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EVOLUTION OF DARK MATTER DENSITY WITH EARLY KINETIC DECOUPLING IN THE CASE OF RESONANT ANNIHILATION*

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We revisit scenario of dark matter (DM) annihilation through an s-channel resonance. The evolution of DM density and temperature is studied by solving a set of coupled Boltzmann equations. We show that the kinetic decoupling is a prolonged process that can happen when DM annihilation is still active in changing DM density. We scan over the parameter space in the resonance region of a vector dark matter model and find that the effects of the early kinetic decoupling can modify DM relic density by up to a factor of two in the area where experimental constraints are satisfied.

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1. Introduction

Numerous observations on various astrophysical and cosmological scales provide evidence for the existence of dark matter (DM) in the Universe, however they come solely from the gravitational interaction of DM with the visible matter [1]. According to the widely studied hypothesis, DM consists of a new type particles whose density was set by the so-called freeze-out mechanism. It is based on the assumption that in the early Universe, DM particles were in the thermal equilibrium with the visible sector and decoupled at some point what resulted in currently observed DM relic density. This mechanism requires that DM couples to the Standard Model (SM) with the strength typical for the weak force but, as DM has not yet been detected, exact form of these interaction is unknown.

Various terrestrial, astrophysical and cosmological probes are used to search for a signal of non-gravitational DM interactions [2], notably direct detection experiments look for events of DM scattering on nucleons. The null result of LUX, XENON1T and PandaX-II constraint strongly the parameter space of many simple WIMP scenarios. However, if DM annihilation

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is enhanced by the s-channel resonance, the coupling of DM to the visible sector must be weaker. Consequently, bounds from direct detection are relaxed. Moreover, scattering processes in the early Universe maintaining DM in kinetic equilibrium are also suppressed, which may imply an annihilation rate that has strong dependence on DM temperature. It leads to an interesting interplay between the evolution of temperature and density of DM annihilating in the resonance region that we study in this paper.

2. DM annihilation through a resonance

If DM mass is close to the half of mediator mass, the annihilation cross section is dictated by the shape of the resonance and depends strongly on the energy of incoming particles. The cross section σ for the s-wave annihilation of DM particles with mass $m_{\rm i}$ and relative velocity $v_{\rm rel}$ can be written in the non-relativistic, Breit–Wigner approximation as

$$\sigma v_{\rm rel} = \sum_{\rm f\neq i} \frac{64\pi\omega}{M^2} \frac{\eta_{\rm i}\eta_{\rm f}\beta_{\rm f}}{\left(\delta + v_{\rm rel}^2/4\right)^2 + \gamma^2}\,, \tag{1}$$

where $\omega = (2s_{\rm R}+1)/(2s_{\rm i}+1)^2$ is a factor that depends on the spin of mediator $s_{\rm R}$ and DM $s_{\rm i}$, M is mediator mass and the sum is over all final states assumed to be SM particles. The shape of the resonance is described by the following dimensionless parameters:

$$\eta_{\rm i/f} \equiv \frac{\Gamma B_{\rm i/f}}{M\bar{\beta}_{\rm i/f}}\,, \qquad \delta \equiv \frac{4m_{\rm i}^2}{M^2} - 1 \qquad \text{and} \qquad \gamma = \frac{\Gamma}{M}\,, \tag{2}$$

where $\beta_{\rm i/f} = \sqrt{1-4m_{\rm i/f}^2/s}$, $\bar{\beta}_{\rm i/f} = \sqrt{1-4m_{\rm i/f}^2/M^2}$, Γ is the total width of the mediator and $B_{\rm i/f}$ are the branching ratios to initial and final states. The parameters $\eta_{\rm i/f}$ describe the couplings of mediator to DM and annihilation products, δ parameterizes the position of the resonance and γ its width. In the resonant scenario, we assume $\delta, \gamma \ll 1$.

