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ELECTROWEAK BOSONIC 2-LOOP CORRECTIONS TO THE Z-POLE PRECISION OBSERVABLES*

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Precision measurements of properties of the electroweak W- and Z-bosons provide strong constraints on the Standard Model (SM) and extensions thereof. This sensitivity crucially depends on the availability of accurate theoretical predictions for these quantities, including higher-order radiative corrections. This contribution gives a brief overview of available calculations and compares the estimate of theory uncertainties with the projected experimental precision of future e^+e^- collider proposals. Then it is shown how numerical Mellin–Barnes integrals can be used to evaluate higher-order loop integrals that depend on three or more independent mass and momentum scales. As a concrete physical application, the complete two-loop corrections to $Z \to b\bar{b}$ are considered.

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1. Status of electroweak precision observables

After the discovery of the Higgs boson, there is overall very good agreement between theory predictions and experimental data for the electroweak SM, see, for example, Ref. [1]. It should be noted that many of the relevant data inputs into such a global fit have been determined with an experimental precision of better than the per-mille level, making the agreement with theory even more impressive. As a result, electroweak precision data puts strong constraints on physics beyond the SM, which could be parametrized in terms of an effective field theory framework [2]. Some of the most constraining quantities include:

- The W-boson mass, which can be predicted from the Fermi constant, G_{F}, of muon decay;
- The total Z-boson width, Γ_Z;
- Various branching fractions of the Z-boson, such as $R_\ell \equiv \Gamma_Z^{\mathrm{had}}/\Gamma_Z^\ell$ and $R_b \equiv \Gamma_Z^b/\Gamma_Z^{\mathrm{had}}$;
- The hadronic Z-peak cross section, σ^0_{had}, which corresponds to the process $e^+e^- \to Z^{(*)} \to$ hadrons;
- The effective weak mixing angle $\sin^2\theta^f_{\mathrm{eff}}$, which is given in terms of the ratio of the vector and axial-vector couplings of the $Z \to f\bar{f}$ vertex, $\sin^2\theta^f_{\mathrm{eff}} = [1 - \mathrm{Re}(g^f_{\mathrm{V}}/g^f_{\mathrm{A}})]/(4|Q_f|)$.

Note that, in general, these are not true observables, but so-called "pseudo-observables," since the effects of QED and QCD radiation have been removed in the definition of G_{F}, $\sin^2\theta^f_{\mathrm{eff}}$ and σ^0_{had}, see, for example, Ref. [3, 4].

Table I shows a selection of important pseudo-observations, together with the current experimental and theoretical uncertainties. It is important to emphasize that the theory errors are estimates based on experience and several well-motivated but somewhat ambigious principles, so that the true magnitude of the missing higher orders could well be larger [5]. Thus, it is generally desirable to have a situation where the theory errors are subdominant compared to the experimental errors.

Over the course of more than 30 years, many groups have contributed to the calculation of SM corrections to these quantities. One-loop corrections have been known for a long time [7]. They have been supplemented by two-loop QCD corrections [8] and partial three- and four-loop corrections of the order of $\mathcal{O}(\alpha_t\alpha_{\mathrm{s}}^2)$ [9], $\mathcal{O}(\alpha_t\alpha_{\mathrm{s}}^3)$ [10], $\mathcal{O}(\alpha_t^2\alpha_{\mathrm{s}})$ and $\mathcal{O}(\alpha_t^3)$ [11], where $\alpha_t = y_t^2/(4\pi)$ and y_t is the top Yukawa coupling. Furthermore, fermionic electroweak two-loop corrections (*i.e.* from diagrams with closed fermion loops) are known for all relevant quantites [6, 12, 13], but full two-loop corrections have been completed so far only for M_W [14], $\sin^2\theta^\ell_{\mathrm{eff}}$ [15] and $\sin^2\theta^b_{\mathrm{eff}}$ [16].

TABLE I

Selected electroweak precision pseudo-observables, together with the current experimental uncertainties [1] and theory error estimates [5, 6]. The last column lists the most important missing orders for the theory error estimate.

