ISOSPIN ASYMMETRY AND NEUTRON STAR PROPERTIES*

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Asymmetric nuclear matter is studied within the relativistic mean field approach. Models with the ω–ρ and σ–ρ cross-interactions, through their remarkable ability to modify the density dependence of the symmetry energy, have been used to analyse the saturation properties of asymmetric nuclear matter.

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1. Introduction

Due to the absence of adequate nuclear physics theory and of experiments, at densities that characterise the interior of a neutron star, the understanding of the nuclear matter equation of state, related to the description of nuclear structures, is still insufficient to correctly solve essential problems in both nuclear physics and astrophysics. Thus, theoretical description of infinite asymmetric nuclear matter and finite nuclear systems bases on models that meet the results of low-energy scattering or saturation properties of nuclear forces. Several theoretical approaches have been developed. It is worth to mention models based on the relativistic Dirac–Brueckner–Hartree–Fock theory (relativistic extension of Brueckner–Hartree–Fock approach [1]) [2, 3] or relativistic quantum hadrodynamics (QHD) [4, 5]. Extensions of the original QHDI model, in which baryons interact by exchange of scalar–isoscalar σ and vector–isoscalar ω mesons, includes additionally the vector–isovector meson ρ [6].

The construction of an acceptable neutron star model requires extrapolation of the available experimental results to higher densities and isospin asymmetry. In such a case, it is expected to find qualitative effects of details

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of the model on the equation of state. There is also possibility to construct models that include different exotic forms of matter [7]. This involves assumptions concerning the general structure of Standard Model and suggests many types of couplings [8, 9]. As neutron star matter is highly asymmetric, there is a need for a detailed description of the isospin-dependent properties of neutron star matter and the neutron star itself. The purpose of this paper is to analyse the properties of nuclear matter and neutron star matter in a wide range of isospin asymmetry. This was done based on a model that has an extended isovector meson sector with the ω–ρ and σ–ρ cross-interactions [10–13].

2. Isospin asymmetric nuclear matter

Nuclear matter equation of state (EoS) considered as the binding energy per nucleon is a function of baryon density $n_b = n_n + n_p$ and the isospin asymmetry δ_a, which refers to the relative neutron excess

$$\epsilon(n_b, \delta_a) = \frac{\mathcal{E}}{n_b} - m_N\,, \qquad \delta_a = \frac{n_n - n_p}{n_n + n_p}\,, \tag{1}$$

where $\mathcal{E}$ denotes the total energy of the system. Neutron (n_n) and proton (n_p) number densities are related to their Fermi momentum k_{F} by the following relation:

$$n_N = \frac{1}{3\pi^2} k^3_{\mathrm{F},N}\,, \qquad N = n, p\,. \tag{2}$$

The mathematical approach that involves the Taylor series approximation for the function which describes such EoS is commonly considered as an effective method to analyse properties of matter under extreme conditions of density and neutron–proton asymmetry

$$\epsilon(n_b, \delta_a) = \epsilon(n_b, \delta_a = 0) + S_2(n_b)\delta_a^2 + S_4(n_b)\delta_a^4 + \ldots \tag{3}$$

Subsequent terms of expansion (3) represent the binding energy of symmetric nuclear matter $\epsilon(n_b, \delta_a = 0)$ and the second and forth order coefficients defined as

$$S_2(n_b) = \frac{1}{2!}\frac{\partial^2 \epsilon(n_b, \delta_a)}{\partial \delta_a^2}\bigg|_{\delta_a=0}\,, \qquad S_4(n_b) = \frac{1}{4!}\frac{\partial^4 \epsilon(n_b, \delta_a)}{\partial \delta_a^4}\bigg|_{\delta_a=0}\,. \tag{4}$$

The representation of $\epsilon(n_b, \delta_a)$ by series (3) is directly related to the problem of finding bounds on this approximation. Analysis that were carried out basing on different theoretical approaches indicate the dominant role of the

