

THERMODYNAMICS AND KINETICS OF GRIBOV–ZWANZIGER PLASMA WITH TEMPERATURE-DEPENDENT GRIBOV PARAMETER

VIKTOR BEGUN

Faculty of Physics, Warsaw University of Technology
Koszykowa 75, 00-662 Warszawa, Poland

WOJCIECH FLORKOWSKI

Institute of Physics, Jan Kochanowski University
Świętokrzyska 15, 25-406 Kielce, Poland

RADOSŁAW RYBLEWSKI

The Henryk Niewodniczański Institute of Nuclear Physics PAN
Radzikowskiego 152, 31-342 Kraków, Poland

(Received January 17, 2017; accepted February 8, 2017)

A Gribov-type dispersion relation is treated as an effective description of interacting gluons forming a hot medium. Temperature dependence of the Gribov parameter is determined from the fit to the lattice Yang–Mills results describing thermodynamic functions. To maintain thermodynamic consistency of the approach, a temperature-dependent bag pressure is introduced. The results obtained for equilibrium functions are generalised in the next step to non-equilibrium conditions. We derive formulas for the bulk and shear viscosity coefficients within the relaxation time approximation. We find evidence for largely enhanced bulk viscosity in the region of the phase transition.

DOI:10.5506/APhysPolB.48.125

1. Introduction

Several phenomenological models of the gluon plasma have been constructed in the past [1, 2], see also [3–5], that describe such a system as an ideal gas of massive, non-interacting quasi-particles with the dispersion relation of the form of

$$E(\mathbf{k}, M) = \sqrt{\mathbf{k}^2 + M^2(T)}. \quad (1)$$

Here, $\mathbf{k}$ is the gluon three-momentum, and E is its energy. A temperature-dependent mass, $M(T)$, accounts for the interaction of originally massless gluons. Thus, in this approximation, the massless interacting gluons become non-interacting massive particles with two possible polarisation directions. We shall continue to call them gluons.

The use of the dispersion relation (1) in standard thermodynamic expressions for the energy density and pressure leads, however, to the violation of the basic thermodynamic identities. As it has been pointed out by Gorenstein and Yang [6], a possible remedy for this situation is the introduction of an additional mean field B , commonly depicted as a bag constant (see also [7, 8]). As a matter of fact, B is not a constant but a function of M and, therefore, it is also a function of T , $B = B(M(T))$. To avoid possible confusion, in the following, we call B the bag pressure. We note that the name of B originates from the renowned MIT bag model [9], see also [10–12].

An alternative description to that using formula (1) is the framework based on the Gribov approach to the quantisation of the Yang–Mills theory that leads to the following dispersion relation [13]:

$$E(\mathbf{k}, \gamma_G) = \sqrt{\mathbf{k}^2 + \frac{\gamma_G^4(T)}{\mathbf{k}^2}}. \quad (2)$$

The Gribov parameter γ_G is, again, a function of the temperature T . At high temperatures, γ_G is known to depend linearly on T . In the region just above the critical temperature, γ_G may be treated as a constant [14, 15]. The use of Eq. (2) has turned out to be quite successful in qualitative description of thermodynamic properties of a purely gluonic system, such as the temperature dependence of the energy density, ε , pressure, P , entropy density, s , and the trace anomaly (interaction measure), I [14–17]. The Gribov dispersion relation has been also used recently to address real-time problems [18, 19].

Clearly, going beyond the approximations that lead to Eq. (2) is quite complicated. Instead, in this work, we propose to fit the temperature dependence of the Gribov parameter directly to the SU(3) gauge theory lattice data [20]. In order to construct a thermodynamically consistent approach, we follow Gorenstein and Yang, and introduce a temperature-dependent bag pressure. We should emphasise that the lattice data [20] give a strong evidence for the first order phase transition at the critical temperature $T_c = 260$ MeV, expected for purely gluonic matter. Since our fits use smooth functions, we are able only to determine an approximate behaviour of physical quantities in the phase transition region. The very character of the phase transition, including its order, is not addressed in our considerations.

Having established a thermodynamically consistent model based on Eq. (2), we continue to realise our main aim, *i.e.*, we construct a kinetic framework allowing for medium-dependent Gribov parameter. In this way, we generalise the results obtained before in Refs. [21, 22], where γ_G was treated as a constant. In particular, we generalise the formulas for the shear and bulk viscosity.

Determination of the kinetic coefficients for strongly interacting matter is of large interest due to possible applications of such results in hydrodynamic modelling of relativistic heavy-ion collisions [23–28]. For example, it was argued in [23] that a sharp rise of the bulk viscosity might cause cavitation and splitting of the plasma into small clusters. The shear and bulk viscosities were calculated in the high- T regime using, for example, perturbation theory [29–32], lattice simulations [33, 34], functional renormalization techniques [35, 36], quasi-particle models [37–42], and a gas of hadrons [43, 44]. For conformal systems, a lower bound on the shear viscosity to entropy density ratio, $\eta/s \geq 1/(4\pi)$, was found [45]. However, breaking of conformal invariance is necessary for the generation of bulk viscosity [46, 47], and a few models have been suggested to reproduce lattice QCD features [48, 49].

