ULTRAVIOLET CUTOFFS AND THE PHOTON MASS*

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The momentum UV cutoff in Quantum Field Theory is usually treated as an auxiliary device allowing to obtain finite amplitudes satisfying all physical requirements. It is even absent (not explicit) in the most popular approach — the dimensional regularization. We point out that if the momentum cutoff is to be treated as a *bona fide* physical scale, presumably equal or related to the Planck scale, the field theory must have a very special features in order not to lead to unacceptable predictions. One of such predictions would be a non-zero mass of the photon. In the naive approach, even with the cutoff equal to the Planck scale, this mass would grossly exceed the existing experimental bounds. We present this danger doing an explicit calculation using a concrete realization of the physical cutoff and speculate about the way to restore gauge symmetry order-by-order in the inverse powers of the cutoff scale.

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In usual applications of Quantum Field Theory (QFT), the momentum cutoff (explicit or, as in the Dimensional Regularization, implicit) is treated as an auxiliary parameter and sent to infinity at the end of the renormalization procedure. However, in the context of a quest for a fundamental theory unifying elementary particle interactions with gravity, QFT models should be viewed as only effective theories with a real momentum cutoff which, as in QFT applications to statistical physics problems, should have a concrete physical interpretation, most probably of the intrinsic scale Λ of the underlying fundamental theory. In this short note, based on the previous work [1] and on the accompanying paper [2], we would like to point out some important aspects of treating the cutoff scale as a *bona fide* physical scale Λ probably related to the Planck scale $M_{\rm Pl}$ (the problem was partly analyzed in connection with quadratic divergences in QFT [3–5]).

A momentum cutoff introduced to regularize a gauge theory obviously spoils its manifest BRST invariance. It is known, however, that the subtraction procedure based on the Quantum Action Principle [6] (see also [7] for a review), which can be implemented with the aid of carefully chosen local counterterms, allows to restore it at the level of the effective action Γ, if the theory is non-anomalous. This is important because, to the best of our knowledge, there does not exist any consistent regularization prescription which would explicitly preserve all symmetries of chiral gauge theories, like the Standard Model (some practical calculations in dimensionally regulated chiral theories done in the QAP framework can be found in [8]). On the other hand, it is important to remember that the mentioned procedure restores the BRST invariance only in the strict limit of removed cutoff (*i.e.* for $\Lambda = \infty$). Here, we would like to point out an important consequence of this simple fact. Namely, treating the momentum cutoff regularizing the effective theory describing low-energy physics as a physical scale (that is, keeping it large but finite) would lead, unless this effective field theory is of a very special form, to generation of the photon mass proportional to inverse powers of Λ. Since the experimental bounds are extremely stringent, even the natural assumption $\Lambda \approx M_{\rm Pl}$ ($M_{\rm Pl}$ being the Planck scale) would lead to unacceptably large photon mass, bigger than the experimental limit. Below, we show this explicitly using a concrete realization of the momentum cutoff. We compute the one-loop vacuum polarization tensor in two simple gauge theories and show that terms suppressed by powers of Λ which violate its gauge invariance (transversality) are analytic in the momentum. This leads us to speculate how the problem could possibly be avoided in the context of an underlying fundamental finite theory.

To define the framework, we consider first renormalization of a general YM theory choosing (out of many other possibilities) the momentum cutoff regularization[1] which consists of modifying *every* derivative in the Lagrangian (including the recursively generated counterterms — see below) according to the rule

$$\partial_\mu \to \exp\left(\partial^2/2\Lambda^2\right)\partial_\mu\,. \tag{1}$$

[1] Were it not for the chiral nature of the known elementary interactions, one could speculate that the finiteness of the putative fundamental theory manifests itself in the low-energy effective theory as a kind of manifestly gauge-invariant lattice regularization with a finite spacing of the order of $1/M_{\rm Pl}$. Since chiral gauge symmetries cannot be preserved on the lattice [9], the problem pointed out in this paper applies also to possible lattice regularizations.

