SOME PROPERTIES OF THE GENERIC 2HDM IN THE ALIGNMENT LIMIT*

Bohdan Grzadkowski

Faculty of Physics, University of Warsaw
Pasteura 5, 02-093 Warszawa, Poland

Odd Magne Ogreid

Western Norway University of Applied Sciences
Postboks 7030, 5020 Bergen, Norway

Per Osland

Department of Physics and Technology, University of Bergen
Postboks 7803, 5020 Bergen, Norway

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We discuss various properties of the generic two-Higgs-doublet extension of the Standard Model focusing on the region of parameter space known as the alignment limit. We emphasize that in the alignment limit in order to retain a possibility of CP violation in the scalar potential, one has to relax the traditional $\mathbb{Z}$ symmetry introduced to prevent flavour changing neutral currents at the tree level in Yukawa couplings. We point out various correlations between properties of non-standard Higgs bosons H_2 and H_3 present in the model and suggest measurements at the LHC that can test the alignment scenario. Spontaneous CP violation in the 2HDM is also discussed in the alignment limit.

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1. Introduction

The Two-Higgs-Doublet Model (2HDM) is an attractive extension of the Standard Model (SM), see [1, 2] and references therein. Its advantages are the extra source of CP violation (CPV) needed for baryogenesis [3], interesting and rich phenomenology that agrees with properties of the observed Higgs boson, and a possibility for flavour-changing neutral interactions in

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the Yukawa couplings at the tree level. In this paper we will discuss a particularly interesting region in the 2HDM parameter space, the so-called alignment limit [4–6] in which the lightest (heavier states might also play that role) Higgs boson has exactly SM couplings to vector bosons and fermions, while other scalars could have masses as low as $\mathcal{O}$(200–400 GeV). Of course, the lightest Higgs boson would have SM-like couplings also in the decoupling limit where all beyond-the-SM states are very heavy. An interesting feature of the 2HDM is that the decoupling is not the only possibility and here we are exploring this option.

The scalar potential of the 2HDM shall be parametrized in the standard fashion

$$\begin{aligned} V(\Phi_1,\Phi_2) &= -\tfrac{1}{2}\left\{m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + \left[m_{12}^2\Phi_1^\dagger\Phi_2 + \text{H.c.}\right]\right\} \\ &+\frac{\lambda_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2 + \lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) \\ &+\tfrac{1}{2}\left[\lambda_5\left(\Phi_1^\dagger\Phi_2\right)^2 + \text{H.c.}\right] + \left\{\left[\lambda_6\left(\Phi_1^\dagger\Phi_1\right) + \lambda_7\left(\Phi_2^\dagger\Phi_2\right)\right]\left(\Phi_1^\dagger\Phi_2\right) + \text{H.c.}\right\}. \end{aligned} \tag{1}$$

Usually, a $\mathbb{Z}_2$ symmetry is imposed on the dimension-4 terms in order to eliminate potentially large flavour-changing neutral currents in the Yukawa couplings. In the present work, we will *not* restrict ourselves by imposing this symmetry, instead we are going to consider the most general scalar potential, keeping also terms that are not allowed by the $\mathbb{Z}_2$ symmetry.

In an arbitrary basis, the vacuum may be complex, and the Higgs doublets can be parameterized as

$$\Phi_j = e^{i\xi_j}\left(\begin{array}{c}\varphi_j^+ \\ (v_j+\eta_j+i\chi_j)/\sqrt{2}\end{array}\right), \qquad j=1,2\,, \tag{2}$$

with the v_j real numbers satisfying $v_1^2+v_2^2=v^2$. The fields η_j and χ_j are real, and the phase difference between the two vevs is given by $\xi\equiv\xi_2-\xi_1$. The vevs may also be written as

$$\langle\Phi_j\rangle = \frac{1}{\sqrt{2}}\left(\begin{array}{c}0 \\ v\hat{v}_j\end{array}\right), \tag{3}$$

where

$$\hat{v}_1 = \frac{v_1}{v}e^{i\xi_1}\,, \qquad \hat{v}_2 = \frac{v_2}{v}e^{i\xi_2}\,. \tag{4}$$

