ON GAMOV STATES OF Σ^+ HYPERONS*

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Both the FINUDA and AMADEUS experiments evidenced a low Σ^+ momentum component when investigating $\Sigma^+\pi^-$ pairs produced in K^- nuclear capture. This component is interpreted as a consequence of the Gamov state formation, with the hyperon trapped in the Coulomb field of the residual nucleus. Description of such states and their participation in the capture reaction is presented. Some consequences are indicated.

1. Introduction

An enhancement of events at low Σ^+ energies, close to the Σ^+ formation threshold, was observed by FINUDA [1], in $\Sigma^+\pi^-$ correlated pairs produced by K^- hadronic captures at rest on ^{6}Li target. Such a phenomenon is absent in the Σ^- momentum spectrum of the corresponding $\Sigma^-\pi^+$ sample and this indicates its electromagnetic origin. Monte Carlo simulations of Σ^+ energy loss in the target does not seem to properly describe the low Σ^+ momentum spectrum. A low Σ^+ momentum peak structure was also measured by AMADEUS (see Ref. [2]) in the reaction

$$K^-\ {}^{12}C \to \Sigma^+\pi^- R\,, \tag{1}$$

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where the residual R is ^{11}B when no fragmentation occurs. The low-energy Σ^+ events amount to some percent of the total $\Sigma^+\pi^-$ sample. The solid carbon fibre target is much thiner in this case, so the Σ^+ energy loss cannot explain the observed phenomenon. Moreover, the low-momentum structure is not observed in [2] when the K^- is absorbed on a solid ^{9}Be target.

In agreement with old measurements (see review [3]), the spectrum of Σ^+ momentum is characterised by a broad distribution centered at ~ 200 MeV/c with an additional threshold enhancement in the 20 ± 15 MeV/c momentum range, which appears very narrow on the nuclear momentum scale. In our analysis, the low-momentum peak is attributed to the interaction of Σ^+ with the residual nucleus. A fraction of the hyperons is trapped into a Gamov state formed by the interplay among an attractive nuclear potential and the repulsive Coulomb barrier. In Section 2, the properties of such states are described. We show that the Gamov state formation offers an explanation to the measured low-momentum enhancement in the Σ^+ distributions.

2. The origin of the anomalous threshold peak in the $\boldsymbol{\Sigma^+}$ momentum

The formation mechanism of the low-momentum peak can be described in terms of a sequence of processes:

— first, the K^- meson undergoes hadronic capture $K^-p \to \Sigma^+\pi^-$, which is described by a transition matrix T. The residual R is considered as a spectator,

— the Σ^+ is trapped by the Coulomb potential of R into a Gamov state,

— the Gamov state decays into R and Σ^+ of low total momentum $\boldsymbol{q}_{\Sigma R}$.

The three-step mechanism indicated above is analysed below in a quasi three-body system consisting of: meson $K(\pi)$ baryon $p(\Sigma)$ and residual system R. We describe the process in the K^--^{12}C centre-of-mass system, the initial ^{12}C nucleus is understood as a bound state of R an p.

The following system of Jacobi coordinates will be used:

— $\boldsymbol{r}$ — which is the relative R — baryon coordinate,

— $\boldsymbol{R}$ — which is the relative coordinate of the meson respect to the R — baryon centre of mass.

Coordinates referring to the final system are marked with primes.

The final state is specified by three momenta. The Gamov state system, consisting of the R–Σ will be denoted by G, the total momentum is then $\boldsymbol{p}_{\mathrm{G}} = \boldsymbol{p}_R + \boldsymbol{p}_\Sigma$ and the relative momentum is

$$\boldsymbol{q}_{\Sigma R} = \alpha' \boldsymbol{p}_\Sigma - \beta' \boldsymbol{p}_R\,, \tag{2}$$

where $\alpha' = m_R/(m_\Sigma + m_R)$ and $\beta' = m_\Sigma/(m_\Sigma + m_R)$. The third momentum which is necessary to determine the state is the pion momentum $\boldsymbol{p}_\pi$. The final wave function is expressed as

