ONSET OF η–NUCLEAR BINDING*

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Recent η few-nucleon stochastic variational method calculations that study the onset of η–nuclear binding are reviewed. The energy dependence of the ηN subthreshold interaction is treated self-consistently. These calculations suggest that a minimum value $\mathrm{Re}\, a_{\eta N} \approx 1$ fm is needed to bind $\eta\,^3$He, whereas $\eta\,^4$He binding requires a minimum value $\mathrm{Re}\, a_{\eta N} \approx 0.7$ fm.

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1. Introduction

The ηN near-threshold ($E_{\mathrm{th}} = 1487$ MeV) interaction is attractive, owing to the $N^*(1535)$ resonance to which the s-wave ηN system is coupled strongly. This was first shown in a πN–ηN coupled channel model [1] and confirmed in fully chiral meson–nucleon coupled channel models that generate dynamically the $\frac{1}{2}^-$ $N^*(1535)$ resonance, *e.g.* [2]. These and other models have been used to calculate η–nuclear quasibound states with widely different predictions. Experimental searches for such states in proton, pion or photon induced η-production reactions are inconclusive. Regarding the onset of η–nuclear binding, Krusche and Wilkin [3] state: "The most straightforward (but not unique) interpretation of the data is that the $\eta\, d$ system is unbound, the $\eta\,^4$He is bound, but that the $\eta\,^3$He case is ambiguous." This ambiguity stems from the strong energy dependence exhibited by the $dp \to \eta\,^3$He production reaction cross section over the first 0.5 MeV excitation, naively suggesting that a nearby S-matrix pole could be in action. However, the $\eta\,^3$He scattering length deduced from a recent fit [4]

$$a_{\eta\,^3\mathrm{He}} = [-(2.23 \pm 1.29) + i(4.89 \pm 0.57)]\ \mathrm{fm}\,, \tag{1}$$

although of the right sign of its real part, does not satisfy the other necessary condition $\mathrm{Re}\, -a > \mathrm{Im}\, a$ for a quasibound state pole.

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With $\eta\,^3$He almost bound, one might expect that the denser ^{4}He nucleus is more likely to exhibit weak binding. However, a recent Faddeev–Yakubovsky evaluation [5] of the scattering lengths $a_{\eta\,^A\mathrm{He}}$ for both He isotopes, $A = 3, 4$, finds this not to be the case, with the denser ^{4}He apparently leading to a stronger reduction of the subthreshold ηN scattering amplitude than in ^{3}He.

The present overview reports and discusses recent few-body stochastic variational method (SVM) calculations of ηNNN and $\eta NNNN$ using several semi-realistic NN interaction models together with two ηN interaction models with strength sufficient to study the onset of η–nuclear binding in the He isotopes [6–8].

2. ηN and NN interaction model input

Figure 1 shows ηN s-wave scattering amplitudes $F_{\eta N}(E)$ calculated in two meson–baryon coupled-channel models across the ηN threshold where $\mathrm{Re}\,F_{\eta N}$ has a cusp. Both amplitudes exhibit a resonance about 50 MeV above threshold, the $N^*(1535)$. The sign of $\mathrm{Re}\,F_{\eta N}$ below the resonance indicates attraction which is far too weak to bind the ηN two-body system. The threshold values $F_{\eta N}(E_{\mathrm{th}})$ are given by the scattering lengths

$$a_{\eta N}^{\mathrm{GW}} = (0.96 + i0.26)\ \mathrm{fm}\,, \qquad a_{\eta N}^{\mathrm{CS}} = (0.67 + i0.20)\ \mathrm{fm}\,, \tag{2}$$

