# COUPLING OF HIDDEN SECTOR* 

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#### Abstract

A hypothetic Hidden Sector of the Universe, consisting of sterile fermions ("sterinos") and sterile mediating bosons ("sterons") of mass dimension 1 (not $2!$ ) - the last described by an antisymmetric tensor field requires to exist also a scalar isovector and scalar isoscalar in order to be able to construct electroweak invariant coupling (before spontaneously breaking its symmetry). The introduced scalar isoscalar might be a resonant source for the diphoton excess of 750 GeV , suggested recently by experiment.


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## 1. Introduction

In the previous decade, we have introduced a specific Hidden Sector of the Universe, consisting of sterile fermions (with mass dimension 3/2) and sterile mediating bosons (of mass dimension 1) described by an antisymmetrictensor field $C_{\mu \nu}$ (denoted before by $A_{\mu \nu}$ ) [1, 2]. In addition to the familiar structure of the Standard Model, we have postulated the existence of an extra scalar isovector $\left(\varphi_{1}, \varphi_{2}, \varphi_{3}\right) \quad(i=1,2,3)$ or

$$
\begin{equation*}
\varphi^{+}=\frac{\varphi_{1}+i \varphi_{2}}{\sqrt{2}}, \quad \varphi^{-}=\frac{\varphi_{1}-i \varphi_{2}}{\sqrt{2}}, \quad \varphi^{0}=\varphi_{3} \tag{1}
\end{equation*}
$$

and also a scalar isoscalar $\varphi$. While the former triplet is conserving, the latter singlet is presumed to break spontaneously the electroweak symmetry, $\langle\varphi\rangle_{\text {vac }} \neq 0$, acting beside the popular Higgs scalar, $\left\langle h^{0}\right\rangle_{\text {vac }} \neq 0$. The introduced sterile fermions $\psi$ and sterile mediating bosons $C_{\mu \nu}$ we will call, for convenience, "sterinos" and "sterons", respectively. The term "hiddons" might be used for hypothetic vector bosons $\chi_{\mu}$ of the hidden sector.

[^0]If the tensor field $C_{\mu \nu}$ is gauged, $C_{\mu \nu}=\partial_{\mu} \chi_{\nu}-\partial_{\nu} \chi_{\mu}$, by a vector field $\chi_{\mu}$ of mass dimension 0 , while this field $\chi_{\mu}$ turns out to be absent outside $C_{\mu \nu}$, then the Lagrangian density is gauge invariant, trivially. In this case, the mass-dimension- 0 vector $\chi_{\mu}(x)$ might play tentatively a fundamental role in the creation of gravitational metric $g_{\mu \nu}(x)$ as a specific condensate of matter

$$
\begin{equation*}
g_{\mu \nu}(x)=\eta_{\mu \nu}+\left\langle\chi_{\mu}(x) \chi_{\nu}(x)\right\rangle_{\mathrm{con}} \tag{2}
\end{equation*}
$$

with the Coulomb-like gauge $\partial_{\nu} \chi^{\nu}=0$ giving $\square \chi_{\mu}=\partial^{\nu} C_{\mu \nu}$, where $C_{\mu \nu}=$ $\partial_{\mu} \chi_{\nu}-\partial_{\nu} \chi_{\mu}$. Here, $C_{\mu \nu}$ is represented later by matrix (10), and a special structure of $C_{\mu \nu}$ can be investigated. If e.g., $C_{i}^{(E) 2}=C_{i}^{(B) 2}(i=1,2,3)$ or only $\sum_{i} C_{i}^{(E) 2}=\sum_{i} C_{i}^{(B) 2}$, then (1/4) $M^{2} C_{\mu \nu} C^{\mu \nu}=0$ (see Eq. (11)) and so, tensor $C_{\mu \nu}$ is effectively massless.