We note that the bulk viscosity in a gluon system with dispersion relation (2) was analysed before in a field theoretic approach of Ref. [50] in the context of the old lattice data [51]. Our approach is more phenomenologically oriented and we use the most recent lattice results for gluons [20]. We also obtain qualitatively and quantitatively different results — the enhancement of the bulk viscosity close to the critical temperature [52, 53]. Such enhancement is particularly interesting due to possible consequences for heavy-ion collisions.

The paper is organised as follows: In Sec. 2, we discuss thermodynamic relations from the point of view of thermodynamic consistency, and determine the temperature dependence of the Gribov parameter and bag pressure. In Sec. 3, we discuss symmetry constraints, the form of the energy-momentum tensor, the kinetic equation in the relaxation time approximation, and the calculation of the bulk and shear viscosities. We conclude in Sec. 4.

Three-vectors are denoted by the bold font, four-vectors are in standard font, the dot denotes the scalar product and $g^{\mu\nu} = \text{diag}(+1, -1, -1, -1)$. The four-vector defining the hydrodynamic flow is denoted by u . In the local rest frame (LRF), $u^\mu = (1, 0, 0, 0)$.

2. Thermodynamics

2.1. Pressure

The starting point for constructing a thermodynamically consistent approach is the definition of equilibrium pressure. The latter is divided into the particle and mean-field parts, hence we write

$$P_{\text{eq}} = P_{\text{GZ}} + P_{\text{mf}} , \quad (3)$$

where

$$P_{\text{GZ}} = -\frac{gT}{(2\pi)^3} \int d^3k \ln \left[1 - \exp \left(-\frac{E(\mathbf{k}, \gamma_{\text{G}})}{T} \right) \right] \quad (4)$$

and

$$P_{\text{mf}} = -B(\gamma_{\text{G}}) . \quad (5)$$

The particle part, P_{GZ} , describes the pressure of the equilibrium Gribov–Zwanziger (GZ) plasma with the dispersion relation (2). The mean-field part is expressed directly by the bag pressure. We note that the Gribov parameter, γ_{G} , appearing in the two terms, depends implicitly on T . In Eq. (4), $g = 16$ is the degeneracy factor connected with the internal degrees of freedom (2 (spin) $\times$ 8 (colour)). By integrating by parts over momentum in (4), we get

$$P_{\text{GZ}} = \frac{g}{(2\pi)^3} \int d^3k \frac{\mathbf{k}^2}{3E} \left(1 - \frac{\gamma_{\text{G}}^4}{\mathbf{k}^4} \right) f_{\text{GZ}} , \quad (6)$$

where we have introduced the equilibrium (Bose–Einstein) distribution function of GZ gluons

$$f_{\text{GZ}} = \left[\exp \left(\frac{E(\mathbf{k}, \gamma_{\text{G}})}{T} \right) - 1 \right]^{-1} . \quad (7)$$

2.2. Entropy density

The mean field does not contribute to the entropy density, hence we may write

$$s_{\text{eq}} = s_{\text{GZ}} . \quad (8)$$

To obtain the particle entropy density, we use the Boltzmann definition

$$s_{\text{GZ}} = -\frac{g}{(2\pi)^3} \int d^3k \Phi[f_{\text{GZ}}] , \quad (9)$$

where the functional Φ has the form appropriate for bosons, namely

$$\Phi[f] = f \ln f - (1 + f) \ln(1 + f) . \quad (10)$$

For $f = f_{\text{GZ}}$, we find

$$\Phi[f_{\text{GZ}}] = \ln \left[1 - \exp \left(-\frac{E}{T} \right) \right] - \frac{E}{T} f_{\text{GZ}} . \quad (11)$$

This gives the equilibrium entropy density in the form of

$$s_{\text{eq}} = \frac{g}{(2\pi)^3} \int d^3k \frac{E(\mathbf{k}, \gamma_G)}{T} f_{\text{GZ}} - \frac{g}{(2\pi)^3} \int d^3k \ln \left[1 - \exp \left(-\frac{E(\mathbf{k}, \gamma_G)}{T} \right) \right] \quad (12)$$

which can be written also as

$$s_{\text{eq}} = \frac{2g}{3(2\pi)^3} \int \frac{d^3k}{E T} \left(2|\mathbf{k}|^2 + \frac{\gamma_G^4}{|\mathbf{k}|^2} \right) f_{\text{GZ}}. \quad (13)$$

The equality of Eqs. (12) and (13) can be verified most easily if one changes to spherical coordinates in the second line of (12) and integrates by parts over the magnitude of the three-momentum.