In the momentum space, this prescription corresponds to the replacement

$$k_\mu \to \mathcal{R}_\mu(k) \equiv \exp\left(-k^2/2\Lambda^2\right) k_\mu \,. \tag{2}$$

For instance, the regularized ghost contribution to the vacuum polarization tensor (diagram C in Fig. 1) reads

$$\widetilde{\Gamma}^{\mu\nu}_{\alpha\beta}(l) = -\mathrm{tr}(e_\alpha e_\beta) \int \frac{\mathrm{d}^4 k}{(2\pi)^4}\, i\, \frac{\mathcal{R}^\mu(k)\mathcal{R}^\nu(k+l)}{[\mathcal{R}(k)]^2\,[\mathcal{R}(k+l)]^2} \,, \tag{3}$$

where e_α are the anti-Hermitian generators of the adjoint representation with included coupling constants (*i.e.* $e_\alpha = g\, T^{\mathrm{ADJ}}_\alpha$ for a simple gauge group).

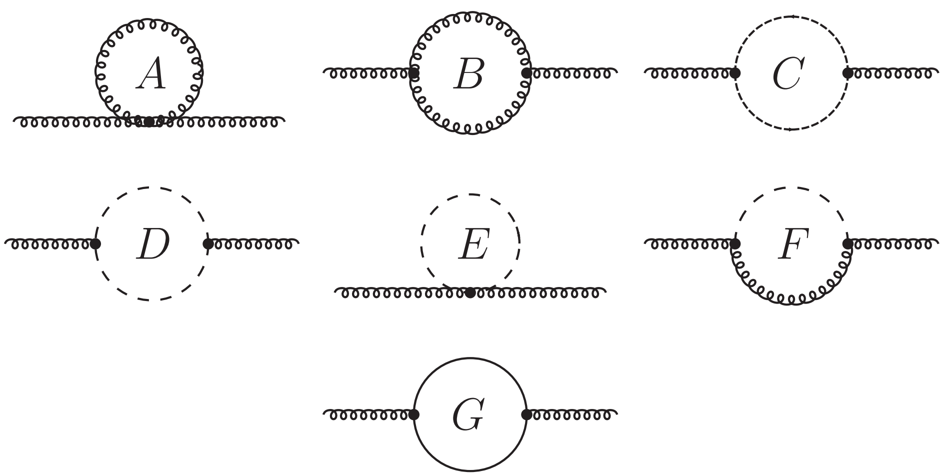

Fig. 1. One-loop contributions to the vacuum polarization.

With replacement (2) the Wick rotation is, strictly speaking, not justified and neglecting the integral over the contour at infinity must be regarded a part of the regularization prescription (alternatively, the prescription can be formulated directly in the Euclidean version of the theory).

As the standard analysis carried out in [2] shows, in regularization (1), all diagrams of a renormalizable theory are convergent with the exception of one-loop vacuum graphs (which anyway cannot appear in physically interesting amplitudes as divergent subdiagrams). Computation of diagrams regularized in this way is more complicated than in the Dimensional Regularization but still manageable. For example, each one-loop diagram can be expressed in terms of the confluent hypergeometric function

$$U(a,b,z) = \frac{1}{\Gamma(a)} \int\limits_0^\infty \mathrm{d}t\, t^{a-1}(1+t)^{b-a-1} \exp(-zt) \,, \tag{4}$$

after applying the expansion

$$\frac{1}{\mathcal{R}^2(k)-m^2}=\frac{e^{k^2/\Lambda^2}}{k^2-m^2}\sum_{n=0}^{\infty}\left[\frac{m^2}{m^2-k^2}\left(1-e^{k^2/\Lambda^2}\right)\right]^n \tag{5}$$

to all regularized propagators. The first term in this expansion bears a very close resemblance to the propagator used in the context of Wilson's exact renormalization group equations [10]. The advantage of our expression is that it is better suited for theories with spontaneous symmetry breaking in which m^2, in general, depends on background scalar fields (which can keep track of vacuum expectation values of dynamical scalar fields). In the Euclidean space, for $k^2 \to -k^2_{\rm E}$, the above series is absolutely convergent. In particular, owing to the growing powers of m^2-k^2 in successive terms, only a finite number of terms of a given one-loop diagram yield integrals which are divergent when the factors $e^{k^2/\Lambda^2}(1-e^{k^2/\Lambda^2})^n$ are omitted. The remaining terms would be integrable without these factors which implies that their contributions vanish in the limit of $\Lambda\to\infty$.