In Eq. (2), there is $\left(\eta_{\mu \nu}\right)=\operatorname{diag}(1,-1,-1,-1)$ due to the normalization of gravity metric $g_{\mu \nu} \rightarrow \eta_{\mu \nu}$, when $\left\langle\chi_{\mu}(x) \chi_{\nu}(x)\right\rangle_{\text {con }} \rightarrow 0$. If we have to do with static approximation, then the Yukawa-type equation $\left(\Delta-m_{\chi}^{2}\right) \chi_{\mu}(x)=$ $\partial^{\rho} C_{\mu \rho}(x)$ holds for hiddons, providing local attraction forces in the case of negative r.h.s.

Since the equation $\square \chi_{\mu}(x)=\partial^{\rho} C_{\mu \rho}(x)$ implies

$$
\begin{align*}
\chi_{\mu}(x) & =\square^{-1} \partial^{\rho} C_{\mu \rho}(x)=\square^{-1}\binom{\operatorname{div} \vec{C}^{(E)}(x)}{\operatorname{rot} \vec{C}^{B}(x)-\partial_{0} \vec{C}^{E}(x)}_{\mu} \\
\text { for } \mu & =\left\{\begin{array}{l}
0 \\
1,2,3
\end{array}\right. \tag{3}
\end{align*}
$$

because of Eq. (10), the gravity metric formula could be given bona fide as

$$
\begin{align*}
& g_{\mu \nu}(x)-\eta_{\mu \nu}=\left\langle\chi_{\mu}(x) \chi_{\nu}(x)\right\rangle_{\mathrm{con}} \\
& =\left\langle\binom{\operatorname{div} \vec{C}^{(E)}(x)}{\operatorname{rot} \vec{C}^{B}(x)-\partial_{0} \vec{C}^{E}(x)}_{\mu} \square^{-2}\binom{\operatorname{div} \vec{C}^{(E)}(x)}{\operatorname{rot} \vec{C}^{B}(x)-\partial_{0} \vec{C}^{E}(x)}_{\nu}\right\rangle_{\mathrm{con}} \tag{4}
\end{align*}
$$

Here, $-\square=\partial_{0}^{2}-\Delta, \vec{\partial}=\left(\partial_{k}\right), \vec{C}^{(E)}=\left(-C_{k}^{(E)}\right), \vec{C}^{(B)}=\left(-C_{k}^{(B)}\right)(k=$ $1,2,3)$ and $\operatorname{div} \vec{C}^{(E)}=\vec{\partial} \cdot \vec{C}^{(E)}, \operatorname{rot} \vec{C}^{(B)}=\vec{\partial} \times \vec{C}^{(B)}$. The relevant condensation mechanism has yet to be discovered.

As it is well-known, the four local functions, one divergence and three components of rotation, play constructive role in physics of spacetime continua [3]. We may expect their importance in the case of gravity metric related hypothetically to sterons and hiddons.

## 2. Coupling of Hidden Sector and Standard Model

The new scalars $\varphi^{+}, \varphi^{-}, \varphi^{0}$ and $\varphi$ with $\langle\varphi\rangle_{\text {vac }} \neq 0$ enable us to define electroweak-symmetry invariant coupling of Hidden Sector to the Standard Model world in the following form

$$
\begin{equation*}
-\frac{1}{2} \sqrt{f}\left(\bar{\psi} \sigma^{\mu \nu} \psi+\xi \varphi_{i} W_{i}^{\mu \nu}+\eta \varphi B^{\mu \nu}\right) C_{\mu \nu} \tag{5}
\end{equation*}
$$