Next, we define orthogonal states

$$\left(\begin{array}{c}G_0 \\ \eta_3\end{array}\right) = \left(\begin{array}{cc}v_1/v & v_2/v \\ -v_2/v & v_1/v\end{array}\right)\left(\begin{array}{c}\chi_1 \\ \chi_2\end{array}\right) \tag{5}$$

and

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} v_1/v & v_2/v \\ -v_2/v & v_1/v \end{pmatrix} \begin{pmatrix} \varphi_1^\pm \\ \varphi_2^\pm \end{pmatrix} . \tag{6}$$

Then G_0 and $G^\pm$ become the massless Goldstone fields, whereas $H^\pm$ are the charged scalars.

The model also contains three neutral scalars, which are linear combinations of the η_i,

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = R \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix} , \tag{7}$$

with the 3×3 orthogonal rotation matrix R satisfying

$$R\mathcal{M}^2 R^{\mathrm{T}} = \mathcal{M}^2_{\mathrm{diag}} = \mathrm{diag}\left(M_1^2, M_2^2, M_3^2\right) , \tag{8}$$

and with $M_1 \leq M_2 \leq M_3$. The rotation matrix R can be parametrized as [7, 8]

$$R = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} = \begin{pmatrix} c_1\, c_2 & s_1\, c_2 & s_2 \\ -(c_1\, s_2\, s_3 + s_1\, c_3) & c_1\, c_3 - s_1\, s_2\, s_3 & c_2\, s_3 \\ -c_1\, s_2\, c_3 + s_1\, s_3 & -(c_1\, s_3 + s_1\, s_2\, c_3) & c_2\, c_3 \end{pmatrix} . \tag{9}$$

Since R is orthogonal, only three of the elements R_{ij} are independent, the rest can be expressed by these through the orthogonality relations. From the potential, one can now derive expressions for the masses of the scalars as well as Feynman rules for scalar interactions. For a general basis as we consider here, these expressions are quite involved and lengthy so, for convenience, we have collected them in Appendix A of [9].

2. Implications of the LHC Higgs signal

Much of our discussion will be phrased in terms of the four physical masses of the model,

$$M_1 \leq M_2 \leq M_3 , \qquad \text{and} \qquad M_{H^\pm} , \tag{10}$$

together with parameters e_i, q_i and q. The e_i parametrize H_iVV couplings [9]

$$e_i \equiv v_1 R_{i1} + v_2 R_{i2} . \tag{11}$$

The q_i are $H_iH^+H^-$ couplings [9]

$$\begin{aligned} q_i \; = \; & \frac{2e_i}{v^2} M_{H^\pm}^2 - \frac{R_{i2}v_1 + R_{i1}v_2}{v_1v_2}\mu^2 + \frac{g_i}{v^2 v_1 v_2} M_i^2 + \frac{R_{i3}v^3}{2v_1v_2}\mathrm{Im}\,\lambda_5 \\ & + \frac{v^2\left(R_{i2}v_1 - R_{i1}v_2\right)}{2v_2^2}\mathrm{Re}\,\lambda_6 - \frac{v^2\left(R_{i2}v_1 - R_{i1}v_2\right)}{2v_1^2}\mathrm{Re}\,\lambda_7 , \end{aligned} \tag{12}$$

where $g_i \equiv v_1^3 R_{i2} + v_2^3 R_{i1}$, while q is the $H^+H^-H^+H^-$ coupling given in [9]. Important parameters are also $f_i \equiv v_1 R_{i2} - v_2 R_{i1} - ivR_{i3}$, they enter gauge couplings that involve Higgs bosons.