$S_2(n_b)$ term at least in the vicinity of the saturation point $(n_b = n_0, \delta_a = 0)$. However, at higher densities and large value of the asymmetry δ_a, the contribution from the quartic term $S_4(n_b)$ should be taken into account. Leaving only terms to the second order in isospin asymmetry, the well-known empirical parabolic approximation is obtained. This enables to divide the EoS of nuclear matter $\epsilon(n_b, \delta_a)$ into two parts — the first that is the energy of symmetric nuclear matter $\epsilon(n_b, \delta_a = 0) \equiv \epsilon(n_b)$ and the second, isospin-dependent one. The latter part encodes information about the symmetry energy whose conventional definition is given by the equation which defines $S_2(n_b)$. Subsequently, the functions $\epsilon(n_b, \delta_a = 0)$ and $S_2(n_b)$ have been expanded in a Taylor series around the equilibrium density n_0 and the following expressions can be obtained:

$$\epsilon(n_b, \delta_a = 0) = \epsilon(n_0) + \frac{1}{2!} 9n_0^2 \left.\frac{\partial^2 \epsilon(n_b)}{\partial n_b^2}\right|_{n_0} u^2 + \ldots \,, \tag{5}$$

$$S_2(n_b) = S_2(n_0) + 3n_0 \left.\frac{\partial S_2(n_b)}{\partial n_b}\right|_{n_0} u + \frac{1}{2!} 9n_0^2 \left.\frac{\partial^2 S_2(n_b)}{\partial n_b^2}\right|_{n_0} u^2 + \ldots \,, \tag{6}$$

where $u = (n_b - n_0)/3n_0$. All derivatives are evaluated at the point $(n_0, 0)$. This point denotes the equilibrium state of isospin-symmetric nuclear matter with minimum energy per nucleon and is characterized by the condition

$$\left.\frac{\partial \epsilon(n_b, \delta_a = 0)}{\partial n_b}\right|_{n_0} = 0 \,. \tag{7}$$

Thus, the linear term in the Taylor expansion (5) vanishes. Equations (3), (5) and (6) can be combined to the following approximated form of the EoS:

$$\epsilon(n_b, \delta_a) = \epsilon(n_0) + E_{\mathrm{sym}}(n_0)\, \delta_a^2 + L \delta_a^2 u + \frac{1}{2!} \left(K_0 + K_{\mathrm{sym}} \delta_a^2\right) u^2 \,. \tag{8}$$

The above equation expresses the nuclear matter EoS through a series of coefficients: binding energy of symmetric nuclear matter $\epsilon(n_0)$, incompressibility defined as

$$K_0 = 9n_0^2 \left.\frac{\mathrm{d}^2 \epsilon(n_b)}{\mathrm{d} n_b^2}\right|_{n_b = n_0} \tag{9}$$

and parameters, which characterize the isospin-dependent part of the EoS. These parameters are: the symmetry energy coefficient $S_2(n_0) \equiv E_{\mathrm{sym}}(n_0)$, the slope (L) and curvature (K_{sym}) of the symmetry energy. The last two parameters are given by the following equations:

$$L = 3n_0 \left.\frac{\mathrm{d} E_{\mathrm{sym}}(n_b)}{\mathrm{d} n_b}\right|_{n_b = n_0} \,, \tag{10}$$

$$K_{\rm sym} = 9n_0^2 \frac{{\rm d}^2 E_{\rm sym}(n_b)}{{\rm d}n_b^2}\bigg|_{n_b=n_0} . \tag{11}$$

Thus, having obtained the equation of state, one can calculate each individual term that enters formula (8).

3. The model

Constituents of this model are neutrons and protons interacting through the exchange of scalar σ and vector ω, ρ meson fields. The Lagrangian function $\mathcal{L}_0$ represents: nucleons, mesons, interaction of scalar and vector mesons with nucleons, and scalar and vector mesons self-interactions

$$\begin{aligned}\mathcal{L}_0 \;=\; & \sum_N \bar{\psi}_N \left(\gamma^\mu i D_\mu - m_{{\rm eff},N}\right)\psi_N + \tfrac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \tfrac{1}{2}m_\sigma^2\sigma^2 \\ & -\tfrac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} - \tfrac{1}{4}\boldsymbol{R}^{\mu\nu}\boldsymbol{R}_{\mu\nu} + \tfrac{1}{2}m_\omega^2\left(\omega^\mu\omega_\mu\right) + \tfrac{1}{2}m_\rho^2\left(\rho^{\mu a}\rho^a_\mu\right) \\ & +\tfrac{1}{4}c_3\left(\omega^\mu\omega_\mu\right)^2 - \tfrac{1}{3}g_2\sigma^3 + \tfrac{1}{4}g_3\sigma^4 .\end{aligned} \tag{12}$$