Here, $f$ and $\xi$ or $\eta$ are massless unknown coupling constants. Form (5) is a subject of spontaneously electroweak-symmetry breaking by $\langle\varphi\rangle_{\mathrm{vac}} \neq 0$, in addition to the Higgs mechanism $\left\langle h^{0}\right\rangle_{\mathrm{vac}} \neq 0$ giving Weinberg-Salam mixing

$$
\begin{align*}
Z_{\mu} & =\cos \theta_{W} W_{\mu}^{0}+\sin \theta_{W} B_{\mu} \\
A_{\mu} & =-\sin \theta_{W} W_{\mu}^{0}+\cos \theta_{W} B_{\mu} \tag{6}
\end{align*}
$$

where $Z_{\mu \nu}=\partial_{\mu} Z_{\nu}-\partial_{\nu} Z_{\mu}$ and $F_{\mu \nu}=\partial_{\mu} A_{\nu}-\partial_{\nu} A_{\mu}$. Then, if we put tentatively $\xi=\eta$, form (5) implies (after breaking) the following neutral part of the electroweak-hidden coupling:

$$
\begin{equation*}
-\frac{1}{2} \sqrt{f}\left[\bar{\psi} \sigma^{\mu \nu} \psi+\xi\left(\varphi^{(F)} F^{\mu \nu}+\varphi^{(Z)} Z^{\mu \nu}\right)\right] C_{\mu \nu} \tag{7}
\end{equation*}
$$

with (valid for $\xi=\eta$ ):

$$
\begin{align*}
\varphi^{(Z)} & =\cos \theta_{W} \varphi^{0}+\sin \theta_{W} \varphi \\
\varphi^{(F)} & =-\sin \theta_{W} \varphi^{0}+\cos \theta_{W} \varphi \tag{8}
\end{align*}
$$

Recall that in contrast to $F_{\mu \nu}=\partial_{\mu} A_{\nu}-\partial_{\nu} A_{\mu}$, the antisymmetric tensor field does not get gauging $C_{\mu \nu}=\partial_{\mu} \chi_{\nu}-\partial_{\nu} \chi_{\mu}$ with mass dimension 2 , when $\chi_{\mu}$ has mass dimension 0 .

Naturally, the kinetic Lagrangian density is now

$$
\begin{equation*}
-\frac{1}{4}\left[\left(\partial_{\lambda} C_{\mu \nu}\right)\left(\partial^{\lambda} C^{\mu \nu}\right)-M^{2} C_{\mu \nu} C^{\mu \nu}\right] . \tag{9}
\end{equation*}
$$

A convenient way to represent the antisymmetric tensor field $C_{\mu \nu}$ reads as the electromagnetic-type matrix [2]

$$
\left(C_{\mu \nu}\right)=\left(\begin{array}{cccc}
0 & C_{1}^{(E)} & C_{2}^{(E)} & C_{3}^{(E)}  \tag{10}\\
-C_{1}^{(E)} & 0 & -C_{3}^{(B)} & C_{2}^{(B)} \\
-C_{2}^{(E)} & C_{3}^{(B)} & 0 & -C_{1}^{(B)} \\
-C_{3}^{(E)} & -C_{2}^{(B)} & C_{1}^{(B)} & 0
\end{array}\right)
$$

The trace of its square is equal to

$$
\begin{equation*}
-\frac{1}{4}\left(C_{\mu \nu} C^{\mu \nu}\right)=\frac{1}{4}\left(C_{\mu \nu} C^{\nu \mu}\right)=\frac{1}{2} \sum_{i}\left(C_{i}^{(E) 2}-C_{i}^{(B) 2}\right) . \tag{11}
\end{equation*}
$$

A practical formula is the square of the matrix $C_{\mu \nu}$ that becomes equal to

