VECTOR-FERMION DARK MATTER MODEL*

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We present a renormalizable vector-fermion dark matter model, where two or three components of the dark sector are stable and hence constitute the observed dark matter relic density. In particular, our model involves an extension of the Standard Model by a dark $\mathrm{U}(1)_X$ gauge symmetry which includes a dark vector X_μ, and two Majorana fermions, ψ_+ and ψ_-. Moreover, we employ the Higgs mechanism in the dark sector to give masses to dark particles; it also provides a second Higgs, h_2. Depending on the masses of these three dark sector particles $(X_\mu, \psi_\pm)$, two or three of them contribute to the dark matter. We have numerically solved a set of coupled Boltzmann equations describing the evolution of density for different DM components.

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1. Introduction

Dark matter (DM) hypothesis has been justified by numerous observations, *e.g.* effects of gravitational lensing, measurements of rotation curves of the galaxy clusters and observations of the cosmic microwave background anisotropies [1, 2]. These effects can be perfectly explained by the hypothesis of WIMPs, *i.e.* weakly interacting massive particles that should form DM [3]. However, there are several discrepancies between simulations based on various collisionless, one-component WIMP models and observations in the galactic scales, for instance, the 'too-big-to-fail' and the 'cusp-core' problems [4]. Since very different DM masses are needed to solve some of these problems, it is reasonable to consider multi-component DM scenarios. In this work, we consider an extension of the Standard Model (SM) that provides two or three (depending on parameters) massive, neutral, stable particles which can constitute the observed DM density. We also present examples of numerical solutions of a set of Boltzmann equations describing the evolution of dark matter density.

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2. The vector-fermion dark matter model

We consider a simple extension of the SM gauge group by an additional $\mathrm{U}(1)_X$ gauge symmetry that does not act on any SM particle. The dark sector of our model has three new fields: X_μ, a gauge vector of the $\mathrm{U}(1)_X$, a Dirac fermion χ, and a complex scalar S. Charges of χ and S under the action of $\mathrm{U}(1)_X$ are $\frac{1}{2}$ and 1, respectively, while they are neutral under the SM gauge group.

Our vector-fermion DM model is described by the following Lagrangian:

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\mathrm{DM}} + \mathcal{L}_{\mathrm{portal}}\,, \tag{1}$$

where $\mathcal{L}_{\mathrm{SM}}$ is the Standard Model Lagrangian, $\mathcal{L}_{\mathrm{DM}}$ is the dark sector Lagrangian, and the $\mathcal{L}_{\mathrm{portal}}$ is the Higgs portal term, which is responsible for all SM–DM interactions. The dark sector Lagrangian has the following form:

$$\begin{aligned}\mathcal{L}_{\mathrm{DM}} \;=\; & -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \left(\mathcal{D}_\mu S\right)^* \mathcal{D}^\mu S + \mu_S^2|S|^2 - \lambda_S|S|^4 \\ & +\bar{\chi}\left(i\not{D} - m_{\mathrm{D}}\right)\chi - \frac{1}{\sqrt{2}}\left(y_{\mathrm{d}} S^* \chi^T \mathcal{C}\chi + \mathrm{h.c.}\right)\,,\end{aligned} \tag{2}$$

where $\mathcal{F}_{\mu\nu} \equiv \partial_\mu X_\nu - \partial_\nu X_\mu$ is the field-strength tensor, $\mathcal{C} \equiv -i\gamma_2\gamma_0$ denotes charge-conjugation operator (with γ_2 and γ_0 being the gamma matrices) and $\mathcal{D}_\mu \equiv \partial_\mu + i g_x q_x X_\mu$ is the covariant derivative connected with $\mathrm{U}(1)_X$, with g_x being the coupling constant and q_x denoting the $\mathrm{U}(1)_X$ charge of the dark particles. There are also following free parameters: μ_S and λ_S, which enter the potential, the dark Yukawa coupling y_{d} and m_{D}, the Dirac mass of field χ. The Higgs portal Lagrangian has a very simple form