The covariant derivative D_μ and nucleon effective mass $m_{{\rm eff},N}$ are given by equations: $D_\mu = \partial_\mu + ig_\omega\omega_\mu + ig_\rho I_N\rho^a_\mu$ (I_N denotes isospin of nucleon), $m_{{\rm eff},N} = m_N - g_\sigma\sigma$, whereas $\Omega_{\mu\nu}$, $\boldsymbol{R}_{\mu\nu}$ are field strength tensors of the ω and ρ mesons. The part of the Lagrangian function that in this model supplements the isospin-dependent sector is considered separately. It includes the cross-interaction between ω–ρ and σ–ρ mesons

$$\mathcal{L}_{\rm iso} = \Lambda_{\rm V}(g_\omega g_\rho)^2(\omega^\mu\omega_\mu)(\rho^{\mu a}\rho^a_\mu) + \Lambda_4\left(g_\sigma g_\rho\right)^2\sigma^2\left(\rho^{\mu a}\rho^a_\mu\right) . \tag{13}$$

Thus, the total Lagrangian function, which describes the dynamic of the system $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\rm iso}$, is the basis for calculating the EoS. The solution of the equations of motion obtained in the mean field approximation in which operators of meson fields are replaced by their expectation values: s_0, w_0, r_0, serves as an input for the calculation of the EoS. Using the energy-momentum tensor $T^{\mu\nu}$, the energy density and pressure are given by the following equations:

$$\begin{aligned}\langle T_{00}\rangle \;\equiv\; & \mathcal{E} = \tfrac{1}{2}m_\omega^2 w_0^2 + \tfrac{1}{2}m_\rho^2 r_0^2 + \tfrac{1}{2}m_\sigma^2 s_0^2 + \tfrac{3}{4}c_3 w_0^4 + 3\Lambda_{\rm V}(g_\omega g_\rho)^2(w_0 r_0)^2 \\ & +\Lambda_4(g_\sigma g_\rho)^2(s_0 r_0)^2 + \tfrac{1}{3}g_2 s_0^3 + \tfrac{1}{4}g_3 s_0^4 + \mathcal{E}_N ,\end{aligned} \tag{14}$$

$$\begin{aligned}\tfrac{1}{3}\langle T_{ii}\rangle \;\equiv\; & P = \tfrac{1}{2}m_\omega^2 w_0^2 + \tfrac{1}{2}m_\rho^2 r_0^2 - \tfrac{1}{2}m_\sigma^2 s_0^2 + \tfrac{1}{4}c_3 w_0^4 + \Lambda_{\rm V}(g_\omega g_\rho)^2(w_0 r_0)^2 \\ & +\Lambda_4(g_\sigma g_\rho)^2(s_0 r_0)^2 - \tfrac{1}{3}g_2 s_0^3 - \tfrac{1}{4}g_3 s_0^4 + P_N ,\end{aligned} \tag{15}$$

where $\mathcal{E}_N$ and P_N are the kinetic parts of the energy density and pressure defined as

$$\mathcal{E}_N = \sum_{N=n,p} \frac{2}{(2\pi)^3} \int\limits_0^{k_{\mathrm{F},N}} k^2 \mathrm{d}k \sqrt{k^2 + m^2_{\mathrm{eff},N}} \tag{16}$$

and

$$P_N = \sum_{N=n,p} \frac{1}{3\pi^2} \int\limits_0^{k_{\mathrm{F},N}} \mathrm{d}k \frac{k^4}{\sqrt{k^2 + m^2_{\mathrm{eff},N}}} \,. \tag{17}$$

Calculations in this paper were done with the use of the TM1 [14] parameter set (Table I).