Since regularization (1) breaks the gauge (more precisely, the BRST) invariance, a special subtraction scheme in necessary in order to arrive at a finite (renormalized) one-particle irreducible effective action Γ (the generator of the strongly connected Green's functions) satisfying *in the limit* of $\Lambda\to\infty$ the requirements of the BRST invariance (embodied in the appropriate functional identity). In the accompanying paper [2], such a subtraction scheme (called Λ-$\overline{\rm MS}$), belonging to the class of mass-independent schemes and adapted to regularization (1), is proposed. It relies on the Quantum Action Principle [6] and is defined recursively in the following way. Having a local action I^Λ_n (with all counterterms up to the order of $\hbar^n$ included and with replacement (1) made in all derivatives), one considers Γ_n — the asymptotic part of $\Gamma^\Lambda_n\equiv\Gamma[I^\Lambda_n]$, obtained from it by omitting all terms that would vanish for $\Lambda\to\infty$ and constructs order $\hbar^{n+1}$ "minimal" counterterms $-\Gamma^{(n+1){\rm div}}_n$ which subtract (appropriately defined) "pure", order $\hbar^{n+1}$, divergences of Γ_n. In the next step, one constructs finite non-minimal counterterms $\delta_\flat\Gamma^{(n+1)}_n$ of the restricted schematic form (A^μ, ϕ and ψ stand, respectively, for generic gauge, scalar and fermion fields)

$$\begin{aligned}\delta_\flat\Gamma^{(n+1)}_n\in\int&(\partial^\mu A_\mu)(\partial^\nu A_\nu)\oplus A_\mu A^\mu\oplus A_\mu\overline{\psi}\gamma^\mu P_{\rm L}\psi\oplus A_\mu\overline{\psi}\gamma^\mu P_{\rm R}\psi\\&\oplus\phi\phi A_\mu A^\mu\oplus A_\mu\partial^\mu\phi\oplus\phi A_\mu\partial^\mu\phi\oplus\phi A_\mu A^\mu\oplus AA\,\partial A\oplus AAAA\,,\end{aligned} \tag{6}$$

so that $I_{n+1}\equiv I_n-\Gamma^{(n+1){\rm div}}_n+\delta_\flat\Gamma^{(n+1)}_n$ leads to Γ_{n+1} — the asymptotic part of $\Gamma^\Lambda_{n+1}\equiv\Gamma[I^\Lambda_{n+1}]$ — which up to terms of the order of $\hbar^{n+1}$ is finite and satisfies the Slavnov–Taylor identities (STIds) following from the required

BRST invariance. In [2], it is shown that choice (6), which is particularly natural (no non-minimal counterterms are generated for terms of the action without gauge fields), satisfies all the requirements and is equivalent to the usual specification of the renormalization conditions. As a result of the procedure sketched above, one constructs the action I_∞

$$I_\infty = I_0 + \sum_{n=0}^{\infty} \left(-\Gamma_n^{(n+1)\mathrm{div}} + \delta_\flat \Gamma_n^{(n+1)} \right) \tag{7}$$

expressed in terms of renormalized parameters and couplings, depending explicitly on Λ (through the counterterms $-\Gamma_n^{(n+1)\mathrm{div}}$) and such that Green's functions obtained from $\Gamma[I_\infty^\Lambda]$ satisfy STIds *in the strict limit* of $\Lambda \to \infty$.