In the first line of (12), we find the energy density of the GZ plasma

$$\varepsilon_{\text{GZ}} = \frac{g}{(2\pi)^3} \int d^3k E(\mathbf{k}, \gamma_G) f_{\text{GZ}}, \quad (14)$$

whereas in the second line of (12), we find P_{GZ} . Therefore, we arrive at the thermodynamic identity

$$s_{\text{eq}} = \frac{\varepsilon_{\text{GZ}} + P_{\text{GZ}}}{T}. \quad (15)$$

Since the bag pressure does not appear in (15), this formula can be used to find the temperature dependence of the Gribov parameter from the temperature dependence of the entropy density found in the lattice simulations. Similar procedure has been applied before in the cases where the quasi-particle approach with the dispersion relation (1) has been used in hydrodynamics, for example, see [54, 55].

2.3. Energy density and thermodynamic identities

Given P_{eq} , the formula for the energy density ε_{eq} is obtained from the thermodynamic identities

$$\varepsilon_{\text{eq}} = T s_{\text{eq}} - P_{\text{eq}}, \quad s_{\text{eq}} = \frac{dP_{\text{eq}}}{dT}. \quad (16)$$

In our case, using Eq. (3), we obtain

$$\varepsilon_{\text{eq}} = \varepsilon_{\text{GZ}} + B - T \Delta \frac{d\gamma_G}{dT} \quad (17)$$

and

$$s_{\text{eq}} = \frac{\varepsilon_{\text{GZ}} + P_{\text{GZ}}}{T} - \Delta \frac{d\gamma_{\text{G}}}{dT}, \quad (18)$$

where we have introduced the notation

$$\Delta = \frac{g}{(2\pi)^3} \int d^3k \frac{2\gamma_{\text{G}}^3}{\mathbf{k}^2 E(\mathbf{k}, \gamma_{\text{G}})} f_{\text{GZ}} + \frac{dB}{d\gamma_{\text{G}}}. \quad (19)$$

Comparing Eqs. (15) and (18), we conclude that the thermodynamic consistency is maintained only if $\Delta = 0$ or, equivalently, if

$$\frac{g}{(2\pi)^3} \frac{d\gamma_{\text{G}}}{dT} \int d^3k \frac{2\gamma_{\text{G}}^3}{\mathbf{k}^2 E(\mathbf{k}, \gamma_{\text{G}})} f_{\text{GZ}} + \frac{dB}{dT} = 0. \quad (20)$$

If the temperature dependence of the Gribov parameter is determined by the fits of the lattice entropy density, Eq. (20) can be used in the next step to determine the temperature dependence of the bag pressure. Thus, in the case of $\Delta = 0$, we obtain the compact expression for the energy density

$$\varepsilon_{\text{eq}} = \varepsilon_{\text{GZ}} + \varepsilon_{\text{mf}}, \quad \varepsilon_{\text{mf}} = B(\gamma_{\text{G}}). \quad (21)$$

The expressions for the energy density and pressure can be used to find the interaction measure (also known as the trace anomaly)

$$I_{\text{eq}} = \varepsilon_{\text{eq}} - 3P_{\text{eq}} = \varepsilon_{\text{GZ}} - 3P_{\text{GZ}} + 4B. \quad (22)$$

2.4. Results for equilibrium

Our starting point is the analysis of the lattice results [20] for the temperature dependence of the scaled trace anomaly $I_{\text{eq}}(T)/T^4$. In order to guarantee that thermodynamic relations are satisfied exactly, we fit only $I_{\text{eq}}(T)$. Subsequently, using the resulting fit function $I_{\text{eq}}^{\text{fit}}(T)$, the equilibrium pressure is calculated from Eq. (2.3) given in Ref. [20], namely

$$\frac{P_{\text{eq}}^{\text{fit}}(T)}{T^4} - \frac{P_{\text{eq}}(T_0)}{T_0^4} = \int_{T_0}^T \frac{dT'}{T'} \frac{I_{\text{eq}}^{\text{fit}}(T')}{T'^4}, \quad (23)$$

where $P_{\text{eq}}(T_0 = 0.7T_c)$ is read off again from Ref. [20]. We have checked that the temperature dependence of pressure resulting from (23) agrees with the lattice data at the level of 1–2%. Having $I_{\text{eq}}^{\text{fit}}(T)/T^4$ and $P_{\text{eq}}^{\text{fit}}(T)/T^4$, we calculate all remaining thermodynamic functions using relations (16) and (22).