$$\mathcal{L}_{\mathrm{portal}} = -\kappa|S|^2|H|^2\,, \tag{3}$$

where H is the Standard Model Higgs field. The portal coupling constant κ can be both positive or negative. Note that the above Lagrangian is invariant under the charge-conjugation symmetry which acts on the dark fields in the following way:

$$\chi \to \chi^{\mathcal{C}} \equiv -i\gamma_2\chi^*\,, \qquad S \to S^{\mathcal{C}} \equiv S^*\,, \qquad X_\mu \to X_\mu^{\mathcal{C}} \equiv -X_\mu\,. \tag{4}$$

2.1. The Higgs sector

Let us analyze now the scalar potential part of the Lagrangian

$$V(H,S) = \underbrace{-\mu_H^2|H|^2 + \lambda_H|H|^4}_{\mathrm{SM}} \underbrace{-\mu_S^2|S|^2 + \lambda_S|S|^4}_{\mathrm{DS}} + \underbrace{\kappa|H|^2|S|^2}_{\mathrm{portal}}\,. \tag{5}$$

The stability of scalar potential enforces following constraints: $\lambda_H > 0$, $\lambda_S > 0$, $\kappa > -2\sqrt{\lambda_H \lambda_S}$. Since the above potential is invariant both under $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y$ symmetry of H and $\mathrm{U}(1)_X$ symmetry of S, therefore, we can choose the vacuum expectation values (vevs) of H and S fields to be real and non-negative without any loss of generality

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} , \qquad \langle S \rangle = \frac{v_x}{\sqrt{2}} , \qquad v, v_x \geq 0 \,. \tag{6}$$

The minimization of the above potential gives

$$v^2 = \frac{2\kappa\mu_S^2 - 4\lambda_S\mu_H^2}{\kappa^2 - 4\lambda_H\lambda_S} , \qquad v_x^2 = \frac{2\kappa\mu_H^2 - 4\lambda_H\mu_S^2}{\kappa^2 - 4\lambda_H\lambda_S} \,. \tag{7}$$

In the unitary gauge, Goldstone bosons of H and S are eaten by the SM gauge bosons $(W^\pm, Z)$ and the dark vector X_μ, respectively. The fluctuations of real parts of neutral components of the H and S fields are h_0 and ϕ_0, respectively, which mix to form the mass eigenstates h_1 and h_2 as

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \mathcal{R}^{-1} \begin{pmatrix} h_0 \\ \phi_0 \end{pmatrix} , \qquad \mathcal{R} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} , \qquad \tan\alpha = \frac{\kappa v v_x}{\lambda_H v^2 - \lambda_S v_x^2} , \tag{8}$$

where α is the mixing angle. Masses of h_1 and h_2 states are

$$m_{1,2}^2 = \lambda_H v^2 (1 \pm \sec 2\alpha) + \lambda_S v_x^2 (1 \mp \sec 2\alpha) \,.$$

We consider h_1 to be the SM-like Higgs particle, therefore, mass of h_1 and vev of H are set to $m_{h_1} = 125$ GeV, $v = 246$ GeV. Mass of h_2 can be either larger or smaller than m_{h_1}.

2.2. *The dark fermionic sector*

After the spontaneous symmetry breaking, the fermionic part of the dark sector Lagrangian can be rewritten in terms of mass eigenstates as

$$\begin{aligned} \mathcal{L}_{\text{DF}} \;=\; & \frac{i}{2} \left(\bar{\psi}_+ \gamma^\mu \partial_\mu \psi_+ + \bar{\psi}_- \gamma^\mu \partial_\mu \psi_- \right) - \frac{1}{2} m_+ \bar{\psi}_+ \psi_+ - \frac{1}{2} m_- \bar{\psi}_- \psi_- \\ & - \frac{i}{4} g_x \left(\bar{\psi}_+ \gamma^\mu \psi_- - \bar{\psi}_- \gamma^\mu \psi_+ \right) X_\mu - \frac{y_\mathrm{d}}{2} \left(\bar{\psi}_+ \psi_+ - \bar{\psi}_- \psi_- \right) \phi_0 \,, \end{aligned} \tag{9}$$