2.1. The alignment limit

The Higgs boson observed at the LHC [10, 11] has SM-like couplings to vector bosons. Therefore, we are going to concentrate here on a region of parameter space such that H_1 couples as the SM Higgs boson, *i.e.* the alignment limit defined formally as

$$e_1 = v\,, \qquad e_2 = e_3 = 0\,. \tag{13}$$

In terms of the mixing angles, the alignment means $\alpha_1 = \beta$ and $\alpha_2 = 0$. Other couplings and parameters are also subject of severe constraints in this limit, *e.g.* $f_1 = 0$, $f_2 = if_3 \equiv \tilde{f} = v(c_3 - is_3)$ and

$$\begin{aligned}
q_1 &= \frac{1}{v}\left(2M_{H^\pm}^2 - 2\mu^2 + M_1^2\right)\,,\\
q_2 &= +c_3\left[\frac{\left(c_\beta^2 - s_\beta^2\right)}{vc_\beta s_\beta}\left(M_2^2 - \mu^2\right) + \frac{v}{2s_\beta^2}\mathrm{Re}\,\lambda_6 - \frac{v}{2c_\beta^2}\mathrm{Re}\,\lambda_7\right] + s_3\frac{v}{2c_\beta s_\beta}\mathrm{Im}\,\lambda_5\,,\\
q_3 &= -s_3\left[\frac{\left(c_\beta^2 - s_\beta^2\right)}{vc_\beta s_\beta}\left(M_3^2 - \mu^2\right) + \frac{v}{2s_\beta^2}\mathrm{Re}\,\lambda_6 - \frac{v}{2c_\beta^2}\mathrm{Re}\,\lambda_7\right] + c_3\frac{v}{2c_\beta s_\beta}\mathrm{Im}\,\lambda_5\,.
\end{aligned} \tag{14}$$

A relevant observation at this moment is that properties/couplings of H_2 and H_3 are correlated in the alignment limit.

2.2. CP violation in the alignment limit

Necessary conditions for having CP violation in the 2HDM were found in terms of invariants with respect to weak-basis transformations more than 20 years ago by Lavoura, Silva and Botella [12, 13]. More recently, it has also been addressed by Branco, Rebelo and Silva-Marcos [14], by Gunion and Haber [15], and by Haber and O'Neil [16]. Independent strategies have been presented both in terms of algebraic invariants [17] and geometric quantities [18–21]. Detailed discussions of CP-violating invariants are also contained in [22]. These invariants are analogous to the Jarlskog invariant J [23] describing CP violation induced by the Yukawa couplings in the Standard Model (SM).

Adopting an alternative invariant J_{30} introduced in [9], the following set of invariants could be adopted to parametrize CP violation present in the model

$$\mathrm{Im}\, J_1 = \frac{1}{v^5}\sum_{i,j,k}\epsilon_{ijk}M_i^2 e_i e_k q_j \,, \tag{15}$$

$$\mathrm{Im}\, J_2 = \frac{2e_1e_2e_3}{v^9}\sum_{i,j,k}\epsilon_{ijk}M_i^4 M_k^2 \,, \tag{16}$$

$$\mathrm{Im}\, J_{30} = \frac{1}{v^5}\sum_{i,j,k}\epsilon_{ijk}q_i M_i^2 e_j q_k \,. \tag{17}$$

One can show that in the alignment limit, the invariants reduce to

$$\mathrm{Im}\, J_1 = \mathrm{Im}\, J_2 = 0\,, \qquad \text{and} \qquad \mathrm{Im}\, J_{30} = \frac{q_2 q_3}{v^4}\left(M_3^2 - M_2^2\right) \,. \tag{18}$$

It turns out that in the $\mathbb{Z}_2$ symmetric versions of the 2HDM models, $q_2{\cdot}q_3 = 0$ as a consequence of consistency conditions. An important implication of this observation [9] is that in the alignment limit in order to allow for CP violation in the potential, one has to relax the $\mathbb{Z}_2$ symmetry and consider the most general version of 2HDM.