Since s_{eq} is independent of the bag pressure, we use it to determine $\gamma_G(T)$. Our functional fit to the lattice points describing entropy density *vs.* temperature is shown in Fig. 1 (solid/red curve). The temperature dependence of the ratio s_{eq}/T^3 shows a rapid increase in the region of the phase transition. Such a behaviour is usually attributed to the increase of the effective number of degrees of freedom in the system. Our model curve follows closely this trend. We note that for the pure Yang–Mills theory studied

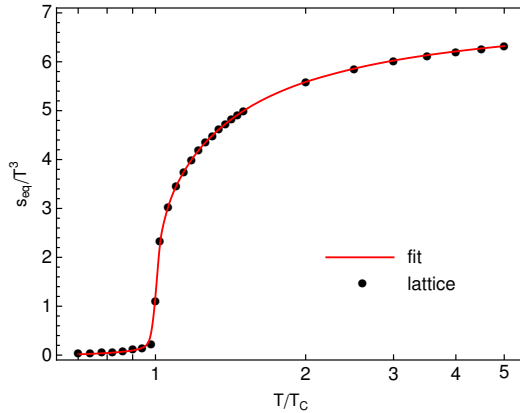


Fig. 1. (Colour on-line) Temperature dependence of the entropy density scaled by T^3 . The lattice data [20] (dots) is compared with our approximation (solid/red line) based on Eq. (12) with the function $\gamma_G(T)$ shown in Fig. 2.

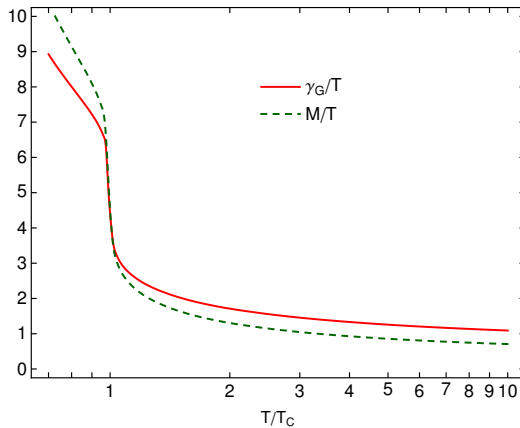


Fig. 2. (Colour on-line) Temperature dependence of the Gribov parameter γ_G obtained from the fit to the lattice results for the entropy density (solid/red line). The dashed/green line represents $M(T)$ obtained in the similar way from the lattice data but with the dispersion relation (1).

in [20], one expects a first order phase transition, hence, the function s_{eq}/T^3 should be discontinuous at T_c . In our model calculation, we rely only on regular approximations to the lattice data and, consequently, do not reproduce exactly the character of the phase transition.

The temperature-dependent Gribov parameter is shown in Fig. 2. The presence of the phase transition is reflected by a sudden decrease of γ_G in the region of $T \approx T_c$. We note that for extremely high temperatures (not shown in the figure), the ratio γ_G/T becomes (to a good approximation) a constant. This means that the system may be treated approximately as conformal in this region. We shall come back to this point discussing the high temperature limit of the bulk viscosity in Sec. 3.4.

Given the function $\gamma_G(T)$, we find the temperature-dependent bag pressure $B(T)$ from identity (20), which guarantees thermodynamic consistency¹. The function $B(T)$ is shown in Fig. 3. We have normalised $B(T)$ in such a way that it is zero at $T = 0$. In the region of the phase transition, $B(T)$ has a strong peak, while for large values of T , it becomes negative [56, 57]. We checked that at asymptotically large T , the bag pressure vanishes. In Figs. 2 and 3, we show also the results obtained with the dispersion relation (1).

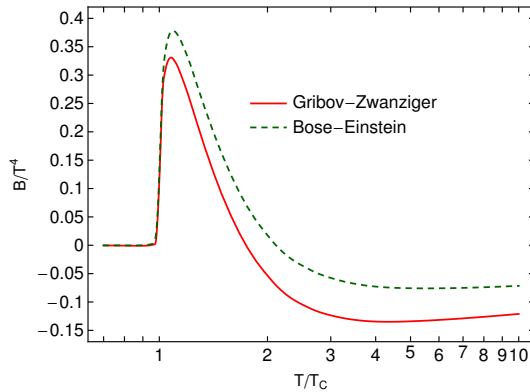


Fig. 3. (Colour on-line) Temperature dependence of the bag pressure obtained from condition (20) that guarantees thermodynamic consistency of our approach with the boundary condition $B(T = 0) = 0$ (solid/red line). The dashed/green line gives $B(T)$ obtained similarly with the dispersion relation (1).

Knowing the functions $\gamma_G(T)$ and $B(T)$, we are in a position to calculate all other thermodynamic functions. In Figs. 4, 5 and 6, we show our results for $\varepsilon_{\text{eq}}(T)/T^4$, $I_{\text{eq}}(T)/T^4$ and $P_{\text{eq}}(T)/T^4$ (solid lines) compared to the lattice data [20] (dots). In all the cases, we obtain a good agreement between our fits and the lattice data, also in the region of the phase transition.

¹ Knowing $\gamma_G(T)$, one may equivalently use Eq. (22).