TABLE I

TM1 parameter set.

$m_\sigma = 511.2$ MeV	$m_\omega = 783$ MeV	$m_\rho = 770$ MeV
$g_\sigma = 10.029$	$g_\omega = 12.614$	$g_\rho = 9.2644$
$g_2 = 7.2325$ fm^{-1}	$g_3 = 0.6183$	$c_3 = 71.0375$

4. Characteristics of asymmetric nuclear matter

Very important feature of the EoS of nuclear matter is the fact that the energy per nucleon, at a given density or Fermi momentum, reaches a minimum. Considering the symmetric nuclear matter, it saturates at density $n_0 = 0.145$ fm^{-3} with the binding energy -16.26 MeV, the incompressibility coefficient $K_0 = 281.16$ MeV, the symmetry energy coefficient $E_{\mathrm{sym}} = 36.89$ MeV, and the symmetry energy slope $L = 110.79$ MeV [15]. These properties of nuclear matter have been obtained for the TM1 parameter set. In order to modify the density dependence of the symmetry energy and to obtain models with softer symmetry energy and lower value of the slope L, Horowitz and Piekarewicz [10] introduced additional ω–ρ and σ–ρ cross-interactions. Such an extended isovector sector is characterised by pairs of parameters $(\Lambda_{\mathrm{V}}, g_\rho)$ or (Λ_4, g_ρ). These pairs of parameters have been selected so as to reproduce $E_{\mathrm{sym}}(n_b) = 25.68$ MeV at the average baryon density n_b, corresponding to $k_{\mathrm{F}} = 1.15$ fm^{-1}, for which the binding energy of ^{208}Pb is reproduced [10, 16]. The values of parameters Λ_{V} and Λ_4 chosen for the purpose of this work are summarised in Table II.

TABLE II

Parameters of the extended isovector sector. These parameters are in the experimentally acceptable range of the binding energy per particle of ^{208}Pb. The last column includes the value of the symmetry energy slope calculated for the adjusted values of $(\Lambda_{\rm V}, g_\rho)$ and (Λ_4, g_ρ).

$\Lambda_{\rm V}$	Λ_4	g_ρ	L [MeV]
0.0	0.0	9.2644	110.79
0.03	0.0	11.0964	55.76
0.0	0.03	13.5927	46.47

The inclusion of the ω–ρ and σ–ρ terms alters the relation for the symmetry energy which now is expressed in terms of effective ρ meson mass

$$E_{\rm sym}(n_b) = \frac{k_{\rm F}^2}{6\sqrt{k_{\rm F}^2 + m_{{\rm eff},N}^2}} + \frac{g_\rho^2}{12\pi^2}\frac{k_{\rm F}^3}{m_{{\rm eff},\rho}^2}\,, \tag{18}$$

where

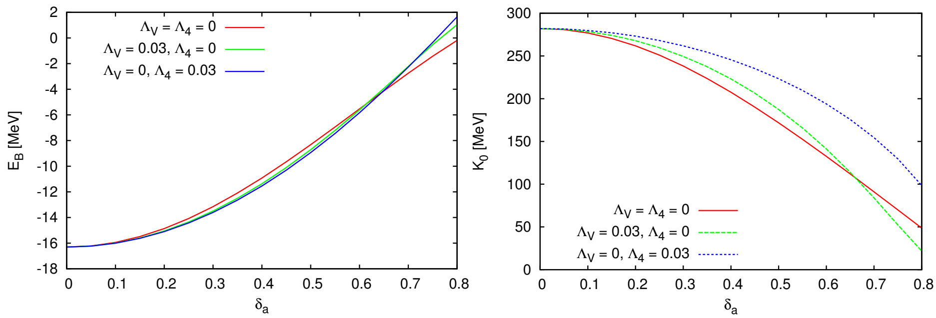
$$m_{{\rm eff},\rho}^2 = m_\rho^2 + 2\left(\Lambda_{\rm V}(g_\omega g_\rho)^2 w_0^2 + \Lambda_4(g_\sigma g_\rho)^2 s_0^2\right)\,. \tag{19}$$