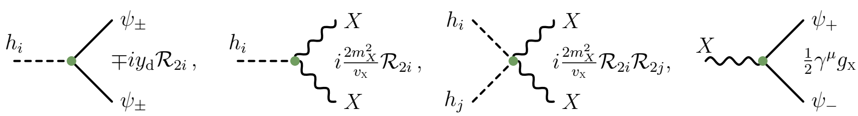
where $\psi_\pm (= \psi_\pm^\mathcal{C})$ are the Majorana (self-conjugate) states, defined as $\psi_+ \equiv \frac{1}{\sqrt{2}}(\chi + \chi^\mathcal{C})$, $\psi_- \equiv \frac{1}{i\sqrt{2}}(\chi - \chi^\mathcal{C})$ with masses $m_\pm = m_\mathrm{D} \pm y_\mathrm{d} v_x$. Without loss of generality, we assume $y_\mathrm{d} > 0$, therefore $m_- < m_+$.

2.3. The dark sector interactions

We can write down the part of Lagrangian that describes interactions involving dark sector particles in terms of mass-eigenstates

$$
\begin{aligned}
\mathcal{L}_{\rm int} \;=\; & -\frac{y_{\rm d}}{2}\left(\bar{\psi}_+\psi_+ - \bar{\psi}_-\psi_-\right)\phi_0 + v_x g_x^2 X^\mu X_\mu \phi_0 + \frac{g_x^2}{2} X^\mu X_\mu \phi_0^2 \\
& -\frac{i}{4} g_x \left(\bar{\psi}_+\gamma^\mu\psi_- - \bar{\psi}_-\gamma^\mu\psi_+\right) X_\mu \,, \qquad (10)
\end{aligned}
$$

where $\phi_0 = h_1 \sin\alpha + h_2 \cos\alpha$. These interactions lead to the following Feynman rules:



(11)

The last vertex gives the only interaction within the dark sector and it allows dark particles to decay. Note that our model respects a discrete $Z_2 \times Z_2'$ symmetry, such that the charges are

Symmetry	X_μ	ψ_+	ψ_-	ϕ_0	SM
Z_2	$-$	$+$	$-$	$+$	$+$
Z_2'	$-$	$-$	$+$	$+$	$+$

Since we assume $m_- < m_+$, therefore, ψ_- particle is always stable. One of the two remaining DM particles, ψ_+ or X_μ, can decay into the other one and ψ_- if this is kinematically allowed.

3. Solving Boltzmann equations and numerical results

The Lagrangian of our model has eight free parameters: g_x, $m_{\rm D}$, $y_{\rm d}$, μ_S, μ_H, λ_S, λ_H, κ. Since h_1 state is assumed to be the SM-like Higgs, its mass and the vev of H are determined, so the number of remaining free parameters is six. We choose the physical basis where free parameters are the masses of the dark sector particles (m_X, m_+, m_-, m_{h_2}), the $\mathrm{U}(1)_X$ coupling g_x, and the Higgs sector mixing angle $\sin\alpha$.