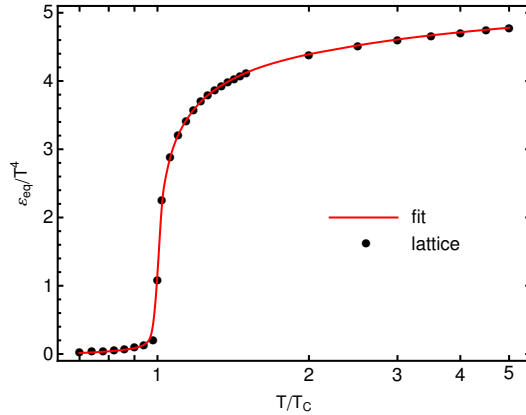


Fig. 4. (Colour on-line) Temperature dependence of the energy density scaled by T^4 . The lattice data [20] (dots) are compared with our approximation based on Eq. (21) with the functions $\gamma_G(T)$ and $B(T)$ depicted in Figs. 2 and 3.

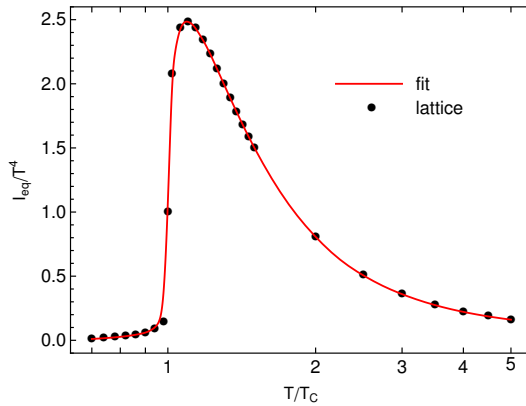


Fig. 5. (Colour on-line) The same as Fig. 4 but for the case of interaction measure. The lattice data are taken from [20].

Finally, in Fig. 7, we show the sound velocity characterising the system (solid/red line) that has been obtained from the formula

$$c_s^2 = \frac{dP_{\text{eq}}}{d\varepsilon_{\text{eq}}} . \quad (24)$$

We note that the quantities P_{eq} and ε_{eq} contain generally contributions from the bag pressure. They disappear if $B = \text{const}$. The results for such a case, with $\gamma_G = 0.7$ GeV, are also shown in the figure (dashed/blue line). We see that the two results are consistent in the region above the phase transition, where γ_G is approximately constant.

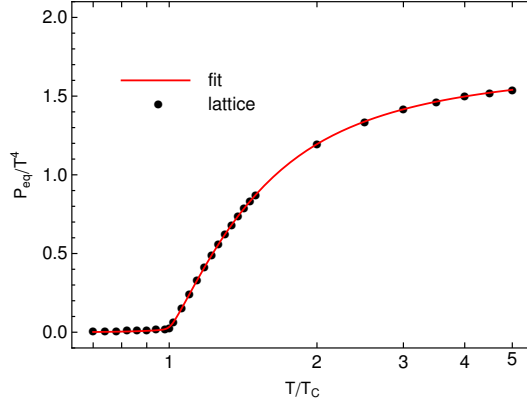


Fig. 6. (Colour on-line) The same as Fig. 4 but for the case of pressure. The lattice data are taken from [20].

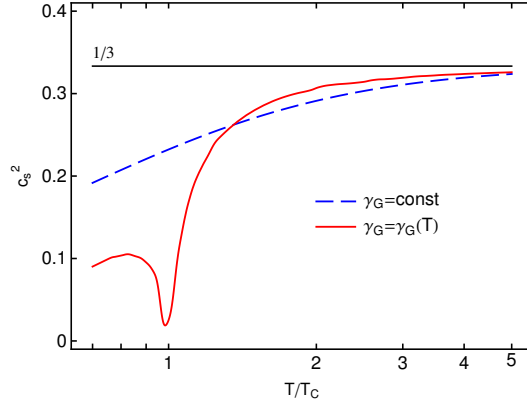


Fig. 7. (Colour on-line) Temperature dependence of the sound velocity squared for the temperature-dependent (solid/red line) and constant (dashed/blue line) Gribov parameter.

The temperature profile of the sound velocity has a dip typical for the phase transition (soft point). For the first order phase transition, the sound velocity drops exactly to zero. Since we use only approximations to the lattice data, our result for c_s^2 is finite at $T = T_c$ and reminds a rapid crossover transition [58].

3. Kinetics

3.1. Lorentz covariance and boost invariance

Having discussed the equilibrium properties of our system, we turn to the study of kinetic properties of the GZ plasma with the temperature-dependent γ_G and B . To calculate the bulk and shear viscosities, we consider a boost-invariant and transversally homogeneous system [59]. These two properties facilitate our considerations — they allow us to formulate a consistent (albeit simple) kinetic model within the relaxation time approximation used for the collision term. We note that although our treatment employs the Bjorken scenario, the final results for the kinetic coefficients are independent of the expansion model, as they depend only on local thermodynamic parameters.

