INTERFERENCE EFFECTS IN A VERY PRECISE MEASUREMENT OF THE MUON CHARGE ASYMMETRY AT FCC-ee* **

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At the future high luminosity electron–positron collider FCC-ee proposed for CERN, the precise measurement of the charge asymmetry in the process $e^-e^+ \to \mu^-\mu^+$ near the Z resonance is of special interest. In particular, such a measurement at $M_Z \pm 3.5$ GeV may provide a very precise measurement of the electromagnetic coupling at the scale $\sim M_Z$, a fundamental constant of the Standard Model. However, this charge asymmetry is plagued by a large trivial contribution from the interference of photon emission from the initial-state electrons and final-state muons. We address the question whether this interference can be reliably calculated and subtracted with the help of a resummed QED calculation.

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1. Introduction

The Future Circular Collider with $e^\pm$ beams (FCC-ee) considered at CERN could produce more than 10^{12} Z bosons per year, a factor of 10^5 more than LEP collider, opening completely new avenues in testing Standard Model (SM) predictions, which may reveal signals on new physics beyond the Standard Model. The pure electromagnetic coupling constant $\alpha_{\rm QED}(M_Z)$,

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will be vitally important in searching for disagreements between the FCC-ee experimental data and SM predictions at a precision level at least a factor of 10 better than in the past. This kind of the measurement was proposed and analyzed in Ref. [1].

Generally, M_Z, G_F, and $\alpha_{QED}(0)$ outweigh other data in the "testing power" in the overall fit of the SM to experimental data. Up to now, $\alpha_{QED}(Q^2=0)$ was ported to $\alpha_{QED}(Q^2=M_Z^2)$ using low-energy QCD data — this limits its usefulness beyond LEP precision. In Ref. [1], it was proposed to use another observable, $A_{FB}(e^+e^-\to\mu^+\mu^-)$ at $\sqrt{s_\pm}=M_Z\pm 3.5$ GeV, because it features a similar "testing profile" in the SM overall fit as $\alpha_{QED}(M_Z^2)$, but could be measured very precisely at a high luminosity FCC-ee[1]. However, A_{FB} varies very strongly near $\sqrt{s_\pm}$, and hence is prone to large QED corrections (for instance, ISR). In particular, away from the Z peak, A_{FB} gets a direct sizable contribution from QED initial–final state interference (IFI). It is, therefore, necessary to rediscuss how efficiently these trivial but large QED effects in A_{FB} can be controlled and/or eliminated.

In this context, the aim is to reduce QED uncertainty to $\delta A_{FB}(e^+e^-\to\mu^+\mu^-)<4\times10^{-5}$. Presently, $\Delta\alpha_{QED}(M_Z)/\alpha_{QED}\simeq 1.1\times10^{-4}$, using low-energy e^+e^- data. Recent studies using the same method of dispersion relations are quoting possible improvements down to $\Delta\alpha/\alpha\simeq(0.5\text{–}0.2)\times10^{-4}$. To be competitive, A_{FB} has to provide $\Delta\alpha/\alpha<10^{-4}$. Using Fig. 4 of Ref. [1], $\Delta\alpha/\alpha<10^{-4}$ translates into $\Delta A_{FB}<4\times10^{-5}$. The LEP-era estimate of the QED uncertainty in A_{FB} outside the Z peak was $\sim 2.5\times10^{-3}$; see Ref. [2]. Its improvement by a factor of 200 or more sounds like a very ambitious goal! However, there was an encouraging precedent: for QED photonic corrections to the Z lineshape ($\sim 30\%$), the uncertainty was reduced down to $\delta\sigma/\sigma\simeq 3\times10^{-4}$; see Ref. [3].

The general features of QED (photonic) corrections in $A_{FB}(e^+e^-\to\mu^+\mu^-)$ are the following: Pure ISR (initial-state radiation) has an indirect influence due to reduction of $\sqrt{s}$. Non-soft higher order missing corrections are under very good control. Pure FSR (final-state radiation) is generally small for a sufficiently inclusive event selection (cut-offs), but cut-off dependence has to be controlled with a high quality MC. The direct contribution of IFI (initial–final state interference) is suppressed at the peak but sizable off-peak. The IFI effect comes from a non-trivial matrix element, even in the soft-photon approximation. The KKMC Monte Carlo program of Refs. [4, 5] is the most sophisticated tool to calculate all the above effects.