The foregoing Breit–Wigner approximation is not always precise. If the mediator couples dominantly to DM, the threshold effect has to be included. The self-energy Σ in the resummed propagator is energy-dependent what in the non-relativistic approximation can be described with velocity-dependent width parameter $\gamma(v)$ [3]

$$\gamma(v) \equiv \frac{\Im\Sigma}{M} = \eta_{\rm i}\frac{v_{\rm rel}}{2} + \sum_{\rm f\neq i}\eta_{\rm f}\bar{\beta}_{\rm f}\,, \tag{3}$$

where the part that comes from decays to the final states (assumed to be far from the threshold) is approximated by a constant.

The annihilation cross section used in the Boltzmann equation is averaged with the thermal distribution which in the non-relativistic approximation reads as follows:

$$\langle \sigma v_{\rm rel} \rangle_x = \frac{x^{3/2}}{2\sqrt{\pi}} \int\limits_0^\infty {\rm d}v_{\rm rel} v_{\rm rel}^2 e^{-x v_{\rm rel}^2/4} \sigma v_{\rm rel} \,. \tag{4}$$

In Fig. 1, it can be seen that $\langle \sigma v_{\rm rel} \rangle$ grows with falling temperature before it saturates at some point. The departure from the Breit–Wigner approximation becomes relevant when $n_{\rm i}^2 \gg |\delta|$ [3].

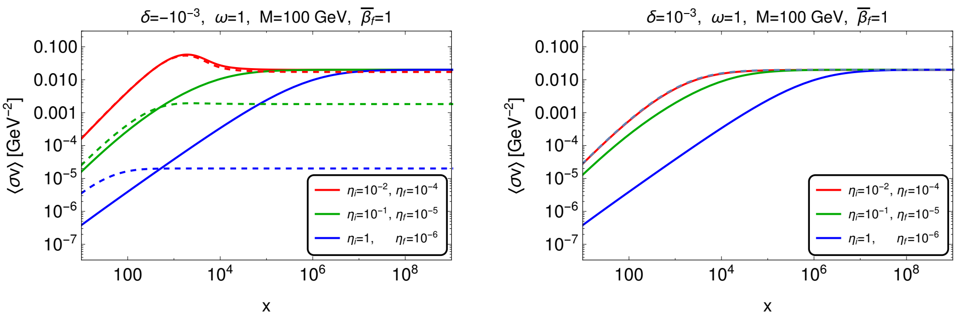

Fig. 1. Thermal cross section $\langle \sigma v_{\rm rel} \rangle$ for negative (left panel) and positive δ (right panel) obtained using energy-dependent width (solid lines) or constant width approximation (dashed lines). In the right panel, the dashed lines coincide along the top curve.

3. Evolution of DM temperature and density

In the standard freeze-out mechanism, it is assumed that DM remains in kinetic equilibrium with the thermal bath of visible sector during the period of active annihilation. This assumption is justified by the frequent DM scatterings on abundant light states in the visible sector which keep DM at the temperature of the SM thermal bath. However, if the annihilation cross section is enhanced by the s-channel resonance, the proper relic abundance requires smaller coupling to the visible sector, whereas similar effect is not present in the case of the t-channel scattering. Therefore, the scattering cross section is typically suppressed. Moreover, as the thermal cross section $\langle \sigma v_{\rm rel} \rangle$ has strong temperature dependence and grows with time, any deviation of DM temperature from the temperature of the SM, even long after DM chemical decoupling, may affect also DM density.

This effect can be studied using the coupled Boltzmann equations that describe the evolution of DM yield Y and temperature parameter y [3, 4]

$$\begin{aligned}
\frac{\mathrm{d}Y}{\mathrm{d}x} &= -\frac{1-\frac{x}{3}\frac{g'_{*s}}{g_{*s}}}{Hx} s\left(Y^2\langle\sigma v_{\rm rel}\rangle_{x_{\rm DM}} - Y_{\rm EQ}^2\langle\sigma v_{\rm rel}\rangle_x\right)\\
\frac{\mathrm{d}y}{\mathrm{d}x} &= -\frac{1-\frac{x}{3}\frac{g'_{*s}}{g_{*s}}}{Hx}\Big\{2M_{\rm DM}c(T)(y-y_{\rm EQ})\\
&-\frac{sy}{Y}\left[Y^2\left(\langle\sigma v_{\rm rel}\rangle_{x_{\rm DM}} - \langle\sigma v_{\rm rel}\rangle_2|_{x_{\rm DM}}\right) - Y_{\rm EQ}^2\left(\langle\sigma v_{\rm rel}\rangle_x - \frac{y_{\rm EQ}}{y}\langle\sigma v_{\rm rel}\rangle_2|_x\right)\right]\Big\}\,.
\end{aligned} \tag{5}$$