	Experiment	Theory error	Main source
M_W	80.385 ± 0.015 MeV	4 MeV	α^3, $\alpha^2\alpha_\mathrm{s}$
Γ_Z	2495.2 ± 2.3 MeV	0.5 MeV	α^2_bos, α^3, $\alpha^2\alpha_\mathrm{s}$, $\alpha\alpha^2_\mathrm{s}$
σ^0_had	41540 ± 37 pb	6 pb	α^2_bos, α^3, $\alpha^2\alpha_\mathrm{s}$
$R_b \equiv \Gamma^b_Z/\Gamma^\mathrm{had}_Z$	0.21629 ± 0.00066	0.00015	α^2_bos, α^3, $\alpha^2\alpha_\mathrm{s}$
$\sin^2\theta^\ell_\mathrm{eff}$	0.23153 ± 0.00016	4.5×10^{-5}	α^3, $\alpha^2\alpha_\mathrm{s}$

Even this impressive body of work will not be sufficient for the level of precision of several proposed e^+e^- colliders, which could run at a center-of-mass energy of about 91 GeV to significantly improve the constraints on electroweak precision observables. These are:

— The GigaZ option of the International Linear Collider (ILC) [17], which is planned to have polarized e^- and e^+ beams and accumulate more than 50 fb^{-1} of data near the Z pole;

— The Future Circular Collider (FCC-ee) [18, 19], which may be able to accumulate about 30 ab^{-1} near the Z pole at each of two detectors;

— The Circular Electron–Positron Collider (CEPC) [20], which like FCC-ee has a ring collider design but a lower integrated target luminosity of 150 fb^{-1} at two detectors.

Owing to the large sample statistics expected at these machines, a much more precise experimental determination of the pseudo-observables in Table I will be possible. The estimated measurement uncertainties are summarized in Table II.

TABLE II

Projected experimental and theoretical uncertainties for some electroweak precision pseudo-observables.

	Measurement error			Intrinsic theory	
	ILC	CEPC	FCC-ee	Current	Future†
M_W [MeV]	3–4	3	1	4	1
Γ_Z [MeV]	0.8	0.5	0.1	0.5	0.2
R_b [10^{-5}]	14	17	6	15	7
$\sin^2\theta^\ell_\mathrm{eff}$	1	2.3	0.6	4.5	1.5

As evident from the table, the currently available theoretical calculations for the SM predictions will not be sufficient in this case. Instead, at least a major part of the three-loop contributions and partial four-loop corrections should be computed to reduce the impact of theory errors. The last column in Table II shows an estimate of the achievable precision if the complete two-loop and $\mathcal{O}(\alpha\alpha_s^2)$ corrections, the fermionic $\mathcal{O}(\alpha^2\alpha_s)$ and $\mathcal{O}(\alpha^3)$ corrections, and the leading four-loop corrections in the large-m_t limit will become available. This estimate is based on an extrapolation from the known orders of the perturbation series, assuming that is approximately behaves as a geometric series [21].

2. Bosonic $\mathcal{O}(\alpha^2)$ corrections

As mentioned in the previous section, the two-loop corrections from diagrams *without* closed fermion loop (henceforth called "bosonic") have not yet been computed for most of the relevant pseudo-observables. Their evaluation begins with several familiar steps that need to be carried out for any loop project and typically heavily rely on (partially) automated computer software tools. In our case, these are the following:

Diagrams and amplitudes are generated automatically with the program FeynArts 3 [22]. The Lorentz and Dirac algebra is performed within the framework of Mathematica [23]. UV divergences are absorbed using on-shell renormalization, with the necessary counterterms defined in Ref. [24]. The electroweak pseudo-observables have been defined such that IR divergences are either absent or can be easily factorized. However, individual diagrams and loop integrals can still be IR divergent, so that a regulator is required, for which we use dimensional regularization.

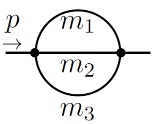
In addition to these standard steps, one needs to evaluate two-loop vertex integrals with up to four independent mass and momentum scales. It is impractical to attempt to solve these integrals analytically, since this would lead to unwieldly expressions and the requirement to define new basis functions. On the other hand, numerical methods are not limited in the number of independent scales, but it is non-trivial to reach sufficient numerical precision to obtain reliable results.

In our collaboration, several numerical techniques have been used to obtain at least two independent evaluations for each two-loop integral. No tensor reduction has been performed, but instead integrals with non-trivial numerator structures are performed directly. For the bosonic two-loop $Z \to f\bar{f}$ corrections, this leads to $\mathcal{O}(1000)$ different integrals, many of which have not been computed before either numerically or analytically.

Two-loop integrals with sub-loop self-energy bubble can be efficiently evaluated using the dispersion-relation technique described in Refs. [25]. More general vertex integrals have been computed with the following two approaches: (a) sector decomposition and (b) Mellin–Barnes (MB) representations. Both methods offer an automated procedure for isolating UV and IR divergencies.

A challenge for both approaches is the numerical stability for Minkowskian kinematics, when one may encounter spurious singularities related to thresholds. In sector decomposition [26] this is addressed through a complex contour deformation of the Feynman parameter integrals [27], which is implemented in several public packages, such as Fiesta [28] and SecDec/pySecDec [29, 30]. Nevertheless, one may encounter numerical stability problems for particular ratios of masses and momenta or for pinched contours [31].

