

# FLUCTUATIONS IN THE STATISTICAL MODEL OF THE EARLY STAGE OF NUCLEUS–NUCLEUS COLLISIONS\*

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Predictions on fluctuations of hadron production properties in central heavy-ion collisions are presented. They are based on the Statistical Model of the Early Stage and extend previously published results by considering the strongly intensive measures of fluctuations. In several of the considered cases, a significant change in collision-energy dependence of calculated quantities as a result of the phase transition is predicted. This provides an opportunity to observe new signals of the onset of deconfinement in heavy-ion collisions experiments.

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## 1. Introduction

Relativistic nucleus–nucleus ( $A + A$ ) collisions provide a unique opportunity to experimentally study different phases of strongly interacting matter and transitions between them, for the recent review, see Ref. [1]. In particular, since the discovery of sub-hadronic particles, quarks and gluons, it was expected that at high temperature and/or pressure, densely packed hadrons will “dissolve” into a new phase of quasi-free quarks and gluons, the quark–gluon plasma (QGP) [2–5].

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Years of experimental and theoretical studies of high-energy  $A + A$  collisions led to the conclusion that the QGP exists in nature. This conclusion is based on a wealth of systematic data on  $A + A$  collisions at very high energies from the CERN Large Hadron Collider (LHC) (see, *e.g.*, Ref. [6]) and the BNL Relativistic Heavy Ion Collider (RHIC) (see, *e.g.*, Ref. [7]) and, very importantly, the evidence of the transition between hadronic matter and QGP (the onset of deconfinement) at the CERN Super Proton Synchrotron (SPS) energies [8, 9].

The experimental search for the onset of deconfinement was motivated [10] by predictions of the Statistical Model of the Early Stage (SMES) [11] of  $A + A$  collisions. According to the model, the onset of deconfinement in central  $A + A$  collisions should lead to a rapid change of the energy dependence of several hadron production properties, all appearing in a common energy domain. The predicted signals in single hadron properties were observed [8, 9, 12]. They indicate that the onset of deconfinement (the beginning of the mixed phase) is located at  $\sqrt{s_{NN}}(\text{OD}) \approx 8$  GeV and the softest point (the end of the mixed phase region) at  $\sqrt{s_{NN}}(\text{SP}) \approx 12$  GeV [13], where  $\sqrt{s_{NN}}$  denotes collision energy per nucleon pair in the center-of-mass system. Several signatures of the deconfinement phase transition were observed. In particular, the energy dependence of the yield ratio of positively charged kaons to pions (*horn*) in central Pb+Pb collisions shows the sharp maximum. This ratio is rather sensitive to masses and degeneracy factors of strange particles in the confined and deconfined phases. It was experimentally found that the ratio increases rapidly with  $\sqrt{s_{NN}}$  at low-collision energies (in the hadron phase), decreases at intermediate-collision energies (in the mixed phase), and is weakly dependent on  $\sqrt{s_{NN}}$  at high-collision energies (in the QGP). Such a behaviour was predicted by the SMES [11]. The model prediction was based on the fact that the mass of strange hadrons is significantly higher than the temperature of the hadron phase, whereas the mass of the strange quarks is small compared to the temperature of the QGP phase.

However, up to now, no convincing signal in fluctuations of event properties was reported [14]. Fluctuations are significantly more difficult to study than single hadron properties. This may explain problems in locating the onset of deconfinement using fluctuation measurements and it asks for experimental and theoretical developments in the study of fluctuations in relativistic nucleus–nucleus collisions.

This work extends the model predictions [15, 16] on fluctuations of hadron production properties in relativistic  $A + A$  collisions related to the phase transition. The predictions have been based on the SMES model of  $A + A$  collisions [11]. The extension is needed because new measures of fluctuations were introduced in the recent years, for reviews, see Refs. [12, 14].

In addition, the paper introduces a more general formalism to model fluctuations in high-energy collisions which may be helpful in future efforts. The calculations are performed within the simplest model of the phase transition in  $A+A$  collisions. This model does not include many effects which should be taken into account for a quantitative comparison with experimental results. Different aspects of fluctuations signals of the deconfinement transition are discussed, *e.g.*, in Refs. [17–22].