Finding constraints on the properties of asymmetric nuclear matter is very important due to the correlations between coefficients of equation (8) and neutron star parameters. Analysing the properties of asymmetric nuclear matter, the coefficients that characterise density dependence of the EoS (8) should be specified. These coefficients calculated at equilibrium density n_0^a depend on the asymmetry parameter δ_a. The equilibrium density itself also depends on δ_a and for more asymmetric matter is shifted to lower density.

The main parameter that specifies the EoS is the binding energy. In Fig. 1 (a), the binding energy at equilibrium density n_0^a is presented. Increasing the asymmetry δ_a, which is equivalent to the increasing number of neutrons in nuclear matter, the binding energy for the fixed value of δ_a has been calculated. In the result weakly bound matter is obtained.

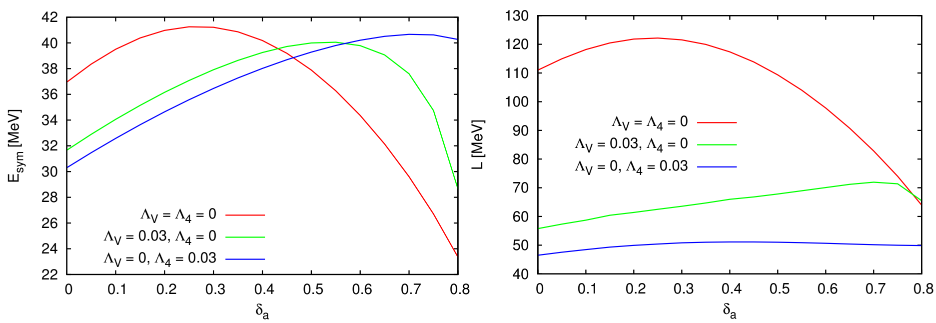
The isospin dependence of incompressibility coefficient $K_0(\delta_a)$ is presented in Fig. 1 (b). The incompressibility decreases with the increasing isospin asymmetry. The asymmetry dependence is rather weak for lower value of δ_a and becoming stronger for more neutron reach matter.

Figures 2 (a) and (b) show changes in the main parameters characterizing the symmetry energy, caused by both the isospin asymmetry and the modification of the model by virtue of nonlinear ω–ρ and σ–ρ couplings. The symmetry energy coefficient $E_{\rm sym}$ for lower values of asymmetry increases and then, after reaching a maximum, decreases. The inclusion of $\Lambda_{\rm V}$ and Λ_4 parameters shifts the maxima towards higher value of the isospin asymmetry. The symmetry energy slope L changes with increasing asymmetry



(a) Binding energy (b) The incompressibility coefficient K_{sym}

Fig. 1. The left panel: the minimized energy per nucleon as a function of the asymmetry parameter δ_a. The right panel: the isospin-dependent incompressibility at equilibrium density.

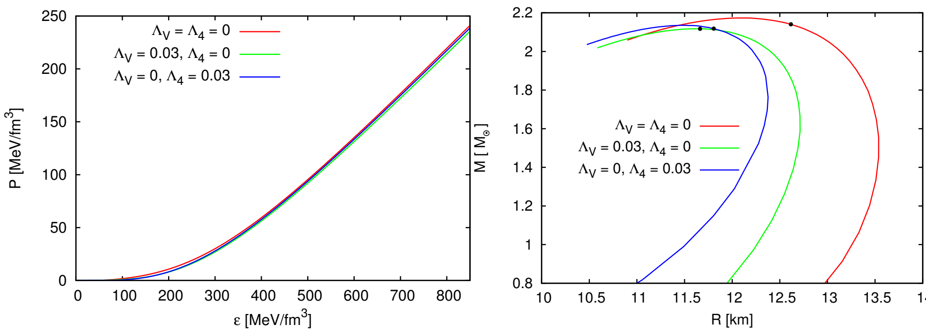


(a) Symmetry energy (b) The slope of the symmetry energy

Fig. 2. The symmetry energy and the slope calculated at equilibrium density as a function of the asymmetry parameter δ_a. Results obtained for different values of Λ_{V} and Λ_4 parameters are included.