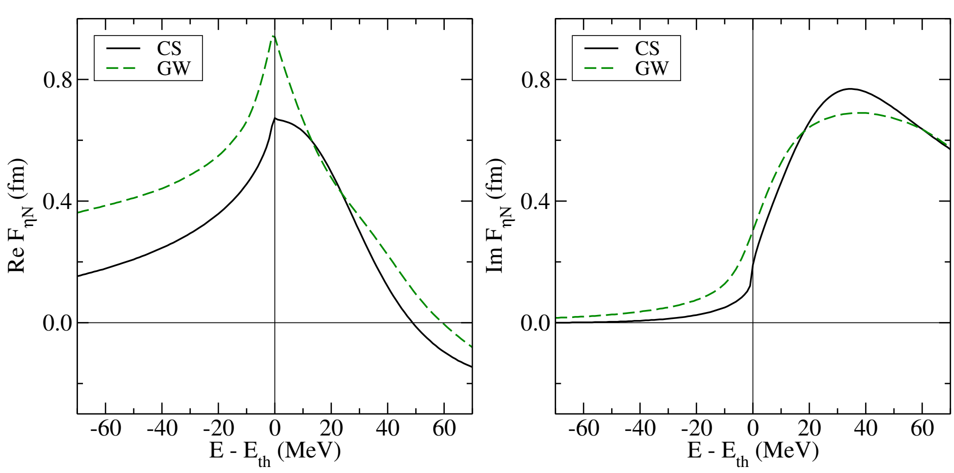

Fig. 1. Real and imaginary parts of the ηN c.m. scattering amplitude near threshold in two meson–baryon coupled-channel $N^*(1535)$ models: GW [9] and CS [10].

with lower values below threshold. Figure 2 shows subthreshold values of the strength function $b_\Lambda(E)$ defined by an effective ηN potential

$$v_{\eta N}(E;r) = -\frac{4\pi}{2\mu_{\eta N}} b_\Lambda(E)\delta_\Lambda(r)\,, \qquad \delta_\Lambda(r) = \left(\frac{\Lambda}{2\sqrt{\pi}}\right)^3 \exp\left(-\frac{\Lambda^2 r^2}{4}\right)\,, \tag{3}$$

derived from the scattering amplitude $F^{\rm GW}_{\eta N}(E)$ of Fig. 1 for several choices of inverse range Λ. The normalized Gaussian functions $\delta_\Lambda(r)$ are perceived in $\not\pi$EFT (pionless EFT) as a single ηN zero-range Dirac $\delta^{(3)}(\boldsymbol{r})$ contact term (CT), regulated by using momentum-space scale parameters Λ. Substituting the underlying short range vector–meson exchange dynamics by a single regulated CT requires that $\Lambda \leq m_\rho$ (~ 4 fm^{-1}).

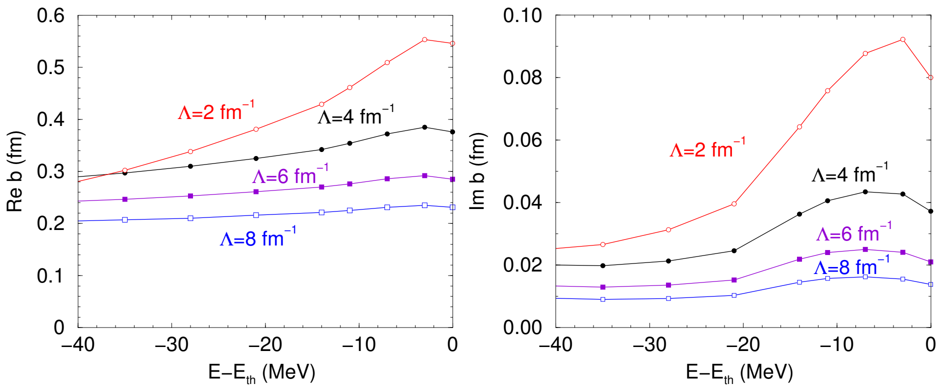

Fig. 2. Real and imaginary parts of the strength function $b_\Lambda(E)$ of the effective ηN potential $v^{\rm GW}_{\eta N}(E)$, Eq. (3), obtained from the scattering amplitude $F^{\rm GW}_{\eta N}(E)$ of Fig. 1 below threshold for four values of the scale parameter Λ.

Similarly, a $\not\pi$EFT $V_{NN}(ij)$ is derived at leading order by fitting a single regulated CT $\sim \delta_\Lambda(r_{ij})$ in each spin–isospin s-wave channel to the respective NN scattering length. To avoid NNN and ηNN Thomas collapse in the limit $\Lambda \to \infty$, one introduces a *universal* three-body regulated CT

$$V_3(ijk) = d_3^\Lambda\, \delta_\Lambda(r_{ij}, r_{jk})\,, \qquad \delta_\Lambda(r_{ij}, r_{jk}) = \delta_\Lambda(r_{ij})\delta_\Lambda(r_{jk}) \tag{4}$$

by fitting to $B_{\rm exp}(^3{\rm He})$. $B_{\rm calc}(^4{\rm He})$ is found in this $\not\pi$EFT version [11] to vary moderately with Λ and to exhibit renormalization scale invariance by approaching a finite value $B_{\Lambda\to\infty}(^4{\rm He}) = 27.8 \pm 0.2$ MeV that compares well with $B_{\rm exp}(^4{\rm He}) = 28.3$ MeV. Using $v^{\rm GW}_{\eta N}(E)$, we find that a potential collapse of $\eta\, d$ has little effect on $B(\eta\, ^A{\rm He})$ values calculated for $\Lambda \leq 4$ fm^{-1} [12].