A general understanding of the genuine IFI effect is the following: In $e^-e^+\to\mu^-\mu^+$, not only is the e^- annihilated, but its accompanying electromagnetic field of charge -1 also gets annihilated, and a new electromagnetic

[1] It is advertised as "determining $\alpha_{QED}(M_Z^2)$ from $A_{FB}(\sqrt{s_\pm})$".

field of charge -1 is recreated along with the μ^-. At a wide scattering angle θ, these two processes are independent sources of real photons. The effect of the cut-off on the photon energy is, therefore, essentially θ-independent. For very small scattering angles, when the μ^- is close to the initial e^-, the μ^- inherits most of its electromagnetic field from the e^-; hence real bremsstrahlung is weaker. For $\theta \to 0$, the effect of the cut-off on the real photons essentially vanishes. On the other hand, in backward scattering ($\theta \to \pi$) replacing the e^- field of charge -1 with a μ^+ field of charge $+1$ is "more violent", and more real photons are produced. The effect of the cut-off on photon energy is then stronger and for sharper cut-off, one gets a pronounced dip in the muon angular distribution.

2. IFI prediction from KKMC

The IFI phenomenon described above is clearly seen in the angular distribution shown in Fig. 1 for $e^-e^+ \to \mu^-\mu^+$ scattering at $\sqrt{s} = 10$ GeV, obtained using the KKMC program. Far from the resonance, the IFI contri-

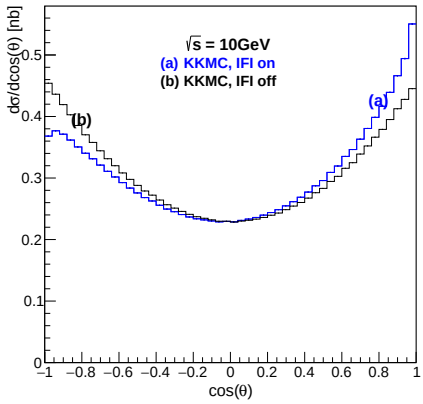


Fig. 1. Angular distribution of $e^-e^+ \to \mu^-\mu^+$ at $\sqrt{s} = 10$ GeV for (a) IFI switched on and (b) switched off.

bution to charge asymmetry $A_{\rm FB}$ is about 3% for a loose cut-off on photon energy $v < v_{\rm max}$, and grows for stronger cut-offs; see line (d) in Fig. 2, right. Line (c) represents $A_{\rm FB}$ at $\sqrt{s} = M_Z$. It illustrates the suppression of $A_{\rm FB}$ by the factor Γ_Z/M_Z at the resonance position due to the time separation between production and decay for any long-lived (narrow) resonance. On the other hand, the two curves (a) and (b) in the RHS plot in Fig. 2, right show the IFI contribution to $A_{\rm FB}(v_{\rm max})$ at $\sqrt{s} = 94.3$ GeV and $-A_{\rm FB}(v_{\rm max})$ at $\sqrt{s} = 87.9$ GeV. As we see, partial Γ_Z/M_Z suppression is still in action. The difference $(a) - (b)$ is also shown, demonstrating a partial cancellation of IFI contributions between those two energies.

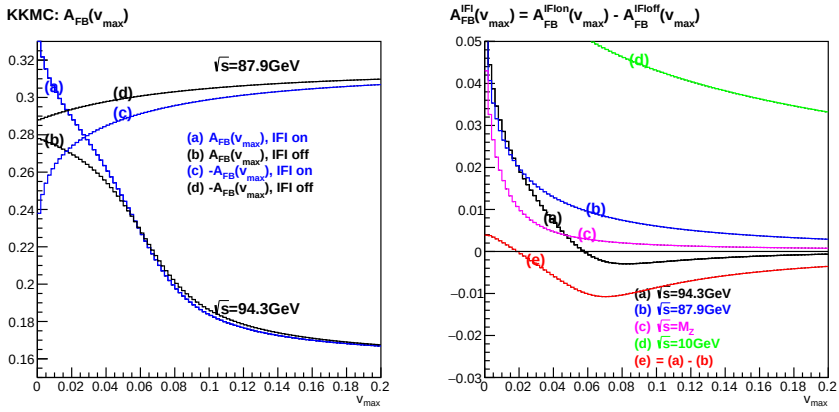


Fig. 2. Charge asymmetry in $e^-e^+ \to \mu^-\mu^+$ at various energies as a function of the cut-off on $v = 1 - M^2_{\mu^+\mu^-}/s$, obtained using KKMC.