2.3. "Heavy" Higgs bosons (H_2, H_3, $H^\pm$) in the alignment limit

As has already been mentioned earlier, the advantage of the alignment limit is that the H_1 may couple exactly in the SM manner, while H_2, H_3 and $H^\pm$ could be relatively light with electroweak-scale masses. Since their couplings are correlated in this region of the parameter space, some observables that can test the alignment scenario can be defined. For instance,

$$\frac{\mathrm{BR}(H_3 \to H^+H^-)}{\mathrm{BR}(H_3 \to H_2H_2)} = \sqrt{\frac{M_3^2 - 4M_{H^\pm}^2}{M_3^2 - 4M_2^2}} \,. \tag{19}$$

On the other hand, some couplings vanish so that various processes are forbidden

$$\mathrm{BR}(H_1 \to W^+W^-, ZZ) = \mathrm{BR}(H_{\mathrm{SM}} \to W^+W^-, ZZ)\,, \tag{20}$$

$$\mathrm{BR}(H_{2,3} \to W^+W^-, ZZ, H_1H_1, H_1Z) = 0\,, \tag{21}$$

$$\mathrm{BR}(H_3 \to H_1H_2) = 0 \tag{22}$$

at tree level.

3. Spontaneous CP violation

In this section, we are going to discuss possibilities for spontaneous CPV (SCPV). Conditions for SCPV in terms of scalar potential parameters have been known since the seminal paper of Lee [24], however conditions formulated in terms of measured physical parameters were unknown until recently. It turns out [25] that the following holds.

Theorem 3.1 *Let us assume that the quantity*

$$D = e_1^2 M_2^2 M_3^2 + e_2^2 M_3^2 M_1^2 + e_3^2 M_1^2 M_2^2 \tag{23}$$

is non-zero[1]*. Then, in a charge-conserving general 2HDM, CP is violated spontaneously if and only if the following three statements are satisfied simultaneously*

— *At least one of the three invariants* $\mathrm{Im}\, J_1, \mathrm{Im}\, J_2, \mathrm{Im}\, J_{30}$ *is non-zero,*

— $M_{H^\pm}^2 = \frac{v^2}{2D}\left[e_1 q_1 M_2^2 M_3^2 + e_2 q_2 M_3^2 M_1^2 + e_3 q_3 M_1^2 M_2^2 - M_1^2 M_2^2 M_3^2\right]$,

— $q = \frac{1}{2D}\Big[(e_2 q_3 - e_3 q_2)^2 M_1^2 + (e_3 q_1 - e_1 q_3)^2 M_2^2 + (e_1 q_2 - e_2 q_1)^2 M_3^2 + M_1^2 M_2^2 M_3^2\Big]$.

An important comment is here in order. The above relation involves all the parameters of the scalar potential, so the issue of SCPV may only be resolved if the whole potential is known.

It turns out that in the alignment limit, the above theorem greatly simplifies

$$M_{H^\pm}^2 = \frac{v q_1 - M_1^2}{2}\,, \qquad q = \frac{1}{2}\left(\frac{q_2^2}{M_2^2} + \frac{q_3^2}{M_3^2} + \frac{M_1^2}{v^2}\right)\,. \tag{24}$$

4. Summary

The 2HDM allows for extra sources of CP violation that might be useful to explain baryon asymmetry. In addition, it allows for the alignment limit in which H_1 couples to vector bosons and fermions in exactly the SM manner. It turns out that the alignment limit does not require the other "heavier" scalar bosons to be heavy, they may remain in the range of the electroweak mass scale. As has been shown, if the alignment limit is combined with a $\mathbb{Z}_2$ that forbids FCNC, then CPV is not allowed by the model. Therefore, one has to consider the generic 2HDM ($\lambda_6 \neq 0$ and/or $\lambda_7 \neq 0$) to retain CPV.

[1] In any meaningful model, we demand that all $M_i^2 > 0$ and that at least one e_i is non-zero, implying $D > 0$.

A consequence of that is a possibility of generating FCNC at the tree level. One can define observables that can test the alignment limit at the LHC.

In the alignment limit complicated conditions for spontaneous CP violation simplify so that this scenario could in principle be tested.

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