The Boltzmann equation describes the time dependence of the number density n of a given kind of particles interacting with others in the expanding Universe. The set of coupled Boltzmann equations for our model takes the following form (for the details see Ref. [5]):

$$
\begin{aligned}
\frac{\mathrm{d}n_X}{\mathrm{d}t} &= -3Hn_X - \left\langle \sigma^{XX\phi\phi'}_{v_{\mathrm{M\o l}}} \right\rangle \left(n_X^2 - \bar{n}_X^2\right) - \left\langle \sigma^{X\psi_+\psi_-h_i}_{v_{\mathrm{M\o l}}} \right\rangle \\
&\times \left(n_X n_{\psi_+} - \bar{n}_X \bar{n}_{\psi_+} \frac{n_{\psi_-}}{\bar{n}_{\psi_-}} \right) - \left\langle \sigma^{X\psi_-\psi_+h_i}_{v_{\mathrm{M\o l}}} \right\rangle \left(n_X n_{\psi_-} - \bar{n}_X \bar{n}_{\psi_-} \frac{n_{\psi_+}}{\bar{n}_{\psi_+}} \right) \\
&- \left\langle \sigma^{Xh_i\psi_+\psi_-}_{v_{\mathrm{M\o l}}} \right\rangle \bar{n}_{h_i} \left(n_X - \bar{n}_X \frac{n_{\psi_+} n_{\psi_-}}{\bar{n}_{\psi_+} \bar{n}_{\psi_-}} \right) - \left\langle \sigma^{XX\psi_+\psi_+}_{v_{\mathrm{M\o l}}} \right\rangle \left(n_X^2 - \bar{n}_X^2 \frac{n_{\psi_+}^2}{\bar{n}_{\psi_+}^2} \right) \\
&- \left\langle \sigma^{XX\psi_-\psi_-}_{v_{\mathrm{M\o l}}} \right\rangle \left(n_X^2 - \bar{n}_X^2 \frac{n_{\psi_-}^2}{\bar{n}_{\psi_-}^2} \right) + \mathit{\Gamma}_{\psi_+\to X\psi_-} \left(n_{\psi_+} - \bar{n}_{\psi_+} \frac{n_X}{\bar{n}_X} \frac{n_{\psi_-}}{\bar{n}_{\psi_-}} \right) ,
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
\frac{\mathrm{d}n_{\psi_-}}{\mathrm{d}t} &= -3Hn_{\psi_-} - \left\langle \sigma^{\psi_-\psi_-\phi\phi'}_{v_{\mathrm{M\o l}}} \right\rangle \left(n_{\psi_-}^2 - \bar{n}_{\psi_-}^2\right) - \left\langle \sigma^{\psi_-\psi_+Xh_i}_{v_{\mathrm{M\o l}}} \right\rangle \\
&\times \left(n_{\psi_-} n_{\psi_+} - \bar{n}_{\psi_-} \bar{n}_{\psi_+} \frac{n_X}{\bar{n}_X} \right) - \left\langle \sigma^{X\psi_-\psi_+h_i}_{v_{\mathrm{M\o l}}} \right\rangle \left(n_X n_{\psi_-} - \bar{n}_X \bar{n}_{\psi_-} \frac{n_{\psi_+}}{\bar{n}_{\psi_+}} \right) \\