$$
\begin{align*}
& -\frac{1}{4}\left(C_{\mu \lambda} C^{\nu \lambda}\right)=\frac{1}{4}\left(C_{\mu \lambda} C^{\lambda \nu}\right) \\
& =\left(\begin{array}{cccc}
\vec{C}_{1}^{(E) 2} & \left(\vec{C}^{(E)} \times \vec{C}^{(B)}\right)_{1} & \left(\vec{C}^{(E)} \times \vec{C}^{(B)}\right)_{2} & \left(\vec{C}^{(E)} \times \vec{C}^{(B)}\right)_{3} \\
-\left(\vec{C}^{(E)} \times \vec{C}^{(B)}\right)_{1} & C_{1}^{(E) 2}-C_{2}^{(B) 2}-C_{3}^{(B) 2} & C_{1}^{(E)} C_{2}^{(E)}+C_{1}^{(B)} C_{2}^{(B)} & C_{3}^{(E)} C_{1}^{(E)}+C_{3}^{(B)} C_{1}^{(B)} \\
-\left(\vec{C}^{(E)} \times \vec{C}^{(B)}\right)_{2} & C_{1}^{(E)} C_{2}^{(E)}+C_{1}^{(B)} C_{2}^{(B)} & C_{2}^{(E) 2}-C_{3}^{(B) 2}-C_{1}^{(B) 2} & C_{2}^{(E)} C_{3}^{(E)}+C_{2}^{(B)} C_{3}^{(B)} \\
-\left(\vec{C}^{(E)} \times \vec{C}^{(B)}\right)_{3} & C_{3}^{(E)} C_{1}^{(E)}+C_{3}^{(B)} C_{1}^{(B)} & C_{2}^{(E)} C_{3}^{(E)}+C_{2}^{(B)} C_{3}^{(B)} & C_{3}^{(E) 2}-C_{1}^{(B) 2}-C_{2}^{(B) 2}
\end{array}\right), \tag{12}
\end{align*}
$$

where $\vec{C}^{(E)}=\left(-C_{i}^{(E)}\right)$ and $\vec{C}^{(B)}=\left(-C_{i}^{(B)}\right)(i=1,2,3)$.
Note that the mass term $(1 / 4) M^{2}\left(C_{\mu \nu} C^{\mu \nu}\right)=0$ of the field $C^{\mu \nu}$ vanishes when $\sum_{i} C_{i}^{(E) 2}=\sum_{i} C_{i}^{(B) 2}$. Here, $C_{i}^{(E)}$ and $C_{i}^{(B)}$ are intrinsic degrees of freedom for a steron described by the field $C_{\mu \nu}$ (in analogy to the spin of a sterino or of another Dirac bispinor field).

## 3. Fermionic versus bosonic coupling

In particle physics, a fundamental role is played by 16 independent Dirac matrices building up 5 Lorentz covariant forms:

$$
\left.\begin{array}{rl}
S_{\mu \nu} & \equiv \frac{1}{2}\left\{\gamma_{\mu}, \gamma_{\nu}\right\} \equiv \eta_{\mu \nu}, \\
V_{\mu} & \equiv \gamma_{\mu}=\left\{\begin{array}{ll}
\beta & \text { for } \mu=0 \\
\beta \alpha_{k} & \text { for } \mu=k
\end{array}, \quad V_{\mu}^{(p)} \equiv \gamma_{\mu} \gamma_{5}\right.
\end{array}\right\} \begin{array}{ll}
i \alpha_{l} & \text { for } \mu=0, \nu=l \\
\varepsilon_{k l m} \sigma_{m} & \text { for } \mu=k, \nu=l \\
-i \alpha_{k} & \text { for } \mu=k, \nu=0 \tag{13}
\end{array},
$$

where $\left\{\gamma_{\mu}, \gamma_{\nu}\right\}=2 \eta_{\mu \nu},(\mu, \nu=0,1,2,3)$ and $\left[\sigma_{k}, \sigma_{l}\right]=2 i \varepsilon_{k l m} \sigma_{m},(k, l, m=$ $1,2,3)$.

These covariant forms determine couplings of mediating bosons (with mass dimension 1) to fermionic pairs (with mass dimension $3 / 2+3 / 2=3$ ). The mass dimension of interaction Lagrangian density is then $1+3=4$, while kinetic Lagrangian density of mediating bosons gets the dimension $2+2=4$.