## 2. Event-by-event fluctuations in SMES

Based on the success of statistical and hydrodynamical models of particle production in high-energy collisions, the SMES assumes that the matter created at the early stage of collisions is in equilibrium. The created matter has zero conserved charges, thus, all chemical potentials are equal to zero, and the fireball energy  $E$  and volume  $V$  are assumed to vary from collision to collision according to the probability distribution function  $P(E, V)$ . Consequently, the energy density,  $\varepsilon = E/V$ , also may change from collision to collision. This leads to changes of other properties of matter which, within the grand canonical ensemble (GCE) used here, can be calculated using its equation of state (EoS). In particular, according to the first and the second principles of thermodynamics, the entropy change  $\delta S$  ( $\delta$  denotes a deviations from average value) is related to energy and volume changes as  $T\delta S = \delta E + p\delta V$ , which provides  $T\delta S = V\delta\varepsilon + (p + \varepsilon)\delta V$ , where  $p$  is the pressure. Using the identity  $TS = E + pV$ , one finds

$$\frac{\delta S}{S} = \frac{1}{1 + p/\varepsilon} \frac{\delta\varepsilon}{\varepsilon} + \frac{\delta V}{V}. \quad (1)$$

In the case of energy and volume being proportional and the energy density being constant,  $\delta\varepsilon = 0$ , Eq. (1) gives:  $\delta S/S = \delta V/V = \delta E/E$ . Thus, the relative fluctuations of entropy are equal to those of energy and volume, and they are insensitive to the EoS. In other cases, the entropy fluctuations depend on the EoS and thus on a form of the created matter.

Similarly to entropy, mean multiplicity of particles of a given type changes with  $E$  and  $V$ . These changes depend on the EoS and particle properties providing they lead to changes of  $\varepsilon$ . In particular, mean multiplicity of light pions or light quarks and massless gluons is approximately proportional to entropy of the produced matter. When crossing the transition region, the effective number of degrees of freedom increases and thus the entropy increases faster with increasing  $\sqrt{s_{NN}}$ . Consequently, entropy fluctuations caused by energy density fluctuations will also be modified. Mean multiplicity of strange hadrons in the confined phase or strange quarks in the deconfined phase is rather sensitive to their masses and effective number of degrees of freedom. These quantities are changed rapidly when crossing

the transition region by increasing energy density. This results in a change of fluctuations of mean multiplicity of strange particles caused by energy density fluctuations.

Within the SMES, the fireball formed in a high-energy collision is treated as a micro-state which belongs to the ensemble of all possible micro-states. Such a behaviour is postulated for the created matter which corresponds to zero values of the conserved charges. The micro-states are thus defined by the EoS and the energy-volume probability density function  $P(E, V)$ . In particular, particle multiplicity is a property of a single event which varies from event to event even for fixed values of  $E$  and  $V$ .

In the GCE, these multiplicity fluctuations follow the Poisson distribution. Four extensive quantities  $E$ ,  $V$  and  $N$ ,  $N_S$  are considered, where  $N$  and  $N_S$  are mean multiplicities of all hadrons and strange hadrons, respectively. The distribution of the first pair,  $P(E, V)$ , is assumed, whereas the distribution of the second pair  $\mathcal{P}(N, N_S)$  follows from fluctuations of ensemble properties and fluctuations of event properties.