&- \left\langle \sigma^{\psi_-h_iX\psi_+}_{v_{\mathrm{M\o l}}} \right\rangle \bar{n}_{h_i} \left(n_{\psi_-} - \bar{n}_{\psi_-} \frac{n_{\psi_+}}{\bar{n}_{\psi_+}} \frac{n_X}{\bar{n}_X} \right) - \left\langle \sigma^{\psi_-\psi_-XX}_{v_{\mathrm{M\o l}}} \right\rangle \left(n_{\psi_-}^2 - \bar{n}_{\psi_-}^2 \frac{n_X^2}{\bar{n}_X^2} \right) \\
&- \left\langle \sigma^{\psi_-\psi_-\psi_+\psi_+}_{v_{\mathrm{M\o l}}} \right\rangle \left(n_{\psi_-}^2 - \bar{n}_{\psi_-}^2 \frac{n_{\psi_+}^2}{\bar{n}_{\psi_+}^2} \right) + \mathit{\Gamma}_{\psi_+\to X\psi_-} \left(n_{\psi_+} - \bar{n}_{\psi_+} \frac{n_{\psi_-}}{\bar{n}_{\psi_-}} \frac{n_X}{\bar{n}_X} \right) ,
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
\frac{\mathrm{d}n_{\psi_+}}{\mathrm{d}t} &= -3Hn_{\psi_+} - \left\langle \sigma^{\psi_+\psi_+\phi\phi'}_{v_{\mathrm{M\o l}}} \right\rangle \left(n_{\psi_+}^2 - \bar{n}_{\psi_+}^2\right) - \left\langle \sigma^{\psi_+\psi_-Xh_i}_{v_{\mathrm{M\o l}}} \right\rangle \\
&\times \left(n_{\psi_+} n_{\psi_-} - \bar{n}_{\psi_+} \bar{n}_{\psi_-} \frac{n_X}{\bar{n}_X} \right) - \left\langle \sigma^{X\psi_+\psi_-h_i}_{v_{\mathrm{M\o l}}} \right\rangle \left(n_X n_{\psi_+} - \bar{n}_X \bar{n}_{\psi_+} \frac{n_{\psi_-}}{\bar{n}_{\psi_-}} \right) \\
&- \left\langle \sigma^{\psi_+h_iX\psi_-}_{v_{\mathrm{M\o l}}} \right\rangle \bar{n}_{h_i} \left(n_{\psi_+} - \bar{n}_{\psi_+} \frac{n_{\psi_-}}{\bar{n}_{\psi_-}} \frac{n_X}{\bar{n}_X} \right) - \left\langle \sigma^{\psi_+\psi_+XX}_{v_{\mathrm{M\o l}}} \right\rangle \left(n_{\psi_+}^2 - \bar{n}_{\psi_+}^2 \frac{n_X^2}{\bar{n}_X^2} \right) \\
&- \left\langle \sigma^{\psi_+\psi_+\psi_-\psi_-}_{v_{\mathrm{M\o l}}} \right\rangle \left(n_{\psi_+}^2 - \bar{n}_{\psi_+}^2 \frac{n_{\psi_-}^2}{\bar{n}_{\psi_-}^2} \right) - \mathit{\Gamma}_{\psi_+\to X\psi_-} \left(n_{\psi_+} - \bar{n}_{\psi_+} \frac{n_{\psi_-}}{\bar{n}_{\psi_-}} \frac{n_X}{\bar{n}_X} \right) ,
\end{aligned}
\tag{14}
$$

