions

In the present work, we have compared the perturbative predictions for four-jet production at the LHC in leading-order collinear and high-energy ($k_T$-)factorization. While we find that there is no significant difference between the collinear and HEF approach for hard central cuts, significant differences show up, especially for DPS, when the cuts on the transverse momenta are lowered. We agree with Ref. [1] that lowering the cut in transverse momenta can significantly enhance the experimental sensitivity to DPS but we also observe that HEF severely tames this effect for symmetric cuts, due to gluon-emission effects which alter the transverse-momentum balance between final state partons. We have found that the damping is not present when cuts are not identical. A more complete treatment of the subjects addressed in this proceeding can be found in [8].

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