For instance, electrons and photons are coupled according to the electromagnetic interaction Lagrangian density

$$
\begin{equation*}
e \bar{\psi} \gamma_{\mu} \psi A^{\mu} \tag{14}
\end{equation*}
$$

Here, $A_{\mu}$ and $F_{\mu \nu}=\partial_{\mu} A_{\nu}-\partial_{\nu} A_{\mu}$ have mass dimension 1 and 2, respectively, while $-(1 / 4) F_{\mu \nu} F^{\mu \nu}=(1 / 2) \sum_{k}\left(E_{k}^{2}-B_{k}^{2}\right)$ carries mass dimension 4 for the kinetic Lagrangian density of photons.

All other fermionic couplings among those given by Eq. (13) are also realized experimentally, except for the Pauli-type antisymmetric tensor coupling

$$
\begin{equation*}
\frac{1}{2} \sqrt{f} \bar{\psi} \sigma_{\mu \nu} \psi C^{\mu \nu} \tag{15}
\end{equation*}
$$

where a new antisymmetric tensor field is introduced (see Eq. (10))

$$
\left(C_{\mu \nu}\right)=\left\{\begin{array}{lll}
C_{l}^{(E)} & \text { for } & \mu=0, \nu=l,  \tag{16}\\
\varepsilon_{k l m} C_{m}^{(B)} & \text { for } & \mu=k, \nu=l, \\
-C_{k}^{(E)} & \text { for } & \mu=k, \nu=0
\end{array}\right.
$$

$(k, l, m=1,2,3)$. The mathematical existence of this tensor isoscalar suggests an experimentally new scalar isovector and new scalar isoscalar.

Arguing for extending fermionic tensor coupling (15) to bosonic tensor coupling, we obtain Eq. (5) as an electroweak-symmetry invariant coupling of Hidden Sector to the Standard Model world (before spontaneously breaking the symmetry).

An actual candidate for diphoton at 750 GeV , discussed recently [4-7], might be a scalar isoscalar participating in the process

$$
\begin{equation*}
\varphi_{\text {phys }}^{(F)} \rightarrow \gamma C \rightarrow \gamma\left\langle\varphi^{(F)}\right\rangle_{\mathrm{vac}} \gamma \rightarrow \gamma \gamma, \tag{17}
\end{equation*}
$$

where $\varphi^{(F)}=\left\langle\varphi^{(F)}\right\rangle_{\mathrm{vac}}+\varphi_{\text {phys }}^{(F)}$ with $\left\langle\varphi^{(F)}\right\rangle_{\mathrm{vac}} \neq 0$. Similarly, for $\varphi^{(Z)}=$ $\left\langle\varphi^{(Z)}\right\rangle_{\text {vac }}+\varphi_{\text {phys }}^{(Z)}$ with $\left\langle\varphi^{(Z)}\right\rangle_{\text {vac }} \neq 0$, we get the process

$$
\begin{equation*}
\varphi_{\text {phys }}^{(Z)} \rightarrow Z C \rightarrow Z\left\langle\varphi^{(F)}\right\rangle_{\mathrm{vac}} \gamma \rightarrow Z \gamma . \tag{18}
\end{equation*}
$$

In Ref. [2], we considered alternatively

$$
\begin{equation*}
C C \rightarrow \varphi_{\text {phys }}^{(F)} \gamma \varphi_{\text {phys }}^{(F)} \gamma \rightarrow \gamma \gamma \tag{19}
\end{equation*}
$$

(here, we use the notation $C$ instead of $A$ ).

## Appendix A

## Free steron versus sterino

With the interaction and kinetic parts of steron Lagrangian density, Eqs. (5) and (9), the Lagrangian field equation for $C^{\mu \nu}$ reads

$$
\begin{equation*}
\left(\square-M^{2}\right) C^{\mu \nu} \equiv-\sqrt{f}\left(\bar{\psi} \sigma^{\mu \nu} \psi+\xi \varphi_{i} W_{i}^{\mu \nu}+\eta \varphi B^{\mu \nu}\right) \tag{A.1}
\end{equation*}
$$