The boost invariance of our approach should be discussed in the context of the Lorentz invariance. In fact, the latter is explicitly broken by Eq. (2) that has been derived in the Coulomb gauge. A solution to this problem has been proposed in Refs. [21, 22]. Since the Coulomb gauge as well as the value of the Gribov parameter are both determined by the procedures defined in the local rest frame of the fluid element, the energy of a gluon E (denoting always the energy determined in LRF) should be treated as a scalar function of three-momentum. The details of this procedure are discussed in greater detail in Ref. [22]. Here, we only explain our notation.

The longitudinal momentum of a particle is denoted by $k_{\parallel}$, the magnitude of the transverse momentum is $k_{\perp} \equiv \sqrt{k_x^2 + k_y^2}$, and k^0 is the magnitude of the three-vector $\mathbf{k} \equiv (k_x, k_y, k_{\parallel})$, namely, $k^0 \equiv |\mathbf{k}|$. Thus, we use the four-vector $k^{\mu} = (|\mathbf{k}|, k_x, k_y, k_{\parallel})$ which has standard Lorentz transformation properties with $k \cdot k = k^2 = 0$. The function $E(\mathbf{k}, \gamma_G)$ defines the energy of the gluon with three-momentum $\mathbf{k}$ in LRF. If the latter has the four-velocity u , the energy of a gluon can be written covariantly as

$$E(k \cdot u, \gamma_G) = \sqrt{(k \cdot u)^2 + \frac{\gamma_G^4}{(k \cdot u)^2}}. \quad (25)$$

For one-dimensional, boost-invariant and transversally homogeneous systems, one may introduce the boost-invariant variables: $\tau = \sqrt{t^2 - z^2}$, $v = k^0 t - k_{\parallel} z = |\mathbf{k}| t - k_{\parallel} z$, $w = k_{\parallel} t - k^0 z = k_{\parallel} t - |\mathbf{k}| z$ [60, 61]. Hence, we find

$$k \cdot u = \frac{v}{\tau} = \sqrt{\frac{w^2}{\tau^2} + k_{\perp}^2} \quad (26)$$

and

$$E(\tau, w, k_{\perp}) = \sqrt{\frac{w^2}{\tau^2} + k_{\perp}^2 + \frac{\gamma_G^4}{\frac{w^2}{\tau^2} + k_{\perp}^2}}. \quad (27)$$

The phase-space distribution function is a Lorentz scalar. This means that in our case, the phase-space distribution function depends only on τ , w , and $k_\perp$

$$f(x, p) = f(\tau, w, k_\perp). \quad (28)$$

Finally, we note that the Lorentz invariant measure in the momentum space expressed in the new variables is $dw d^2k_\perp/\tau$.

3.2. Energy-momentum tensor and energy-momentum conservation law

As long as the system is in local equilibrium, its energy-momentum tensor has the perfect-fluid form

$$T_{\text{eq}}^{\mu\nu} = (\varepsilon_{\text{eq}} + P_{\text{eq}})u^\mu u^\nu - P_{\text{eq}}g^{\mu\nu}, \quad (29)$$

where the flow vector has the form $u^\mu = (t, 0, 0, z)/\tau$. For boost-invariant systems which are additionally homogeneous in the transverse plane, the general form of the energy-momentum tensor that holds also *out of equilibrium* is

$$T^{\mu\nu} = (\varepsilon_{\text{tot}} + P_\perp^{\text{tot}})u^\mu u^\nu - P_\perp^{\text{tot}}g^{\mu\nu} + (P_\parallel^{\text{tot}} - P_\perp^{\text{tot}})z^\mu z^\nu. \quad (30)$$

Here, ε_{tot} is the total energy density, $P_\perp^{\text{tot}}$ is the total transverse pressure, $P_\parallel^{\text{tot}}$ is the total longitudinal pressure, and $z^\mu = (z, 0, 0, t)/\tau$. We note that the structure of Eq. (30) follows directly from definitions of the components of $T^{\mu\nu}$. The total quantities include the particle and the field parts altogether, namely, we have

$$\varepsilon_{\text{tot}} = \varepsilon + B, \quad P_\parallel^{\text{tot}} = P_\parallel - B, \quad P_\perp^{\text{tot}} = P_\perp - B, \quad (31)$$

where [62]

$$\varepsilon = g \int \frac{dw d^2k_\perp}{(2\pi)^3 \tau} E(\tau, w, k_\perp) f, \quad (32)$$

$$P_\parallel = g \int \frac{dw d^2k_\perp}{(2\pi)^3 \tau} \frac{w^2 \left[1 - \frac{\gamma_G^4}{(w^2/\tau^2 + k_\perp^2)^2} \right]}{\tau^2 E(\tau, w, k_\perp)} f, \quad (33)$$

$$P_\perp = g \int \frac{dw d^2k_\perp}{(2\pi)^3 \tau} \frac{k_\perp^2 \left[1 - \frac{\gamma_G^4}{(w^2/\tau^2 + k_\perp^2)^2} \right]}{2 E(\tau, w, k_\perp)} f. \quad (34)$$

The standard pressure is then defined as $P = (2P_\perp + P_\parallel)/3$.