Specially, the scaled variance of particle multiplicity can be presented [16] as a sum of the scaled variance induced by the initial  $E$  and  $V$  fluctuations,  $\omega_0$ , and the scaled variance of particle multiplicity at fixed  $E$  and  $V$  values, which is equal to unity for the assumed Poisson distribution

$$\omega[N] = 1 + \omega_0[N], \quad \omega[N_S] = 1 + \omega_0[N_S]. \quad (2)$$

For the purposes of this work, it is sufficient to characterize the distribution  $P(E, V)$  by its five parameters which include its first and second moments

$$\langle \varepsilon \rangle, \quad \langle V \rangle, \quad \frac{\sqrt{\langle (\delta \varepsilon)^2 \rangle}}{\langle \varepsilon \rangle}, \quad \frac{\sqrt{\langle (\delta V)^2 \rangle}}{\langle V \rangle}, \quad \frac{\langle \varepsilon V \rangle - \langle \varepsilon \rangle \langle V \rangle}{\sqrt{\langle (\Delta \varepsilon)^2 \rangle \langle (\Delta V)^2 \rangle}}, \quad (3)$$

the latter three — two scaled dispersions and the correlation coefficient — are dimensionless.

The SMES formulation and parameter values (unless otherwise stated) used in the original papers [11, 15, 16] are adopted here. Thus, the ideal gas EoS is used to model the confined phase, the bag model EoS is used for the QGP phase, and the first-order phase transition between them is assumed.

### 3. Strongly intensive measure of fluctuations

Since event-by-event volume fluctuations cannot be eliminated in experimental studies of  $A + A$  collisions, it is important to minimize their effect by defining suitable fluctuation measures. It was shown within the model of independent sources that one can construct fluctuation measures from the

first and second moments of two extensive event quantities,  $A$  and  $B$ , which are independent of the source number distribution. The measures were referred to as strongly intensive quantities [23]. The first measure of this type was introduced in Ref. [24], and then the concept was generalized [23] and extended to third [25] and higher moments [26]. Here, the predictions of the SMES model will be calculated for strongly intensive quantities which include the first and second moments of  $A$  and  $B$ .

Two families of strongly intensive quantities can be constructed [23]

$$\Delta[A, B] = \frac{1}{C_\Delta} [\langle B \rangle \omega[A] - \langle A \rangle \omega[B]] , \quad (4)$$

$$\Sigma[A, B] = \frac{1}{C_\Sigma} [\langle B \rangle \omega[A] + \langle A \rangle \omega[B] - 2(\langle AB \rangle - \langle A \rangle \langle B \rangle)] , \quad (5)$$

where  $\omega[X] \equiv (\langle X^2 \rangle - \langle X \rangle^2) / \langle X \rangle$ . The normalization factors  $C_\Delta$  and  $C_\Sigma$  are required to be proportional to first moments of any extensive quantities. Note that  $\Sigma[A, B]$  includes the correlation term  $\langle AB \rangle - \langle A \rangle \langle B \rangle$ , whereas  $\Delta[A, B]$  does not. Another quantity  $\nu_{\text{dyn}}[A, B]$  frequently used to characterize the fluctuations of particle multiplicities  $A$  and  $B$  was introduced in Ref. [27]. The  $\nu_{\text{dyn}}[A, B]$  measure can be expressed through  $\Sigma[A, B]$  and first moments  $\langle A \rangle$  and  $\langle B \rangle$ . This measure is inversely proportional to the systems size,  $\nu[A, B] \sim 1/\langle V \rangle$ , and, therefore, it is not an intensive quantity.

Two selections of the  $C_\Delta$  and  $C_\Sigma$  normalization factors are used in the present paper. Firstly, the normalization factors equal to mean of the second argument are assumed

$$C_\Delta = C_\Sigma = \langle B \rangle . \quad (6)$$

As pointed out in Ref. [14], this normalization within the statistical model of the ideal Boltzmann gas in the GCE (IB-GCE) formulation and  $B \sim V$  leads to

$$\Delta[A, B] = \Sigma[A, B] = \omega^*[A] , \quad (7)$$

where  $\omega^*[A]$  is the scaled variance of  $A$  for a fixed system volume.

Secondly, the normalization

$$C_\Delta = \langle N \rangle - \langle N_S \rangle , \quad C_\Sigma = \langle N \rangle + \langle N_S \rangle \quad (8)$$

will be used for particle multiplicities [28]. It provides  $\Delta[N, N_S] = \Sigma[N, N_S] = 1$  in the IB-GCE.