In the limit of $f \rightarrow 0$, we obtain a free equation for $C^{\mu \nu},\left(\square-M^{2}\right) C^{\mu \nu}=0$, getting the plane-wave solutions

$$
\begin{equation*}
C_{a}^{\mu \nu}(x)=\frac{1}{(2 \pi)^{3 / 2}} \frac{1}{\sqrt{2 k_{0}}} e_{a}^{\mu \nu} e^{-i k x}, \tag{A.2}
\end{equation*}
$$

where

$$
\begin{equation*}
k_{e}=\sqrt{\vec{k}^{2}+M^{2}} \tag{A.3}
\end{equation*}
$$

and

$$
\left(e_{a}^{\mu \nu}\right)=\left(e^{\mu \nu}\right)_{a}=\left(\begin{array}{cccc}
0 & e_{1}^{(E)} & e_{2}^{(E)} & e_{3}^{(E)}  \tag{A.4}\\
-e_{1}^{(E)} & 0 & -e_{3}^{(B)} & e_{2}^{(B)} \\
-e_{2}^{(E)} & e_{3}^{(B)} & 0 & -e_{1}^{(B)} \\
-e_{3}^{(E)} & -e_{2}^{(B)} & e_{1}^{(B)} & 0
\end{array}\right)_{a}
$$

due to Eq. (10). Here, three independent polarizations $\vec{e}_{a}^{(E, B)}=\left(e_{k}^{(E, B)}\right)_{a}$ ( $a=1,2,3$ and $k=1,2,3$ ) are chosen ortonormal, separately for $E$ and $B^{a}$

$$
\begin{equation*}
\vec{e}_{a}^{(E, B)} \cdot \vec{e}_{b}^{(E, B)}=\delta_{a b}, \quad \sum_{a=1}^{3} e_{k a}^{(E, B)} e_{l a}^{(E, B)}=\delta_{k l} \tag{A.5}
\end{equation*}
$$

Now, we may impose a priori a hypothetic relation between polar and axial polarizations, $\vec{e}_{a}^{(E)}$ and $\vec{e}_{a}^{(B)}$, within solution (A.2), putting the constraint

$$
\begin{equation*}
\vec{e}_{1,2,3}^{(E)} \times \vec{e}_{3,1,2}^{(E)}=\vec{e}_{2,3,1}^{(B)}, \tag{A.6}
\end{equation*}
$$

besides the identity

$$
\begin{equation*}
\vec{e}_{1,2,3}^{(E)} \times \vec{e}_{2,3,1,}^{(E)}=(+ \text { or }-) \vec{e}_{3,1,2}^{(E)} \tag{A.7}
\end{equation*}
$$

(in the right- or left-handed frame of reference, respectively). Then, constraint (A.6) can be presented trivially as

$$
\begin{equation*}
\vec{e}_{a}^{(B)}=(+ \text { or }-) \vec{e}_{a}^{(E)} \tag{A.8}
\end{equation*}
$$

and so,

$$
\begin{equation*}
\vec{e}_{a}^{(B) 2}=\vec{e}_{a}^{(E) 2} \tag{A.9}
\end{equation*}
$$

$(a=1,2,3)$. Thus, from Eqs. (A.4) and (A.9), the trace of matrix $e_{a}^{\mu \nu}$ squared (minus) is

$$
\begin{equation*}
e_{\mu \nu a} e_{a}^{\mu \nu}=2\left(\vec{e}_{a}^{(B) 2}-\vec{e}_{a}^{(E) 2}\right)=0 \tag{A.10}
\end{equation*}
$$

the last step being valid when one uses constraint (A.9). Then, we can infer that the effective mass term $(1 / 4) M^{2} C_{\mu \nu} C^{\mu \nu}$ of the steron field $C^{\mu \nu}$ vanishes (see Eq. (11)).