In our case, the energy-momentum conservation law, $\partial_\mu T^{\mu\nu} = 0$, simplifies to a single equation

$$\frac{d\varepsilon_{\text{tot}}}{d\tau} = -\frac{\varepsilon_{\text{tot}} + P_{\parallel}^{\text{tot}}}{\tau}. \quad (35)$$

Taking into account (31), this is equivalent to

$$\frac{d\varepsilon}{d\tau} + \frac{dB}{d\tau} = -\frac{\varepsilon + P_{\parallel}}{\tau}. \quad (36)$$

Using (27), (32) and (33), one obtains from (36)

$$\begin{aligned} \frac{dB}{d\tau} + g \int \frac{dw d^2 k_{\perp}}{(2\pi)^3 \tau} \frac{2\gamma_G^3}{\left(\frac{w^2}{\tau^2} + k_{\perp}^2\right) E(\tau, w, k_{\perp})} \frac{d\gamma_G}{d\tau} f \\ + g \int \frac{dw d^2 k_{\perp}}{(2\pi)^3 \tau} E(\tau, w, k_{\perp}) \frac{\partial f}{\partial \tau} = 0. \end{aligned} \quad (37)$$

Before we analyse non-equilibrium features of Eq. (37), it is interesting to discuss the case of local equilibrium. In this case, Eq. (35) can be written as

$$\frac{d\varepsilon_{\text{eq}}}{d\tau} = -\frac{\varepsilon_{\text{eq}} + P_{\text{eq}}}{\tau}. \quad (38)$$

Using thermodynamic identities $dP_{\text{eq}} = s_{\text{eq}} dT$ and $d\varepsilon_{\text{eq}} = T ds_{\text{eq}}$, we obtain from (38) the Bjorken scaling solution for the entropy density [59]

$$s_{\text{eq}}(\tau) = \frac{\tau_0 s_{\text{eq}}(\tau_0)}{\tau}. \quad (39)$$

This allows us to express the sound velocity of system (24) by the formula

$$c_s^2 = \frac{dP_{\text{eq}}}{d\varepsilon_{\text{eq}}} = \frac{\frac{dP_{\text{eq}}}{dT}}{\frac{d\varepsilon_{\text{eq}}}{dT}} = \frac{s_{\text{eq}}}{T \frac{ds_{\text{eq}}}{dT}} = -\frac{d \ln T}{d \ln \tau}. \quad (40)$$

3.3. Mean field and kinetic equations

If considered in equilibrium, the first two terms of Eq. (37) agree with Eq. (20). This suggests that they may be treated as a non-equilibrium extension of Eq. (20). Therefore, as the first dynamic equation for a non-equilibrium system we take²

$$g \int \frac{dw d^2 k_{\perp}}{(2\pi)^3 \tau} \frac{2\gamma_G^3}{\left(\frac{w^2}{\tau^2} + k_{\perp}^2\right) E(\tau, w, k_{\perp})} \frac{d\gamma_G}{d\tau} f + \frac{dB}{d\tau} = 0. \quad (41)$$

² We note that this is a heuristic argument, which suggests subsequently the form of Eqs. (42) and (43). Alternatively, we may postulate first the relaxation time equations (42) and (43), as their form is frequently used, and conclude that Eq. (41) should be satisfied.

On the other hand, from the third term of Eq. (37), we conclude that the distribution function may satisfy the standard relaxation time approximation equation of the form of [62–66]

$$\frac{\partial f(\tau, w, k_{\perp})}{\partial \tau} = \frac{f_{\text{GZ}}(\tau, w, k_{\perp}) - f(\tau, w, k_{\perp})}{\tau_{\text{GZ}}(\tau)}, \quad (42)$$

where

$$\int \frac{dw d^2 k_{\perp}}{\tau} E(\tau, w, k_{\perp}) f_{\text{GZ}}(\tau, w, k_{\perp}) = \int \frac{dw d^2 k_{\perp}}{\tau} E(\tau, w, k_{\perp}) f(\tau, w, k_{\perp}). \quad (43)$$