$$\Psi_{\mathrm{F}} = \exp[i\boldsymbol{R}'(\boldsymbol{p}_\pi + \boldsymbol{p}_G - \boldsymbol{p}_K)] \int \mathrm{d}\boldsymbol{R}\, \mathrm{d}\boldsymbol{r}\, G(\boldsymbol{r}', \boldsymbol{r})\, T\, \delta(\boldsymbol{r}_{Kp})\, \Phi_K(\boldsymbol{R}, \boldsymbol{r})\, \Phi_N(\boldsymbol{r})\,, \tag{3}$$

where G is Green's function describing propagation of the ΣR pair. For in flight K^- captures, the total c.m. system is not fixed, $\boldsymbol{p}_K$ represents K^- meson momentum in centre-of-mass system and $\Phi_K = \exp(i\alpha \boldsymbol{r} \boldsymbol{p}_K)$. For atomic captures, $\boldsymbol{p}_K = 0$ and Φ_K becomes an atomic function $\Phi_L(\boldsymbol{R})$ for a given angular momentum L state. The operator T describes transfer of strangeness. It is assumed to be of zero range, as the KN force range is known to be very short, and depends on the invariant mass of the meson–baryon pair measured in terms of the final momenta. Thus, $T \equiv T_{Kp\Sigma\pi}(M_{\Sigma\pi})$ and the reaction in question offers a chance to study this energy dependence. The energy in the KN centre-of-mass system is a sum of the bound nucleon and kaon energies reduced by recoil of the pair with respect to the residual system R. The upper kinematic limit of the $M_{\Sigma\pi}$ spectrum is given by $M_N - B_N + E_K$, where B_N in the last nucleon binding energy, E_K is the kaon kinetic energy. For atomic captures, the kinematic limit is 1416 MeV in carbon, for the in-flight capture the corresponding kinematic limit is pushed up of about 14 MeV for $p_K = 120$ MeV (typical momenta of the charged kaons produced at the DAΦNE factory). This covers the profile of $\Lambda(1405)$ resonance which dominates the K^-p interaction.

Green's function G is built in terms of two solutions, regular Φ and outgoing Φ^+, of the Schrödinger equation involving Σ–R interaction potential. We split this potential into long and short ranged parts

$$V = V_{\mathrm{l}} + iW_{\mathrm{s}} = V_{\mathrm{cul}} + V_{\mathrm{o}}\rho(r) + iW_{\mathrm{o}}\rho(r) \tag{4}$$

and solve for G in the standard way of the *two potentials problem*. The short-ranged imaginary iW_{s} part describes nuclear absorption of the hyperon. The long-ranged part V_{l} is composed of the Coulomb and the nuclear interactions. For low-Z nuclei and low-energy hyperon, only S-wave solutions matter. Concerning the potential V_{l}, two wave function are found. The radial function $\phi = \Phi/r$ is obtained by solving the Schrödinger equation

$$-\frac{1}{2\mu}\phi(r)'' + V_{\mathrm{l}}\phi = E\phi\,, \tag{5}$$

where in Eq. (5) μ is the reduced mass of the Σ–R system, and the boundary condition at origin is $\phi(0) = 0$, $\phi'(0) = 1$. The second solution of Eq. (5) (denoted ϕ^+) fulfils the asymptotic condition of an outgoing Coulomb wave. In practice, we cut V_{cul} at large distance and $\phi^+ \sim \exp(iq_{\Sigma R}r)$. The Green function for S-wave is given by

$$G_1\left(E, r, r'\right) = \frac{2\mu}{4\pi} \frac{[\phi^+(r_>)\phi(r_<)]}{[rr'W_1(\phi^+, \phi)]}, \tag{6}$$

where $W_1(\phi^+, \phi)$ is the Wronski determinant, $r_>(r_<)$ denote larger(smaller) of the two cordinates. The no-interaction limit (or high-momentum limit) of Φ^+ is the spherical Hankel function and similar limit for Φ is the Bessel function. Calculations are performed with a Coulomb potential of uniformly charged sphere, cut (at 30 fm). The outgoing solution is normalised at $r_\infty = 50$ fm to its asymptotic form.