in the same way as the symmetry energy. However, the most significant modifications are due to the presence of ω–ρ and σ–ρ couplings. This lowers considerably the slope and causes softening of the symmetry energy. The isospin dependence of the properties of nuclear matter is of particular importance in astrophysical applications and especially in modelling neutron star interiors. In this case, variations in the isospin asymmetry is related to the change of the model, specifically to the distinction between parametrizations: ordinary TM1 parameter set and TM1 with the extended isovector sector. EoSs for neutron stars have been constructed basing on the presented models of asymmetric nuclear matter. This allows one to analyse how the properties of asymmetric nuclear matter translate into modification of the properties of neutron star matter. In Fig. 3 (a), EoSs obtained for TM1 parametrization are depicted. The inclusion of ω–ρ and σ–ρ couplings increases the asymmetry of the system and thus leads to softer EoSs. The softening is not very impressive.

Changes of the EoSs are reflected in the mass–radius relations which are shown in Fig. 3 (b). Models with softer EoSs give as a results lower values of the maximum mass. Dots on the individual M–R curves show the behaviour of selected neutron star configurations which have the same baryon number. Thus, the differences in masses are due to the difference in the asymmetry of matter and through this in the symmetry energy for these particular neutron star configurations.

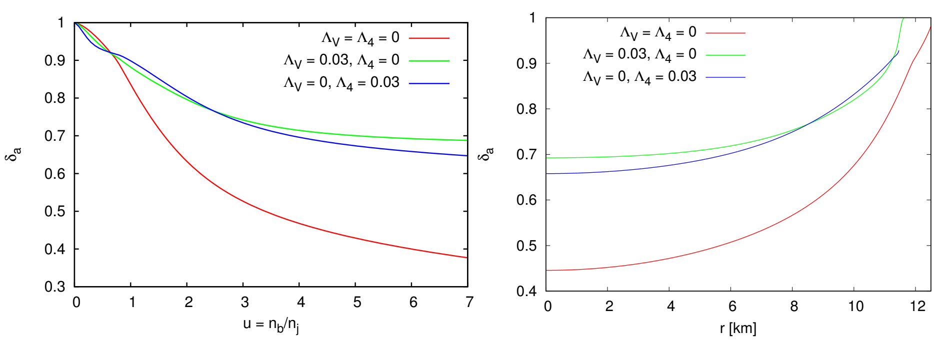


(a) Neutron star matter equations of state

(b) Mass–radius relations

Fig. 3. EoSs and M–R relations obtained for different values of parameters $\Lambda_{\rm V}$ and Λ_4.

The density dependence of the isospin asymmetry of neutron star matter, for the considered models, is given in Fig. 4 (a). This result is consistent with the behaviour of the incompressibility of asymmetric nuclear matter. In Fig. 4 (b), the radial dependence of the isospin asymmetry for the selected neutron star configurations are presented. In general, models with $\Lambda_{\rm V}$ and Λ_4 different from zero lead to more asymmetric neutron star matter, especially in the core of a neutron star.

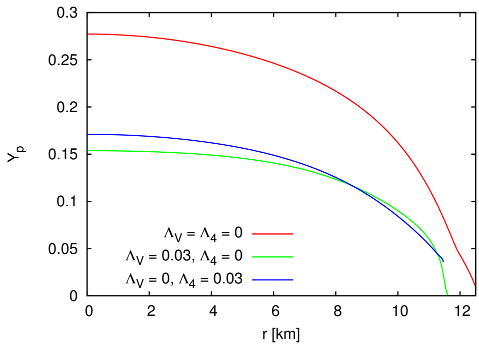


(a) Density dependence of the isospin asymmetry δ_a

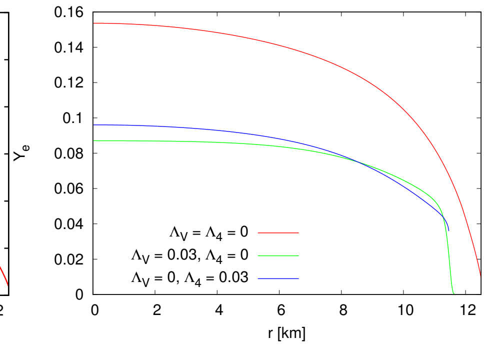
(b) Radial dependence of the isospin asymmetry δ_a for selected neutron star configurations, marked on the M–R diagram