3. Energy-independent $\not\pi$EFT η–nuclear few-body calculations

Figure 3 shows η separation energies B_η from $\not\pi$EFT SVM calculations of $\eta\,^3$He and $\eta\,^4$He using energy-independent ηN potentials $v_{\eta N}(E=E_{\rm th})$ fitted to given real values of $a_{\eta N}$ for chosen values of Λ. The figure suggests that binding $\eta\,^3$He ($\eta\,^4$He) requires that $a_{\eta N} \geq 0.55$ fm (0.45 fm), compatible with an effective value ${\rm Re}\,a'_{\eta N} = 0.48 \pm 0.05$ fm derived for a nearly bound $\eta\,^3$He [4]. The figure does not show that once $\eta\,d$ becomes bound, beginning at $a_{\eta N} \approx 1.2$ fm for $\Lambda = 4$ fm^{-1} [6], values of $B_\eta^{A=3,4}(\Lambda > 4\ {\rm fm}^{-1})$ diverge [12].

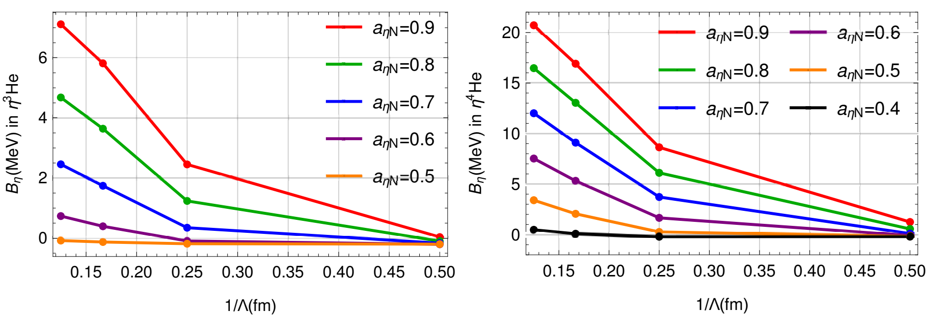

Fig. 3. Separation energies B_η obtained in SVM calculations of $\eta\,^3$He and $\eta\,^4$He using $\not\pi$EFT NN and ηN real interactions, the latter fitted to values of $a_{\eta N} < 1$ fm, plus a universal NNN and ηNN three-body CT (4), as a function of $1/\Lambda$.

4. Energy dependence in η–nuclear few-body systems

The $N^*(1535)$ resonance induces strong energy dependence of the scattering amplitudes $F_{\eta N}(E)$, Fig. 1, requiring the use of energy-dependent potentials $v_{\eta N}(E_{\rm input})$ in η–nuclear few-body calculations. It is shown in Ref. [8] that this generates a continuous ηN two-body energy distribution in the subthreshold region, with output expectation value

$$\langle E_{\rm output}\rangle = E_{\rm th} - \frac{B}{A} - \xi_N \frac{1}{A}\langle T_N\rangle + \frac{A-1}{A}\langle E_\eta\rangle - \xi_A \xi_\eta \left(\frac{A-1}{A}\right)^2 \langle T_\eta\rangle\ , \quad (5)$$

where $\xi_{N(\eta)} = m_{N(\eta)}/(m_N+m_\eta)$, $\xi_A = Am_N/(Am_N+m_\eta)$, T_N and T_η are nuclear and η kinetic energy operators in appropriate Jacobi coordinates, B is the total binding energy and $E_\eta = (H - H_N)$ with each Hamiltonian defined in its own c.m. frame. Self consistency requires $\langle E_{\rm output}\rangle = E_{\rm input}$, satisfied after a few iterations in the ηN subthreshold regime. Applications of self consistency (sc) to meson–nuclear systems are reviewed in Ref. [13]. For recent K^--atom and K^--nuclear applications see Refs. [14, 15].

5. Results and discussion

Our fully self-consistent ηNN, ηNNN and $\eta NNNN$ bound-state calculations [6–8] use the following nuclear core models: *(i)* $\not\pi$EFT with a three-body contact term [11], *(ii)* AV4p, a Gaussian basis adaptation of the Argonne AV4' NN potential [16], and *(iii)* MNC, the Minnesota soft core NN potential [17]. The $N^*(1535)$ models GW [9] and CS [10] were used to generate energy-dependent ηN potentials which prove to be too weak to bind any ηNN system when using AV4p or MNC for the nuclear core model. Calculated η separation energies B_η are shown in Figs. 4 and 5.