In a similar way, we get the sterino Lagrange field equation

$$
\begin{equation*}
\left(\gamma^{\mu} i \partial_{\mu}-\frac{1}{2} \sqrt{f} \sigma^{\mu \nu} C_{\mu \nu}-m_{\psi}\right) \psi=0 \tag{A.11}
\end{equation*}
$$

when we apply the energy and kinetic parts of sterino Lagrangian density, Eq. (5) and the term

$$
\begin{equation*}
\bar{\psi}\left(\gamma^{\mu} i \partial_{\mu}-m_{\psi}\right) \psi \tag{A.12}
\end{equation*}
$$

respectively. Due to the identity

$$
\begin{equation*}
\frac{1}{2} \sigma^{\mu \nu} C_{\mu \nu}=i \vec{\alpha} \cdot \vec{C}^{(E)}+\vec{\sigma} \cdot \vec{C}^{(B)} \tag{A.13}
\end{equation*}
$$

following from formulae (10) and (13), we can rewrite the sterino field equation (A.11) as

$$
\begin{equation*}
\left[\gamma^{\mu} i \partial_{\mu}-m_{\psi}-\sqrt{f}\left(i \vec{\alpha} \cdot \vec{C}^{(E)}+\vec{\sigma} \cdot \vec{C}^{(B)}\right)\right]=0 \tag{A.14}
\end{equation*}
$$

where $\vec{C}^{(E, B)}=\left(C_{k}^{(E, B)}\right)$.
A physically interesting case might be a sterino $\psi(x)$ embedded in the uniform steron field $\vec{C}^{(E)}=\overrightarrow{\mathrm{const}}=(0,0, C)$ and $\vec{C}^{(B)}=\overrightarrow{\mathrm{const}}=\left(0,0, C^{\prime}\right)$. Then, from Eq. (A.14) we infer that

$$
\begin{equation*}
\left[E-\vec{\alpha} \cdot \vec{p}-\beta m_{\psi}-\sqrt{f} \gamma_{3}\left(i C+\gamma_{5} C^{\prime}\right)\right] \psi=0 \tag{A.15}
\end{equation*}
$$

for $i \partial_{\mu} \psi(\vec{p})=p_{\mu} \psi(\vec{p})$ with $\vec{p}$ denoting the momentum of sterino, while $E(\vec{p})$ is its energy spectrum. Multiplying Eq. (A.15) on the l.h.s. by $[E+\alpha \cdot \vec{p}+$ $\left.\beta m_{\psi}+\sqrt{f} \gamma_{3}\left(i C+\gamma_{5} C^{\prime}\right)\right]$, we get sterino quadratic spectrum

$$
\begin{equation*}
\left[E^{2}-\vec{p}^{2}-m_{\psi}^{2}-f\left(C^{2}+C^{2}\right)-2 \sqrt{f} C^{\prime}\left(\sigma_{3} m_{\psi}+i \gamma_{1} p_{2}-i \gamma_{2} p_{1}\right)\right] \psi=0 \tag{A.16}
\end{equation*}
$$

We solve this spectrum in terms of sterino momenta $p_{1}, p_{2}, p_{3}$, finding the eigenvalues of the complete set of independent observables

$$
\begin{equation*}
\frac{1}{2} \sigma_{3} \psi=m_{s} \psi \equiv \pm \frac{1}{2} \psi \tag{A.17}
\end{equation*}
$$

and

$$
\begin{equation*}
\left(i \gamma_{1} p_{2}-i \gamma_{2} p_{1}\right) \psi= \pm \sqrt{\left(i \gamma_{1} p_{2}-i \gamma_{2} p_{1}\right)^{2}} \psi \equiv \pm \sqrt{p^{2}{ }_{1}+p^{2}{ }_{2}} \psi \tag{A.18}
\end{equation*}
$$

with

$$
\begin{equation*}
E^{2}=\vec{p}^{2}+m_{\psi}{ }^{2}+f\left(C^{2}+C^{\prime 2}\right)+2 \sqrt{f} C^{\prime}\left( \pm m_{\psi} \pm \sqrt{p^{2}{ }_{1}+p^{2}{ }_{2}}\right) . \tag{A.19}
\end{equation*}
$$