Within the IB-GCE and provided  $A$  and  $B$  are uncorrelated in a fixed volume, one finds that  $\omega^*[A]$  and  $\omega^*[B]$  can be expressed via  $\Sigma[A, B]$  and  $\Delta[A, B]$ . Let us introduce the quantities

$$\Omega[A, B] \equiv \frac{1}{2} [\Sigma[A, B] + \Delta[A, B]] = \omega[A] - \frac{\langle A B \rangle - \langle A \rangle \langle B \rangle}{\langle B \rangle}, \quad (9)$$

$$\Omega[B, A] \equiv \frac{1}{2} [\Sigma[B, A] + \Delta[B, A]] = \omega[B] - \frac{\langle A B \rangle - \langle A \rangle \langle B \rangle}{\langle A \rangle}. \quad (10)$$

Here, the normalization of  $\Sigma$  and  $\Delta$  is given by Eq. (6). Then, one finds [23, 26]

$$\omega^*[A] = \Omega[A, B], \quad \omega^*[B] = \Omega[B, A]. \quad (11)$$

Predictions for  $\Omega$  quantities will be also calculated in this paper.

Four extensive event quantities  $E$ ,  $V$  and  $N$ ,  $N_S$  define their six pairs

$$[E, V], \quad [N, V], \quad [N_S, V], \quad [N, E], \quad [N_S, E], \quad [N_S, N], \quad (12)$$

for which the strongly intensive fluctuation measures are calculated. Within the model, the results for the  $[E, V]$  pair depend only on the assumed distribution  $P(E, V)$ . The remaining pairs include at least one extensive event quantity whose fluctuations are dependent on the EoS. The applications of the strongly intensive quantities to the analysis of the data, and, particularly, a role of the finite experimental acceptance can be found in the recent review [14] and references therein.

Let us consider two examples of the  $P(E, V)$  distribution. For the most central  $A + A$  collisions, one may expect  $P(E, V) \cong P(E, V = \text{const})$ , *i.e.*,  $\varepsilon$  fluctuates from event to event, and  $V$  remains constant. On the other hand, a strong correlation between  $E$  and  $V$  with  $P(E, V) \cong P(E \sim V)$  is expected for collisions with different centralities. In this case, the energy density  $\varepsilon$  is similar for all events. Within the IB-GCE, these fluctuations do not influence the strongly intensive measures  $\Delta$  and  $\Sigma$ .

Let us note that the volume fluctuations are present in most central collisions as well. Particularly, these volume fluctuations strongly affect the scaled variances of particle multiplicity fluctuations. However, the role of these volume fluctuations is much smaller for the strongly intensive measure. If the energy density fluctuations are absent, the volume fluctuations do not contribute to  $\Delta$  and  $\Sigma$  within the model. However, in the general case of arbitrary  $E$  and  $V$  fluctuations, the volume fluctuations contribute to  $\Delta$  and  $\Sigma$  by the term proportional to  $1 + \langle (\delta V)^2 \rangle / \langle V \rangle^2$ . For central Pb+Pb collisions, this term can be estimated as  $1 + 1/\langle N_p \rangle$ , where  $\langle N_p \rangle$  is the average number of nucleon participants that is much larger than 1. Therefore, we use approximation  $1 + 1/\langle N_p \rangle \cong 1$  and the contributions of volume fluctuations to  $\Delta$  and  $\Sigma$  measures are neglected.

#### 4. Results of SMES calculations

Based on the SMES model [11], the following numerical values of the parameters and their collision-energy dependence are assumed. Mean volume and mean energy density at  $\sqrt{s_{NN}} = 10$  GeV are set to be  $\langle V \rangle = 350 \text{ fm}^3$  and  $\langle \varepsilon \rangle = 3.2 \text{ GeV/fm}^3$ , respectively. Their dependence on the collision energy is taken to be  $\langle V \rangle \sim 1/\sqrt{s_{NN}}$  and  $\langle \varepsilon \rangle \sim \sqrt{s_{NN}} (\sqrt{s_{NN}} - 2 m_N)$ . Scaled dispersion of volume fluctuations is set to  $\sqrt{\langle (\delta V)^2 \rangle} / \langle V \rangle = 0$ , which is a good approximation for central Pb+Pb collisions. Three values of the  $\sqrt{\langle (\delta \varepsilon)^2 \rangle} / \langle \varepsilon \rangle$  parameter are used: 0.17 (solid line), 0.13 (dashed line), and 0 (dotted line). The value  $\sqrt{\langle (\delta \varepsilon)^2 \rangle} / \langle \varepsilon \rangle = 0.17$  is considered to be the upper limit based on the UrQMD and HSD simulations [29]. The correlation between  $E$  and  $V$  is set to zero.