The KKMC program of Refs. [4, 5] provides the best state-of-the-art calculation of the IFI contribution available today. It includes an $\mathcal{O}(\alpha^2)$ QED photonic matrix element[2], $\mathcal{O}(\alpha^1)$ electroweak corrections, soft-photon resummation at the amplitude level[3], amplitude-level resummation of the $\sim \ln^n(\Gamma_Z/M_Z)$ QED effects due to the Z resonance[4]. Moreover, since KKMC is regular Monte Carlo event generator, it provides predictions for arbitrary experimental event selections, cut-offs, and observables.

The "problem" with KKMC predictions for IFI (and other observables) is that there is no other calculation of comparable quality in order to check for missing higher-order contributions and/or technical biases at the very high experimental precision anticipated at FCC-ee.

3. New program KKFoam for testing/calibrating KKMC

In order to address the above problem of calibrating and testing the KKMC predictions for IFI in $A_{\rm FB}$, another new program KKFoam was recently developed. It is based on soft-photon resummation including resonance effects as in Refs. [7–9], integrating analytically over photon angles, and using the FOAM Monte Carlo simulator/integrator [10], to integrate the remaining four photon energy variables and muon scattering angle. KKFoam can calculate distributions only for a very limited class of experimental cuts, on v and $\cos\theta$.

[2] Except for the non-logarithmic parts of the $\mathcal{O}(\alpha^2)$ IFI penta-boxes.
[3] A generalization of the Yennie–Frautschi–Suura exponentiation of Ref. [6].
[4] A generalization of the soft-photon resummation near a resonance pioneered by the Frascati group; see Refs. [7–9].

A detailed description of the QED distributions used in KKFoam will be available in a forthcoming publication [11] — here, we will only briefly describe its soft photon content in a simplified form. It reads as follows:

$$\begin{aligned}\sigma(s,v_{\max}) &= \frac{3\sigma_0(s)}{16}\sum_{V,V'}\int_0^1 \mathrm{d}v_{\mathrm{I}}\,\mathrm{d}v_{\mathrm{F}}\,\mathrm{d}r\,\mathrm{d}r' \\ &\times\int \mathrm{d}\cos\theta\mathrm{d}\phi\;\theta\left(v_{\max}-v_{\mathrm{I}}-r-r'-v_{\mathrm{F}}\right) \\ &\times\rho(\gamma_{\mathrm{I}},v_{\mathrm{I}})\;\rho(\gamma_X,r)\;\rho\left(\gamma_X,r'\right)\;\rho(\gamma_{\mathrm{F}},v_{\mathrm{F}})\;e^{Y(p_i,q_i)} \\ &\times\frac{1}{4}\sum_{\varepsilon\tau}\Re\Big\{e^{\alpha\Delta B_4^V(s(1-v_{\mathrm{I}}-r))}\mathfrak{M}^V_{\varepsilon\tau}\big(v_{\mathrm{I}}+r,\cos\theta\big) \\ &\times\left[e^{\alpha\Delta B_4^{V'}(s(1-v_{\mathrm{I}}-r'))}\mathfrak{M}^{V'}_{\varepsilon\tau}\left(v_{\mathrm{I}}+r',\cos\theta\right)\right]^*\Big\}, \end{aligned} \tag{1}$$

with

$$\begin{aligned}\rho(\gamma,v)\equiv F(\gamma)\gamma v^{\gamma-1} &= \frac{v^{\gamma-1}e^{-\gamma C_E}}{\Gamma(\gamma)}, \\ \gamma_{\mathrm{I}}(s) &= \int\frac{\mathrm{d}^3k}{k^0}S_{\mathrm{I}}(k)\delta\left(\frac{2k^0}{\sqrt{s}}-1\right), \\ \gamma_{\mathrm{F}}(s(1-v_{\mathrm{I}})) &= \int\frac{\mathrm{d}^3k}{k^0}S_{\mathrm{F}}(k)\delta\left(\frac{2k^0}{\sqrt{s}}-1\right), \\ \gamma_X(\cos\theta) &= \int\frac{\mathrm{d}^3k}{k^0}S_X(k)\delta\left(\frac{2k^0}{\sqrt{s}}-1\right), \end{aligned} \tag{2,3}$$

and the classic YFS form factor

$$Y(s,t)=2\alpha\Re B_4(s,t,m_\gamma)+\int_{2k^0<\sqrt{s}}\frac{\mathrm{d}^3k}{k^0}\left[S_{\mathrm{I}}(k)+2S_X(k)+S_{\mathrm{F}}(k)\right], \tag{4}$$