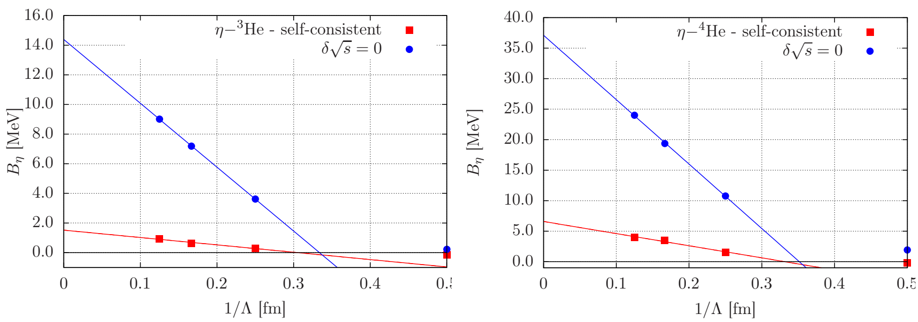

Fig. 4. $B_\eta(\eta\,^3\mathrm{He})$ and $B_\eta(\eta\,^4\mathrm{He})$ as a function of $1/\Lambda$ in $\not\pi$EFT few-body calculations using $v^{\rm GW}_{\eta N}$, with (squares) and without (circles) imposing self consistency.

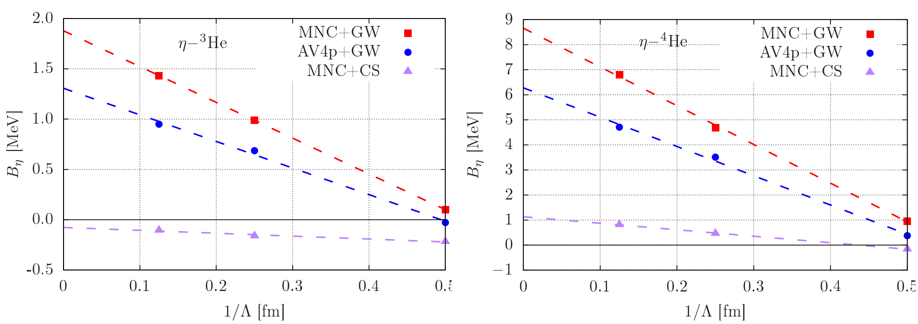

Fig. 5. $B_\eta(\eta\,^3\mathrm{He})$ and $B_\eta(\eta\,^4\mathrm{He})$ as a function of $1/\Lambda$ in self consistent few-body calculations using sets of NN and ηN interaction models, as marked.

Figure 4 demonstrates in $\not\pi$EFT the moderating effect that imposing self-consistency, using $v^{\rm GW}_{\eta N}(E_{\rm sc})$ rather than the threshold values $v^{\rm GW}_{\eta N}(E_{\rm th})$, bears on the calculated B_η values and their Λ scale dependence.

Figure 5 demonstrates the dependence of B_η, calculated self-consistently, on the choice of NN and ηN interaction models. For physically acceptable scale values, $\Lambda \leq 4$ fm^{-1}, this model dependence is quite weak.

The B_η values shown here were calculated assuming real Hamiltonians, justified by $\mathrm{Im}\, v_{\eta N} \ll \mathrm{Re}\, v_{\eta N}$ from Fig. 2. This approximation is estimated to add near threshold less than 0.3 MeV to B_η. Perturbatively-calculated widths Γ_η of weakly bound states amount to only few MeV, outdating those reported in Ref. [6]. Focusing on the AV4p results in Fig. 5, which are close to the $\not\pi$EFT results in Fig. 4, we conclude that $\eta\,^3$He becomes bound for $\mathrm{Re}\, a_{\eta N} \sim 1$ fm, as in model GW, while $\eta\,^4$He binding requires a lower value of $\mathrm{Re}\, a_{\eta N} \sim 0.7$ fm, almost reached in model CS. These $\mathrm{Re}\, a_{\eta N}$ onset values, obviously, are *larger* than those estimated in Section 3 upon calculating with $v_{\eta N}(E = E_{\mathrm{th}})$ threshold input. Finally, $\mathrm{Re}\, a_{\eta N} < 0.7$ fm if $\eta\,^4$He is unbound, as might be deduced from the recent WASA-at-COSY search [18].

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