In the MB approach, threshold singularities are reflected by bad convergence behaviour of the MB integrals. For example, the basic two-loop sunset integral can be cast into the MB representation



$$
\begin{aligned}
= \frac{-1}{(2\pi i)^3} \int \mathrm{d}z_1 \mathrm{d}z_2 \mathrm{d}z_3 \left(m_1^2\right)^{-\varepsilon - z_1 - z_2} \left(m_2^2\right)^{z_2} \left(m_3^2\right)^{1-\varepsilon+z_1-z_3} \\
\times \left(-p^2\right)^{z_3} \Gamma(-z_2)\Gamma(-z_3)\Gamma(1+z_1+z_2)\Gamma(z_3-z_1) \\
\times \frac{\Gamma(1-\varepsilon-z_2)\Gamma(\varepsilon+z_1+z_2)\Gamma(\varepsilon-1-z_1+z_3)}{\Gamma(2-\varepsilon+z_3)} \,.
\end{aligned}
$$

Here, the integrations are along contours parallel to the imaginary axis. The p^2-dependent term then becomes

$$
\left(-p^2\right)^{z_3} = \left(p^2\right)^{z_3} \, e^{-i\pi \, \mathrm{Re}\, z_3} \, e^{\pi \, \mathrm{Im}\, z_3} \,, \tag{1}
$$

where the last term blows up for $\mathrm{Im} z_3 \to \infty$. One can mitigate this behaviour by rotating the contour in the complex plane [32]. In this case, the z_3 integration may be parametrized as $z_3 = z_3^0 + (a+ib)t$, where z_3^0, a and b are real constants and t runs from $-\infty$ to ∞. The expression

$$
\left(-p^2\right)^{z_3} = \left(p^2\right)^{z_3^0 + ibt} \, e^{-i\pi\left(z_3^0 + at\right)} \, e^{at \log p^2 + \pi bt} \tag{2}
$$

then contains two terms in the last exponential, which may be arranged to compensate each other through suitable choices of a and b. However, there are cases where this method does not work, for example, if $p^2 = m_1^2 = m_2^2 = m_3^2$, in which case all power terms with exponent z_3 cancel except for $(-1)^{z_3}$.

In this case, however, one can alternatively apply shifts of the integration contour(s) parallel to the real axis [4, 16, 33, 34]. Whichever poles are crossed by this contour, shift need to be accounted for by explicitly adding back their residues. One can oftentimes arrange the direction of the shift of each variable of the multi-dimensional MB integral, such that the remaining MB integral is numerically of much smaller magnitude than the original one, thus reducing the demand for numerical precision of the integration. In addition, the shifts can improve the asymptotic behaviour of the integrand, since the rate of decay of a gamma function, $\Gamma(x+iy)$, for $|y| \to \infty$ depends on the value of x.

The generation of MB integrals from a Feynman integral and the isolation of UV and IR divergencies can be performed automatically with the help of the computer algebra tools MB [35], MBresolve [36], barnesroutines [37], Ambre [38] and PlanarityTest [39]. These programs can be used both for planar and non-planar two-loop topologies. The contour rotation and shift techniques are applied by a new code MBnumerics [40].

By comparing SecDec 3 with MBnumerics, we find that the MB method with contour shift oftentimes yield much higher precision with less computing time than sector decomposition. However, the availability of two different methods is still important for independent cross-checks. Furthermore, while SecDec is highly automatized, the application of MBnumerics is automatic for certain classes of integrals, but requires additional user input for adaptation to new integral classes.

These numerical methods have been applied recently to the calculation of the bosonic two-loop contributions to the $Z \to b\bar{b}$ vertex [16]. The correction to the effective weak mixing angle for this coupling, $\sin^2\theta^b_{\rm eff}$, was found to be -0.22×10^{-4}, which is of the typical size expected for an electroweak NNLO effects. While this is small compared to the experimental uncertainty for this quantity (0.016) [41], it is noteworthy that the bosonic two-loop contributions are not much smaller than the fermionic ones (0.86×10^{-4}) [13]. The detailed results for the bosonic two-loop corrections to $\sin^2\theta^b_{\rm eff}$ have been published in Ref. [16] in terms of a simple fit formula that captures the dependence on the input parameters M_W, M_H and m_t.

The calculation of the corresponding corrections for the $Z \to f\bar{f}$ vertices, where f stands for leptons or first- and second-generation quarks, is currently underway [42]. This will also allow one to determine the bosonic corrections to branching ratios and the total Z-peak cross section.

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