From Eqs. (4), (5), (9), one gets  $\Sigma[E, V] = \Delta[E, V] = \Sigma[E, V] = \omega^*[E] = \langle (\delta E)^2 \rangle / \langle E \rangle$  shown in Fig. 1 as a function of  $\sqrt{s_{NN}}$  for the model parameter values given above. This result is evidently insensitive to the EoS.

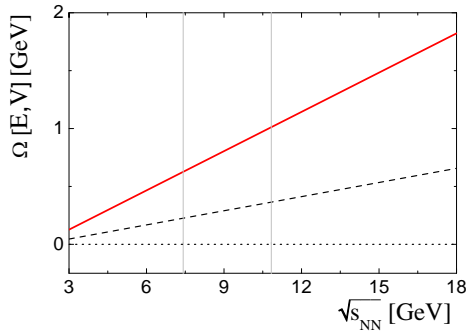


Fig. 1.  $\Sigma[E, V] = \Delta[E, V] = \Sigma[E, V] = \omega^*[E] = \langle (\delta E)^2 \rangle / \langle E \rangle$  as a function of  $\sqrt{s_{NN}}$  in central Pb+Pb collisions at the CERN SPS energy range. See the text for the numerical values of the model parameters and their  $\sqrt{s_{NN}}$  dependence. The normalization factors  $C_\Delta = C_\Sigma = \langle V \rangle$  are used. The solid, dashed, and dotted lines correspond to  $\sqrt{\langle (\delta \varepsilon)^2 \rangle} / \langle \varepsilon \rangle$  equal to 0.17, 0.13, and 0, respectively. The vertical lines indicate the beginning (the onset of deconfinement) and end (the softest point) of the mixed phase region,  $\sqrt{s_{NN}}(\text{OD}) \cong 7.4 \text{ GeV}$  and  $\sqrt{s_{NN}}(\text{SP}) \cong 10.8 \text{ GeV}$ , respectively.

The energy density fluctuations modify fluctuations of event properties like particle multiplicity and these modifications are dependent on the EoS and particle type. On the other hand, the EoS and particle properties change significantly when crossing the phase transition region. Thus, one expects that the collision-energy dependence of properly selected fluctuation measures may signal the transition region. The fluctuation measures which are

sensitive to the EoS are plotted in Figs. 2–4 as functions of collision energy in the range which includes the phase transition region. Note that the  $N$  and  $N_S$  fluctuations for  $\sqrt{\langle(\delta V)^2\rangle}/\langle V\rangle = 0$  and  $\sqrt{\langle(\delta\varepsilon)^2\rangle}/\langle\varepsilon\rangle = 0$  are assumed to be Poissonian and uncorrelated.