where $\langle \sigma^{ijkl}_{v_{\mathrm{M\o l}}} \rangle \equiv \langle \sigma^{ijkl} v_{\mathrm{M\o l}} \rangle$ is the thermal averaged cross section for the process $ij \to kl$. Above $h_i = h_1, h_2$ and $\phi\phi'$ denote all the allowed SM particles, including h_1, h_2.

The Boltzmann equations form a complicated coupled set, therefore to solve them, we employ a dedicated C++ code. We compared our numerical results for two-component case with those of micrOMEGAs [6][1].

Below, we present a two-component DM case (Fig. 1), where ψ_+ and ψ_- are stable, and a three-component DM case (Fig. 2), where X_μ, ψ_+ and ψ_- are stable[2]. The (dashed) solid curves show our results for the (equi-

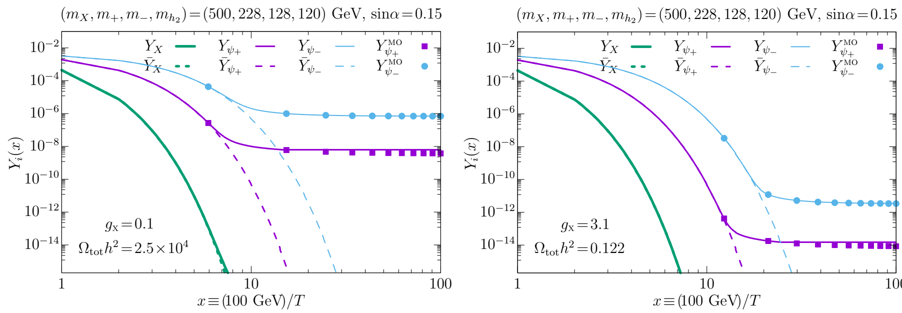


Fig. 1. These plots show two-component DM scenario, where ψ_+ and ψ_- are stable particles. The left and right panels correspond to $g_x = 0.1$, and 3.1, respectively, whereas other model parameters are shown in the legends. Note the very good agreement between our results (solid curves) and micrOMEGAs (points). The right panel satisfies the correct total relic density of DM, $\Omega_{\rm tot}h^2 \equiv (\Omega_{\psi_+} + \Omega_{\psi_-})h^2$, observed by Planck [2] and direct detection limits by LUX2016 [8].

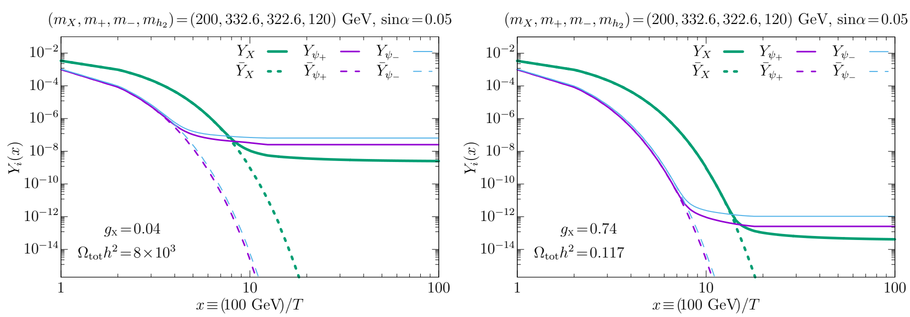


Fig. 2. These plots show our results for three-component DM scenario, where X_μ, ψ_+ and ψ_- are stable. Parameters of the plots are shown in the legends. The left panel is for $g_x = 0.04$ and the right panel is for $g_x = 0.74$. Note that the right panel satisfies the correct relic density of DM, $\Omega_{\rm tot}h^2 \equiv (\Omega_X + \Omega_{\psi_+} + \Omega_{\psi_-})h^2$, observed by Planck [2] and direct detection limits by LUX2016 [8].

[1] micrOMEGAs [6] is limited to only two-component DM.

[2] To simplify calculations, we have used following variables: $x \equiv (100\ \text{GeV})/T$, where T is the temperature of the thermal bath, and $Y_i \equiv n_i/s$, where s is the entropy density of the Universe.

librium) yields of corresponding dark species, whereas the points represent values obtained from micrOMEGAs[3]. The most visible effect is the increase of annihilation cross sections with g_x, what leads to later freeze-out and, in consequence, to smaller relic density of DM particles. Moreover, it turns out that the dark gauge coupling g_x plays a critical role in the dynamics of dark matter species, in particular it allows to have semi-annihilation, conversion and decay processes. A more complete analysis of our vector-fermion DM model, including parameter scans, is presented in Ref. [5].

4. Conclusions

In this work, we have presented a simple renormalizable extension of the SM by an Abelian gauge symmetry. The dark sector of our model contains a dark vector boson X_μ, two dark Majorana fermions $\psi_\pm$ and a second Higgs h_2 which serves as a messenger to the visible sector. Depending on the masses of dark particles $X_\mu, \psi_\pm$, we have two- or three-component DM scenarios. We have developed a code providing the numerical solution to the set of three coupled Boltzmann equations for the densities of DM components, which takes into account not only annihilations into the SM, but also semi-annihilations, conversions and possible decays within the dark sector. Here, we presented two sample cases where two (ψ_+ and ψ_-) and three (X_μ, ψ_+ and ψ_-) components are stable and contribute to the total DM relic density.

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