Being mass-independent, the proposed scheme introduces, similarly as the ordinary $\overline{\mathrm{MS}}$ scheme, an auxiliary mass scale μ. We have verified by explicit one-loop calculations in a general renormalizable model (with a non-anomalous fermionic representation) that the proposed subtraction scheme is equivalent to the standard $\overline{\mathrm{MS}}$ scheme based on the Dimensional Regularization with the anti-commuting γ^5 matrix (the so-called naive prescription): the one-loop 1PI effective action in Λ-$\overline{\mathrm{MS}}$ can be obtained from its $\overline{\mathrm{MS}}$ counterpart via a reparametrization (a "finite renormalization") of fields and couplings. Furthermore, we have proved recursively, that the finite effective action $\Gamma[I_\infty^\Lambda]$ and the action I_∞^Λ itself satisfy the Renormalization Group Equations (RGE) which ensure independence of physical result of the auxiliary mass scale μ. (The one-loop equivalence of the $\overline{\mathrm{MS}}$ and Λ-$\overline{\mathrm{MS}}$ schemes allowed us to obtain in [2] the two-loop RGE for the Λ-$\overline{\mathrm{MS}}$ scheme parameters.) Finally, we have performed some two-loop consistency checks as well.

Established RG invariance of I_∞^Λ consisting of the regularized original action I_0^Λ and the constructed counterterms, in which replacement (1) is made (as pointed out in [2], this is necessary for consistency of the entire scheme), allowed to show that, despite not having the same functional form as I_0 (*e.g.* each interaction term depending on the gauge fields A^μ is multiplied by a different series of renormalized couplings with coefficients divergent as $\Lambda \to \infty$), it does wind up into a bare action I_B which depends on Λ only through the appropriately defined bare parameters and through the regularizing exponential factors (1) accompanying derivatives. This (technically non-trivial in the case of a gauge symmetry violating regularization) result opens the possibility to view I_∞^Λ (after expanding the exponentials, so that they give rise to infinite sum of higher and higher dimension operators) as a part of the complete Lagrangian density of an effective field theory which in the perturbative expansion reproduces results of some *finite* fundamental theory of all interactions. The scale Λ should be then identified with an intrinsic physical scale of the putative fundamental theory rather than with the scale introduced by the Wilsonian procedure of integrating out some high-energy degrees of freedom. For this interpretation to be possible it is, however, indispensable to address the problem of the residual breaking of the gauge (BRST) invariance by terms suppressed by inverse powers of Λ of which one of the consequences is the photon mass generation.

To illustrate the problem we consider here, using regularization (2), the one-loop contribution to the standard gauge field self energy (vacuum polarization) tensor $\widetilde{\Gamma}^{\mu\nu}_{\alpha\beta}(l)$ contracted with the four-momentum l_μ. Before making subtractions (as indicated by the superscript $1B$), we find (in the Landau gauge, using a developed Mathematica package described in [2])

$$
\begin{aligned}
l_\mu \widetilde{\Gamma}^{\mu\nu}_{\alpha\beta}(l)^{(1B)} &= -3g_s^2\,\delta_{\alpha\beta}\,\frac{l^\nu}{(4\pi)^2} \\
&\qquad \times\left\{-\Lambda^2 - \frac{5}{24}l^2 - \frac{7}{384}\frac{l^4}{\Lambda^2} + \frac{1}{1536}\frac{l^6}{\Lambda^4} + \mathcal{O}\left(\Lambda^{-6}\right)\right\}
\end{aligned}
$$

in “QCD without quarks” (diagrams A, B and C in Fig. 1), and

$$
\begin{aligned}
l_\mu \widetilde{\Gamma}^{\mu\nu}(l)^{(1B)} &= e^2\,\frac{l^\nu}{(4\pi)^2}\left\{-\frac{2}{3}\left(l^2 - 3m^2 + 3\Lambda^2\right)\right. \\
&\qquad \left. + \frac{1}{\Lambda^2}\left[-\frac{11}{96}l^4 + l^2m^2 + 6m^4\ln\frac{m^2}{(0.37\Lambda)^2}\right] + \mathcal{O}\left(\Lambda^{-4}\right)\right\}
\end{aligned}
$$

in QED with a single charged lepton of mass m (diagram G).