DM yield $Y = n/s$ is defined as DM density n divided by entropy density $s = g_{*s}(T)\frac{2\pi^2}{45}T^3$, where T is temperature of the SM thermal bath. The parameter y and its kinetic equilibrium value $y_{\rm EQ}$ are defined as [4]

$$y \equiv \frac{M_{\rm DM}T_{\rm DM}}{s^{2/3}} \equiv \frac{g_{\rm i}}{Y s^{5/3}}\int\frac{\mathrm{d}^3p}{2\pi^3}\boldsymbol{p}^2 f(\boldsymbol{p}) \quad \text{and} \quad y_{\rm EQ} \equiv \frac{M_{\rm DM}T}{s^{2/3}}\,. \tag{6}$$

This definition is general, but $T_{\rm DM}$ can be interpreted as the DM temperature only if $f(p)$ is a thermal distribution. In this work, we assume this is the case due to effective self-scatterings which are also enhanced by the resonance[1]. In chemical equilibrium $Y = Y_{\rm EQ} = \frac{45}{2\pi^4}\sqrt{\frac{\pi}{8}}\frac{g_{\rm i}}{g_*}x^{3/2}e^{-x}$, where $g_{\rm i}$ is the number of DM degrees of freedom, $x = M_{\rm DM}/T$ and $x_{\rm DM} = M_{\rm DM}/T_{\rm DM}$. The averaged cross section $\langle\sigma v_{\rm rel}\rangle_2$ reads

$$\langle\sigma v_{\rm rel}\rangle_2|_x = \frac{x^{3/2}}{4\sqrt{\pi}}\int_0^\infty \mathrm{d}v_{\rm rel}\sigma v_{\rm rel}\left(1+\frac{1}{6}v_{\rm rel}^2 x\right)v_{\rm rel}^2 e^{-v_{\rm rel}^2 x/4}\,, \tag{7}$$

and the scattering rate $c(T)$ can be written as

$$\begin{aligned}
c(T) &= \frac{1}{12(2\pi)^3 M_{\rm DM}^4 T}\\
&\times\sum_{\rm f}\int \mathrm{d}k k^5\omega^{-1}g^{\pm}\left(1\mp g^{\pm}\right)|\mathcal{M}_{\rm f}|^2_{t=0;s=M_{\rm DM}^2+2M_{\rm DM}\omega+M_{\rm f}^2}\,,
\end{aligned}$$

where $\mathcal{M}_{\rm f}$ is the matrix element for the DM scattering on a state 'f' from the thermal bath with equilibrium distribution function $g^{\pm} = (e^{\omega/T}\pm 1)^{-1}$ that has momentum (ω, k).

4. Resonance region in the model of vector dark matter

To analyze quantitatively the effect of kinetic decoupling on the calculation of DM relic density, we study the resonance region of the Abelian vector dark matter model (VDM) [6, 7] following the notation of [3, 7]. A dark

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[1] The departure from thermal distribution is studied in [5].

matter candidate Z' interacts with the SM via one of the scalar Higgs mediators, the SM-like Higgs h_1 with mass 125 GeV or the second scalar whose mass is a free parameter of the model.

In Fig. 2, we present an exemplary solution of the Boltzmann equations (5). For a given set of parameters, the scatterings cease to be effective in keeping DM in the kinetic equilibrium at $x_{\rm KD} \approx 100$. However, at this point, y does not follow the solution for kinetically decoupled DM (depicted with bottom solid/blue curve), which cools down as the non-relativistic gas (for which $T_{\rm DM} \sim T^2$). The exact solution of equations (2) (middle solid/orange curve) shows that DM temperature decreases slower. It comes from the velocity dependence of the annihilation rate. The cross section is larger for smaller velocities, therefore, annihilation increases the average kinetic energy and temperature of DM.