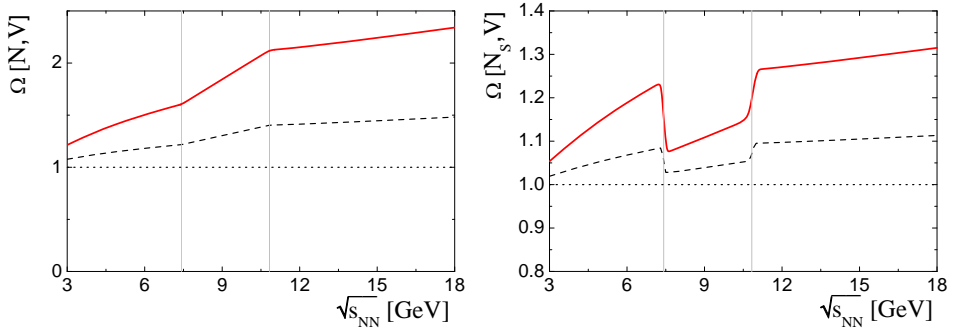


Fig. 2. The same as in Fig. 1 but for  $\Sigma[N, V] = \Delta[N, V] = \Sigma[N, V] = \omega^*[N]$  (left) and  $\Sigma[N_S, V] = \Delta[N_S, V] = \Sigma[N_S, V] = \omega^*[N_S]$  (right). The normalization factors  $C_\Delta = C_\Sigma = \langle V \rangle$  are used.

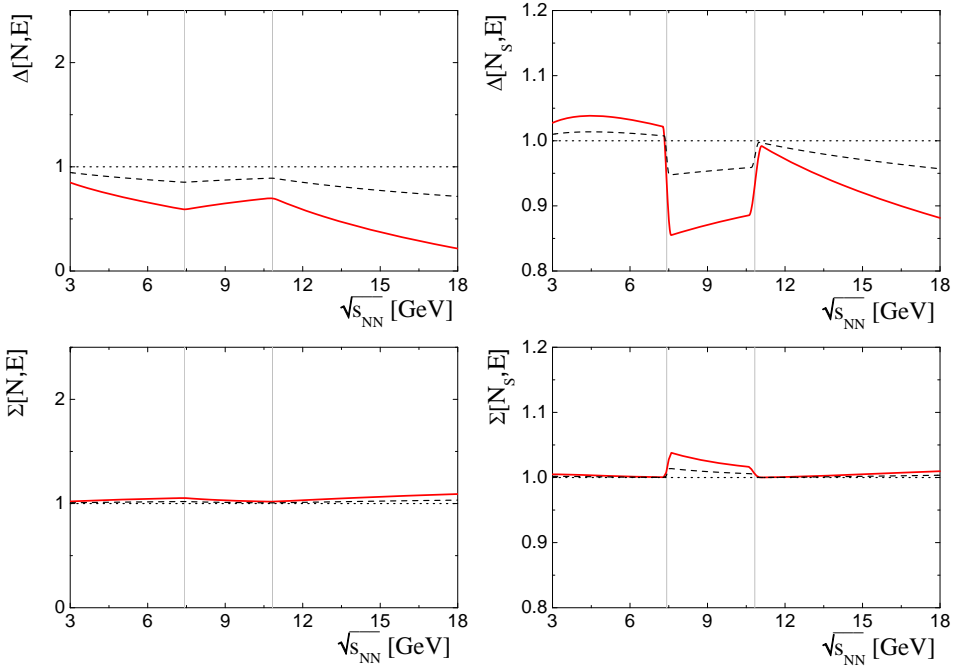


Fig. 3. The same as in Figs. 1 and 2 but for  $\Delta[N, E]$  and  $\Sigma[N, E]$  (left), and  $\Delta[N_S, E]$  and  $\Sigma[N_S, E]$  (right). The normalization factors  $C_\Delta = C_\Sigma = \langle E \rangle$  are used.



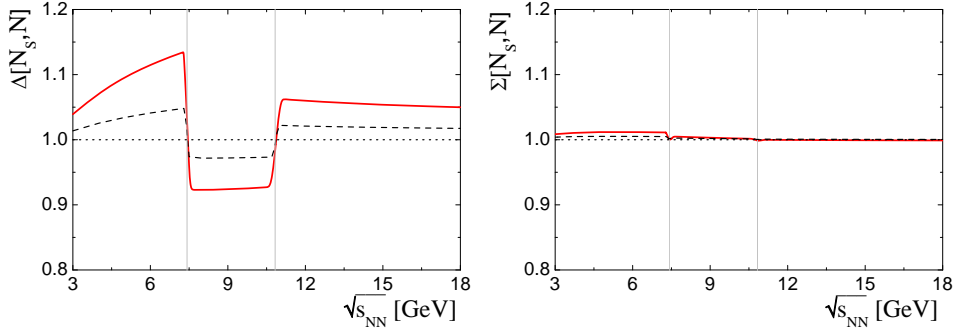


Fig. 4. The same as in Figs. 1–3 but for  $\Delta[N_S, N]$  (left) and  $\Sigma[N_S, N]$  (right). The normalization factors  $C_\Delta = \langle N \rangle - \langle N_S \rangle$  and  $C_\Sigma = \langle N \rangle + \langle N_S \rangle$  are used.