Terms that survive in the limit of $\Lambda \to \infty$ clearly show that the gauge symmetry is badly broken by the regularization prescription (1) but are removed by the subtraction procedure based on the QAP. The remaining terms are suppressed by the inverse powers of Λ and are, therefore, harmless in the standard approach to renormalization in which the limit $\Lambda \to \infty$ is taken. However, if Λ is treated as a physical scale, and the limit $\Lambda \to \infty$ is not taken, they imply a contribution to the photon mass m_γ of the order of $(\alpha_{\rm EM}/4\pi)^{1/2} M^2_{\rm top}/\Lambda$. Because of the experimental limit on the photon mass ($m_\gamma < 1.7 \times 10^{-22}$ GeV arising from analyses of the dispersion relations of light emitted from pulsars [11] and $m_\gamma < 10^{-27}$ GeV from combination of all data [12]) such a breaking is excluded even for Λ as high as the Planck scale which would give

$$
|m_\gamma| \approx \left(\frac{\alpha_{\rm EM}}{4\pi}\right)^{1/2}\frac{M^2_{\rm top}}{M_{\rm Pl}} \approx 10^{-18}\ {\rm GeV}\,. \tag{8}
$$

In fact, the situation is even worse since m_γ^2 generated in the above example not only grossly exceeds the experimental limits but has also the wrong sign.

Therefore, if the cutoff Λ is to be treated as a *finite* physical scale of an underlying finite fundamental theory, one has to assume that the *complete* bare action $I_{\rm B}^{\rm complete}$ of the effective QFT, which reproduces all results (including those depending on the gravitational sector) of the latter theory has also additional, as compared to the local action $I_{\rm B} = I_{\infty}^{\Lambda}$ (counter)terms suppressed by inverse powers of Λ which conspire to restore exact BRST invariance of the amplitudes. The structure of the residual gauge symmetry breaking revealed by the above two examples suggests that such a solution may be viable: as the Λ suppressed terms which must be subtracted do not involve logarithms of momenta, $I_{\rm B}^{\rm complete}$ can still be analytic. Additional terms with higher derivatives which must be present in $I_{\rm B}^{\rm complete}$ would complement those which implementing regularization (2) reflect finiteness of the underlying theory — the assumption that higher derivative terms of $I_{\rm B}^{\rm complete}$ combine solely to exponential factors (1) of an otherwise *renormalizable* action is certainly too simplistic for the action of an effective field theory corresponding to the fundamental theory of all interactions. In turn, the presence of logarithms of masses (which are, in general, dependent on the background fields), simply indicates that in order to restore BRST-invariance of all amplitudes for finite Λ, the $I_{\rm B}^{\rm complete}$ should depend on non-polynomial functions of fields (similarly as the action of the field theory describing the Ising lattice model depends on $\cosh(\phi)$, where ϕ is the order parameter field), which after expansion around the background (vacuum expectation values) give rise to vertices with arbitrary numbers of scalar fields.

The ultimate structure of $I_{\rm B}^{\rm complete}$ would, therefore, be such as can naturally be expected on the basis of the general principles of constructing effective theories. In the case considered here, it is tempting to assume that the limit of $\Lambda \to \infty$ corresponds in the effective theory to complete neglect of a gravitational sector, which for finite Λ, is entangled with the other sectors and is indispensable for consistency.

Summarizing, we conjecture that even if the cutoff scale Λ is a real physical scale, it is possible to introduce local counterterms to the bare action (in the spirit of oversubtractions introduced by Symanzik [13] in the context of lattice regularizations) that restore the requisite identities and render the vanishing of the photon mass order-by-order in the inverse powers of the scale Λ.

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