Here, $\vec{\gamma}=\beta \gamma_{5} \vec{\sigma}=\beta \vec{\alpha},(i \vec{\gamma})^{\dagger}=i \vec{\gamma}$ and $\left(i \gamma_{1}\right)^{2}=\mathbf{1}=\left(i \gamma_{2}\right)^{2}$ as well as $\left\{i \gamma_{1}, i \gamma_{2}\right\}=0$. Thus,

$$
\begin{equation*}
\left[\sigma_{3},\left\{i \gamma_{k}, i \gamma_{l}\right\}\right]=2 \delta_{k l}\left[\sigma_{3}, \mathbf{1}\right]=0 \tag{A.20}
\end{equation*}
$$

so, squares of $i \gamma_{1}$ and $i \gamma_{2}$ are independent of $\sigma_{3}$.

## Appendix B

## Maxwell's hidden equations

When sterons $C_{\mu \nu}$ are Coulomb-like gauged,

$$
C_{\mu \nu}=\partial_{\mu} \chi_{\nu}-\partial_{\nu} \chi_{\mu} \quad \text { or } \quad\left\{\begin{array}{l}
\vec{C}^{(E)}=-\partial_{0} \vec{\chi}-\vec{\partial} \chi_{0}  \tag{B.1}\\
\vec{C}^{(B)}=\operatorname{rot} \vec{\chi}
\end{array}\right.
$$

with $\partial^{\nu} \chi_{\nu}=0$, when $C_{\mu \nu}$ and $\chi_{\mu}$ are of mass dimension 1 and 0 , respectively, then from Eq. (B.1) it follows that

$$
\begin{align*}
\operatorname{rot} \vec{C}^{(E)}+\partial_{0} \vec{C}^{(B)} & =0, \\
\operatorname{div} \vec{C}^{(B)} & =0 . \tag{B.2}
\end{align*}
$$

On the other hand, formula (3): $\partial^{\nu} C_{\mu \nu}=\square \chi_{\mu}$ gives (see Eq. (10))

$$
\begin{align*}
\operatorname{rot} \vec{C}^{(B)}-\partial_{0} \vec{C}^{(E)} & =\square \vec{\chi}, \\
\operatorname{div} \vec{C}^{(E)} & =\square \chi_{0}, \tag{B.3}
\end{align*}
$$

when hiddons $\chi_{\mu}$ through $\square \chi_{\mu}$ are responsible for sources of sterons $C_{\mu \nu}$.

The four formulae (B.2) and (B.3) are Maxwell-type equations, acting on sterons $C_{\mu \nu}$ with mass dimensions 1 ("Maxwell's hidden equations"), defining "hidden electromagnetism". In the world of Standard Model, electroweak symmetry is actually active (plus hypothetic new scalar isovector and scalar isoscalar fields recently introduced [1, 2]).

There is also a cross-coupling between the Hidden Sector and Standard Model world, cf. Eq. (5), the latter being electroweak-symmetry invariant (before spontaneously breaking the electroweak symmetry). As a result of coupling (5), the conventional Maxwell's equations (already electroweakly unified) become extended ("Maxwell's extended equations").

## REFERENCES

[1] W. Królikowski, Acta Phys. Pol. B 40, 2767 (2009).
[2] W. Królikowski, Acta Phys. Pol. B 41, 1277 (2010).
[3] Cf. e.g. L.D. Landau, A.E.M. Lifshitz, The Classical Theory of Field, Pergamon Press, New York 1977; Teoria Pola, PWN, Warszawa 2009 (in Polish).
[4] G. Aad et al. [ATLAS Collaboration], ATLAS-CONF-2015-081, (2015).
[5] V. Khachatrian et al. [CMS Collaboration], CMS-EXO-15-004, (2015).
[6] V. Khachatrian et al. [CMS Collaboration], Phys. Lett. B 750, 494 (2015).
[7] G. Aad et al. [ATLAS Collaboration], Phys. Rev. D 92, 032004 (2015).


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