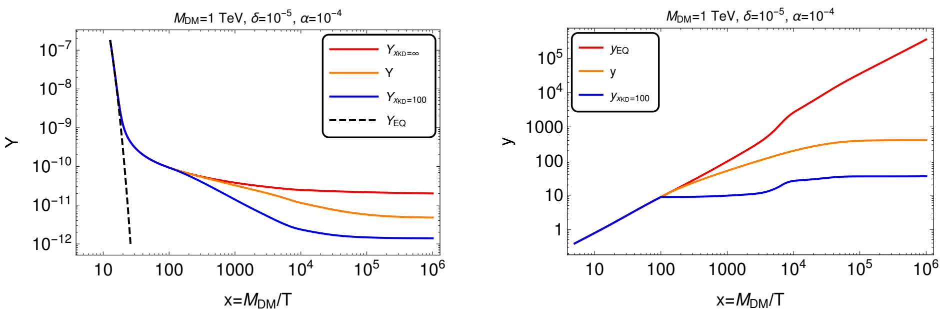

Fig. 2. (Color online) Evolution of dark matter yield Y and temperature parameter y given by Eqs. (5) (middle solid/orange curve) or for kinetically coupled DM (top solid/red) and DM instantaneously decoupled at $x_{\rm KD} = 100$ (bottom solid/blue).

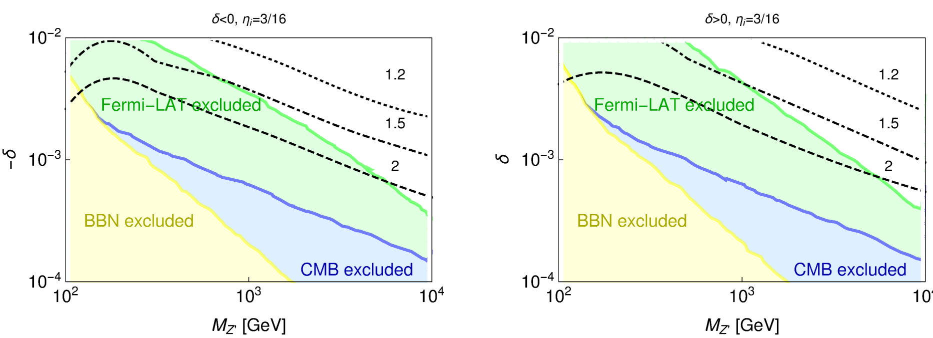

Fig. 3. Experimental constraints in the resonance region ($2M_{\rm DM} \approx M_{h_2}$) of vector dark matter model. Along the dotted, dash-dotted and dashed lines, relic abundance without taking into account the effect of kinetic decoupling is larger by factor 1.2, 1.5 or 2, respectively.

The parameter space of the resonance region ($2M_{\rm DM} \approx M_{h_2}$) of the VDM model is presented in Fig. 3. It can be seen that scenario with highly tuned resonance ($\delta \ll 1$) is excluded by strong bounds on enhanced DM annihilation at small velocities coming from FERMI-LAT, primordial nucleosynthesis (BBN) or measurements of cosmic microwave background (CMB) [3]. In the allowed region, we present curves along which the relic density without considering kinetic decoupling is larger by factor 1.2, 1.5 or 2.

5. Summary and conclusions

We have studied the relevance of DM kinetic decoupling for the calculation of DM relic density in the scenario with enhanced annihilation through the s-channel resonance. We indicate that the Breit–Wigner approximation does not give a proper description of the annihilation cross section if the mediator couples strongly enough to DM. We solve the system of Boltzmann equations for the evolution of DM temperature and density, and show that the kinetic decoupling cannot be treated as an instantaneous process. Finally, we determine the allowed parameter space of the Abelian vector DM model and modifications in the calculation of DM relic abundance due to the early kinetic decoupling.

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