Fig. 4. The asymmetry parameter δ_a as a function of baryon density (left panel) or radius (right panel) calculated for different values of $\Lambda_{\rm V}$ and Λ_4 parameters.

The isospin asymmetry includes information about proton concentration in neutron star matter and through the charge neutrality condition about the electron concentration. Both, proton and electron relative concentrations calculated for different values of $\Lambda_{\rm V}$ and Λ_4 are shown in Fig. 5 (a) and Fig. 5 (b). These figures depict the radial dependence of Y_p and Y_e calculated for the selected neutron star configurations. The obtained results indicate that models with the extended isovector sector lead to considerable lower proton and electron concentrations in neutron star interiors.



(a) Radial dependence of proton concentrations



(b) Radial dependence of electron concentrations

Fig. 5. The relative concentrations of protons $Y_p = n_p/n_b$ and electrons $Y_e = n_e/n_b$ calculated for different values of $\Lambda_{\rm V}$ and Λ_4 parameters. Results obtained for the selected neutron star configurations (dots on the M–R curves).

5. Conclusions

Neutron star matter is characterized by a high value of asymmetry, so it is very important to understand how the properties of nuclear matter depend on the value of isospin asymmetry. The results obtained in this paper are consistent with solutions find in the paper by Chen *et al.* [17] where the analytical expressions for the saturation properties of asymmetric nuclear matter have been derived. Isospin-dependent modifications of the properties of nuclear matter such as the binding energy, incompressibility, symmetry energy and the symmetry energy slope influence neutron star matter EoS and the structure of a neutron star.

REFERENCES

[1] R.L. Becker, M.R. Patterson, *Nucl. Phys. A* **178**, 88 (1971).

[2] M.R. Anastasio, L.S. Celenza, W.S. Pong, C.M. Shakin, *Phys. Rep.* **100**, 327 (1983).

[3] R. Machleidt, *Adv. Nucl. Phys.* **19**, 189 (1989).

[4] B.D. Serot, J.D. Walecka, *Adv. Nucl. Phys.* **16**, 1 (1986).

[5] B.D. Serot, J.D. Walecka, *Int. J. Mod. Phys. E* **6**, 515 (1997).

[6] J. Boguta, A.R. Bodmer, *Nucl. Phys. A* **292**, 413 (1977).

[7] I. Bednarek, R. Manka, *J. Phys. G* **36**, 095201 (2009).

[8] J. Sładkowski, M. Zralek, *Phys. Rev. D* **45**, 1701 (1992).

[9] J. Sładkowski, *Int. J. Theor. Phys.* **35**, 2075 (1996).

[10] C.J. Horowitz, J. Piekarewicz, *Phys. Rev. Lett.* **86**, 5647 (2001).

[11] I. Bednarek, R. Manka, *Int. J. Mod. Phys. D* **10**, 607 (2001).

[12] I. Bednarek, M. Pienkos, J. Sladkowski, J. Syska, *Mon. Not. R. Astron. Soc.* **468**, 1931 (2017).

[13] S.J. Gmuca, *J. Phys. G* **17**, 1115 (1991).

[14] Y. Sugahara, H. Toki, *Progr. Theor. Phys.* **92**, 803 (1994).

[15] M. Dutra *et al.*, *Phys. Rev. C* **90**, 055203 (2014).

[16] B.K. Sharma, S. Pal, *Phys. Lett. B* **682**, 23 (2009).

[17] L.-W. Chen *et al.*, *Phys. Rev. C* **80**, 014322 (2009).