The collision-energy dependence of  $\Sigma[N, V] = \Delta[N, V] = \Sigma[N, V] = \omega^*[N]$  and  $\Sigma[N_S, V] = \Delta[N_S, V] = \Sigma[N_S, V] = \omega^*[N_S]$  is shown in Fig. 2 for central Pb+Pb collisions at the CERN SPS energies. With  $\sqrt{\langle(\delta\varepsilon)^2\rangle}/\langle\varepsilon\rangle$  decreasing to zero, the results approach unity as expected in the IB-GCE.  $\Delta$  and  $\Sigma$  are equal to each other for pairs of quantities  $[N, V]$  and  $[N_S, V]$ . It results from the used approximations:  $N$  and  $N_S$  are uncorrelated in the fixed volume,  $\langle N \rangle \sim V$  and  $\langle N_S \rangle \sim V$ , and  $\langle(\delta V)^2\rangle/\langle V \rangle \cong 0$ . The overall increasing trend of  $\Sigma[N, V]$  and  $\Sigma[N_S, E]$  with  $\sqrt{s_{NN}}$  seen in Fig. 2 is due to non-zero values of  $\sqrt{\langle(\delta\varepsilon)^2\rangle}/\langle\varepsilon\rangle$ . The modifications of this energy dependence are observed in the region of the phase transition. However, measurements of system volume (or quantity which is proportional to volume) are likely to be experimentally difficult or even impossible.

Figure 3 presents the fluctuation measures  $\Delta[N, E]$  and  $\Sigma[N, E]$  as well as  $\Delta[N_S, E]$  and  $\Sigma[N_S, E]$  as a function of collision energy. Similarly to the results presented in Fig. 2, the overall trend caused by the assumed energy density fluctuations is modified in the region of the phase transition. The most pronounced modification is observed for  $\Delta[N_S, E]$ .

Finally, the collision-energy dependence of  $\Delta[N_S, N]$  and  $\Sigma[N_S, N]$  is shown in Fig. 4. Normalization (8) is implied in this case. The most pronounced modification of the overall trend in the phase transition region is observed for  $\Delta[N_S, N]$ , and a weak one for  $\Sigma[N_S, N]$ .

## 5. Summary

In summary, predictions on the collision-energy dependence of fluctuations of hadron production properties in central heavy-ion collisions in the range of the phase transition are presented. They are based on the Statistical Model of the Early Stage and extend previously published results by

predictions for the strongly intensive quantities. They are calculated for six pairs of event quantities

$$[E, V], \quad [N, V], \quad [N_S, V], \quad [N, E], \quad [N_S, E], \quad [N_S, N], \quad (13)$$

where  $E$  and  $V$  stand for the system energy and volume, whereas  $N$  and  $N_S$  for multiplicities of all and strange particles. In several considered cases, the collision-energy dependence is significantly modified in the phase transition region. A non-monotonic dependence on the collision energy is clearly seen for  $\Sigma[N_S, V]$  (Fig. 2 (right)),  $\Delta[N_S, E]$  (Fig. 3 (right)), and  $\Delta[N_S, N]$  (Fig. 4 (left)). These fluctuation measures include multiplicity of strange particles. The non-monotonic behaviour is related to significantly different masses and degeneracy factors of strange particles in the confined and deconfined phases. In absence of the transition, all fluctuation measures are monotonic functions of  $\sqrt{s_{NN}}$ .  $\Delta[N_S, N]$  shown in Fig. 4 (left) shows a high sensitivity to the phase transition. This opens a possibility to observe a transition signal in data on fluctuations of hadron production properties.

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