For attractive V_l, localized solutions may exist at negative energies. For positive energies, there might exists quasi-localized states due to the Coulomb barrier *i.e.* Gamov states. Such solutions are characterised by minute, damped by many orders, waves outside the barrier and happen at discreet momenta Q_G and energies in a narrow region centered at values $E_G = (Q_G)^2/(2\mu)$. The discrete values of momenta characterise the properties of the observed peaks better than the energies E_G, which are more directly related to resonances that would be observed in Σ–R scattering states. Zeroes of the Wronski determinant in the complex energy plane describe the position of Green's function singularities at the Gamov levels $E_G - i\Gamma_G/2$. For the low Z systems in question, well-localized solutions may exist. If the spacial densities $\rho = |\Phi_G(r)/\Phi(0)|^2$ in such a state are cut at r_∞, one can define r.m.s. radii of the Gamov system. In addition, if one requires $\rho(r_\infty) < 10^{-6}$, one obtains radii less than 10 fm and the Gamov levels in the $(0 < E_G < 0.4)$ MeV range. These states are coupled by T to the initial K meson capture states. Their widths are very small $(0.5 < \Gamma_G < 20)$ keV. The region of E_G indicated above sets the limits for the depth of V_o potential well $(-19.3 < V_o < -18.0)$ MeV. In light nuclei, the Gamov states may exist provided there are no bound states. As no Σ hyper-nuclear states have been found in the nuclei of interest, the Σ-nuclear potential is not under theoretical control. The experimental investigation of K^- induced reactions in nuclear matter will furnish the real Gamov state energy and real V_l.

The Gamov widths are very narrow, in the keV region, while the experimental widths are about 0.2 MeV. A natural question arises if the experimental widths are related to the imaginary part of the nuclear potential which describes the $\Sigma \to \Lambda$ conversion. The W_s contributes to full Green's function G given by the "two potential" integral equation which reads in the

operator form

$$G = G_1 + G_1 i W_{\mathrm{s}}, \qquad G = G_1[1 + i W_{\mathrm{s}} G]. \tag{7}$$

At this stage, one has to realize the presence of a third body, the π meson. If one tries to solve Eq. (7) in the three-body context, say by iterations, one finds a propagator projecting on the Gamov state $|\Phi\rangle\langle\Phi|/[E - E_{\mathrm{G}} + i\Gamma_{\mathrm{G}}/2 - E^{\pi}(q_{\pi})]$. An integration over intermediate pion momenta smears the Gamov singularity and results in a small effect due to small overlap of the Gamov state with the nucleus. Thus, the "near singularity" close to the real energy axis matter in Eq. (7) only in Green's function G_1. We use this approximation and determine G, and the factor $[1 + i W_{\mathrm{s}} G]$ from the Schrödinger equation (5) solved with the full potential V. Such an equation also offers discreet quasi-localized solutions, but only in the far non-physical region for $E' - i\Gamma'/2$, where the widths are large $\Gamma' \sim W_{\mathrm{o}}$, that is in a few MeV range. Thus, the main effect of absorption is the $i W_{\mathrm{s}} G$ term which renormalizes the strength of coupling to the Gamov state. With potential depth $W_{\mathrm{s}}(0) \simeq -15$ MeV, characteristic of hyperonic atoms [5], one finds $\langle|1 + i W_{\mathrm{s}} G|\rangle^2 \simeq 0.6$. This number describes the loss of Σ due to conversion. We conclude that the widths of experimentally observed states are not given by nuclear absorption and do not test the potential W_{s} directly. These widths are due to the π meson emission and follow the distribution of $|\boldsymbol{p}_{\Sigma} = \boldsymbol{Q}/\alpha + \boldsymbol{p}_{\pi}\beta/\alpha|$ given by Eq. (2). For low-momentum hyperons, the pion momentum is almost constant (~ 170 MeV/c) and the width of the peak is determined by the distribution of the $\boldsymbol{p}_{\pi}, \boldsymbol{Q}$ angle allowed by the phase space. With $Q \to 0$, the width of peak reduces to a non-unmeasurably small Γ_{G}.