which is finite in the $m_\gamma\to 0$ limit. Here, θ is angle the between the momenta of the e^- and μ^-, σ_0 is the point-like cross section, $S_{\mathrm{I}}, S_{\mathrm{F}}, S_X$ are the usual eikonal factors [4] for photon emission from the initial state, final state, and their interference, B_4 is the standard virtual Yennie–Frautschi–Suura form factor [6] and $\mathfrak{M}^V_{\varepsilon\tau}$, $\varepsilon,\tau=\pm1$ are Born spin amplitudes. The additional form factor $\Delta B_4^Z(s)$ due to the Z resonance is that of Eq. (94) in Ref. [4], while $\Delta B_4^\gamma(s)=0$. The structure of the above distribution is illustrated schematically in Fig. 3. The remarkable feature is that there are independent photon energy variables r and r' for the matrix element $\mathfrak{M}$ and its complex conjugate! A more detailed description of the QED distributions in KKFoam will be provided in Ref. [11].

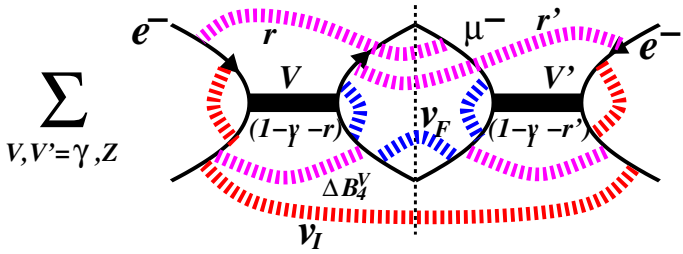


Fig. 3. Exponentiated multiple photon emission from initial and final fermions including ISR, FSR and IFI in the resonant process, as in Eq. (1). Dashed lines represent multiple real and/or virtual photons.

4. Testing IFI from KKMC using KKFoam

With KKFoam in hand, we can compare the predictions for the IFI contribution to $A_{\rm FB}$ to those from KKMC. An example of such a comparison is shown in Fig. 4, where the muon charge asymmetry is displayed as a function of the total photon energy cut-off $v_{\rm max}$. With IFI on, the difference (line (c) in the RHS plot) is of the order of 4×10^{-4} and vanishes as it should at $v_{\rm max} \to 0$. This difference is a factor of 10 smaller than any QED uncertainty due to IFI quoted in the LEP era, and it represents definite progress. However, it is still far (by another factor 10) from what is needed for the FCC-ee experiments. Nevertheless, a new avenue is now opened towards this ambitious goal.

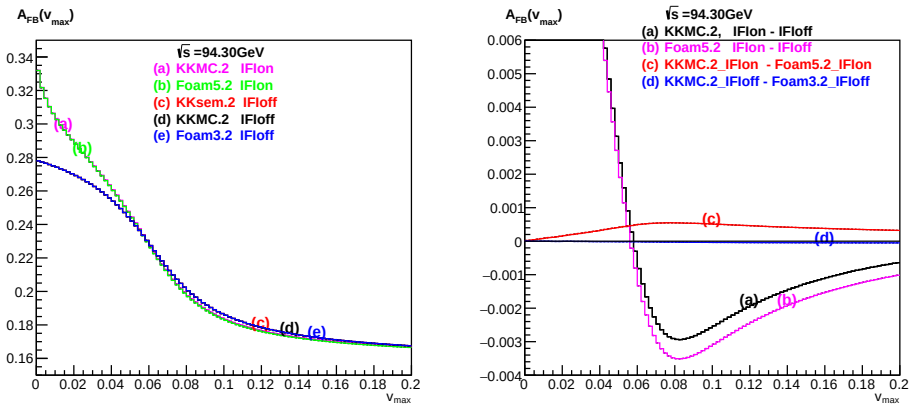


Fig. 4. Muon charge asymmetry at $\sqrt{s} = 94.3$ GeV as a function of the cut-off $v_{\rm max}$ on the total photon energy. Results are obtained using KKMC and KKFoam.

5. Summary

We have presented an example of a promising new calculation of the initial–final QED interference contribution to charge asymmetry in the $e^-e^+ \to \mu^-\mu^+$ process near the Z resonance, which brings us closer to mastering this effect at the precision level needed in the FCC-ee project — that is, a factor of 10 better than the LEP-era state of the art.

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