3. Amplitudes, spectral functions

The transition amplitude generated by the wave function (5) becomes

$$A = \frac{T}{W_1} \int \mathrm{d}\boldsymbol{r}\Phi(r)\Phi_p(r)\Phi_K(\alpha r)\exp(-i\alpha\boldsymbol{p}_{\pi}\boldsymbol{r}) \tag{8}$$

and requires wave function for proton $\Phi_p(r)$ (taken from Ref. [4]) and kaon Φ_K. For captures in-flight, the latter is assumed to be a plane wave. For atomic captures, one needs to know the distribution over atomic states. The atomic transition terminates at $L = 2$ [6] which apparently is the dominant angular momentum at the capture. The distribution of main quantum numbers is not known but it is not relevant as the absolute capture rates are not measured.

In the next step, the spectrum of hyperon momentum is obtained in standard way integrating over $d\rho = d_3 p/E(p)$ for each particle

$$P(p_\Sigma) dp_\Sigma = \int d\rho^\pi d\rho^\Sigma d\rho^R \delta(\boldsymbol{p}_i - \boldsymbol{p}_f) \delta(E_i - E_f) |A|^2. \qquad (9)$$

The shape of the peak is determined by the Wronski determinant. An excellent approximation $|W(q_{\Sigma R})|^2 \simeq \delta(Q - q_{\Sigma R})\omega(Q)$ (whith ω a normalization constant) helps the integration over the three-body phase space.

Figure 1 displays the results obtained for the K^- mesons captured in flight. This case is the easiest as the initial mesonic state is known and the initial K-nucleus interaction is of moderate strength (it was neglected). The high-momentum spectrum was calculated for several versions of resonant (Breit–Wigner) transition amplitude T modelled to simulate the $\Lambda(1405)$. The dependence on the position of resonance is moderate. The position of the peak observed by AMADEUS [2] is obtained with $Q \simeq 15$ MeV$/c$. For all these amplitudes, one finds the low peak of right magnitude of about 3% of the large peak. The shape of the Gamov peak cannot be tested because the momentum resolution in this region is of several MeV$/c$. The distribution has a very peculiar structure determined by the Gamov singularity in the Wronski determinant, folded over limitations induced by the corner of the available phase space where the peak is located.
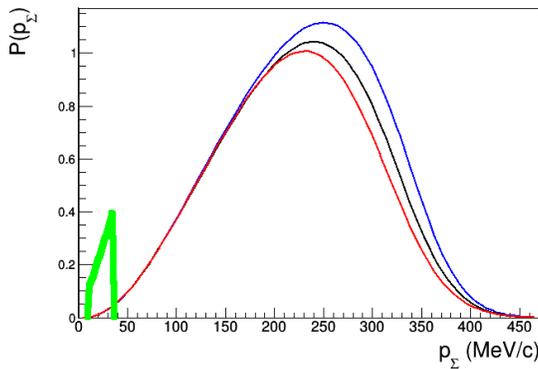
Fig. 1. (Colour on-line) Momentum distribution of Σ^+ following K^- capture in-flight on carbon nuclei. The thick grey/green distribution represents the Gamov peak. Other curves describe the dependence of the spectrum on the position of a 40 MeV wide resonance with $E_r = 1410$ MeV (red distribution), $E_r = 1420$ MeV (black distribution), $T \equiv 1$ (blue distributions) (from bottom to top). Normalisations are arbitrary, as only profiles are measured.

4. Conclusions

The description of the anomalous low-energy momentum distribution $P(p_\Sigma)$ in terms of the Gamov state gives consistent description of the FINUDA and AMADEUS data. The mere fact of existence of the low-energy peak puts strong limitation on the hyperon nucleus potential. The peak strength is more complicated to analyse as it involves: peak position, KN amplitude and nuclear absorption of Σ hyperons.

These two pieces of information are, to some extent, a substitute to (apparently nonexistent) Σ hypernuclei. The FINUDA data are related to hyperon ^{5}He system. The only hypernucleus discovered is $^4\text{He}_\Sigma$ [7]. It would be very interesting to check whether the Gamov states of Σ^+ can also be formed in heavier nuclei. Positive answer would open a new branch of hypernuclear spectroscopy.

The full understanding of p_Σ spectrum, in particular the weight of the anomalous peak, relative to the standard spectrum, might be more informative than the Σ hypernuclear spectroscopy would be. New experiments would be very helpful.

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