The quantity τ_{GZ} in (42) is the relaxation time, which specifies the timescale at which the distribution function f approaches the equilibrium form f_{GZ} . The present approach allows for a τ -dependent relaxation time, however, τ_{GZ} should be momentum-independent. In Eq. (43), we recognize the Landau matching condition for the energy. We note that Eq. (42) has a formal solution [62]

$$f(\tau, w, k_{\perp}) = f_0(w, k_{\perp}) D(\tau, \tau_0) + \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_{\text{GZ}}(\tau')} D(\tau, \tau') f_{\text{GZ}}(\tau', w, k_{\perp}) \quad (44)$$

with the damping function $D(\tau_2, \tau_1)$ defined as

$$D(\tau_2, \tau_1) = \exp \left[- \int_{\tau_1}^{\tau_2} d\tau'' \tau_{\text{GZ}}^{-1}(\tau'') \right]. \quad (45)$$

Solution (44) can be used to analyse the system's real off-equilibrium dynamics. However, in this work, we concentrate on the kinetic coefficients solely, hence, it is sufficient to study Eq. (42) in the linear response approximation. The latter applies for small deviations from local equilibrium.

3.4. Bulk and shear viscosities

In order to determine the viscous corrections, we seek the solution of the kinetic equation (42) in the form of

$$f \approx f_{\text{GZ}} + \tau_{\text{GZ}} \delta f + \dots \quad (46)$$

which gives

$$\delta f = -\frac{d f_{\text{GZ}}}{d\tau} = -\frac{E}{T\tau} \left[\frac{w^2}{E^2\tau^2} \left(1 - \frac{\gamma_G^4}{(w^2/\tau^2 + k_\perp^2)^2} \right) + \frac{\tau dT}{T d\tau} \left(1 - \frac{2\gamma_G^3 T}{(w^2/\tau^2 + k_\perp^2) E^2} \frac{d\gamma_G}{dT} \right) \right] f_{\text{GZ}} (1 + f_{\text{GZ}}) . \quad (47)$$

From the definition of the bulk viscosity $\zeta = (P_{\text{GZ}} - P)/\partial_\mu u^\mu$ [67], using Eq. (47) and the Landau matching condition (43) in the linearised form

$$\int dw d^2 k_\perp E \delta f = 0 , \quad (48)$$

we find

$$\zeta = -\frac{2g\gamma_G^4\tau_{\text{GZ}}}{3T(2\pi)^3} \int \frac{dw d^2 k_\perp}{\tau (w^2/\tau^2 + k_\perp^2)} \left[\frac{w^2}{E^2\tau^2} \left(1 - \frac{\gamma_G^4}{(w^2/\tau^2 + k_\perp^2)^2} \right) - c_s^2(T) \left(1 - \frac{2\gamma_G^3 T}{(w^2/\tau^2 + k_\perp^2) E^2} \frac{d\gamma_G}{dT} \right) \right] f_{\text{GZ}} (1 + f_{\text{GZ}}) . \quad (49)$$

Here, we have used the Bjorken, leading-order solution and replaced the term $-\tau dT/(T d\tau)$ by the sound velocity squared, according to Eq. (40). Changing integration variables: $w/\tau = k_\parallel$, $k_\parallel = k \cos \theta$, $k_\perp = k \sin \theta$, and $y = k/\gamma_G$, and integrating over θ from 0 to π , Eq. (49) can be cast into the form of

$$\zeta = \frac{g\gamma_G^5\tau_{\text{GZ}}}{3\pi^2 T} \int_0^\infty dy \left[c_s^2 \left(1 - \frac{2}{y^4 + 1} \frac{d \ln \gamma_G}{d \ln T} \right) - \frac{1}{3} \frac{y^4 - 1}{y^4 + 1} \right] f_{\text{GZ}} (1 + f_{\text{GZ}}) , \quad (50)$$

with the distribution function $f_{\text{GZ}} = [\exp(\gamma_G \sqrt{y^2 + y^{-2}}/T) - 1]^{-1}$. We note that for $\gamma_G = \text{const}$, Eqs. (49) and (50) agree with the formulas derived in Refs. [21, 22].

It is interesting also to compare (50) with the expression for the bulk viscosity which is valid for the case where the dispersion relation (1) is used. Following [37], we find

$$\zeta_M = \frac{gM^5\tau_{\text{rel}}}{6\pi^2 T} \int_0^\infty dy y^2 \left[c_s^2 \left(1 - \frac{1}{y^2 + 1} \frac{d \ln M}{d \ln T} \right) - \frac{1}{3} \frac{y^2}{y^2 + 1} \right] f (1 + f) , \quad (51)$$

where τ_{rel} is the appropriate relaxation time and f is the equilibrium distribution of the form of $f = [\exp(M \sqrt{y^2 + 1}/T) - 1]^{-1}$.

Our results showing temperature dependence of the bulk viscosity (50) are presented in Fig. 8 (solid/red line). The most prominent feature of this behaviour is a sharp peak appearing in the region of the phase transition. The results obtained with the constant value $\gamma_G = 0.7$ GeV [21, 22] (dashed/blue line) agree with the present calculation in the region above the phase transition but they miss the peak structure. The latter appears in the calculation using the dispersion relation (1) but its height and width are significantly smaller.

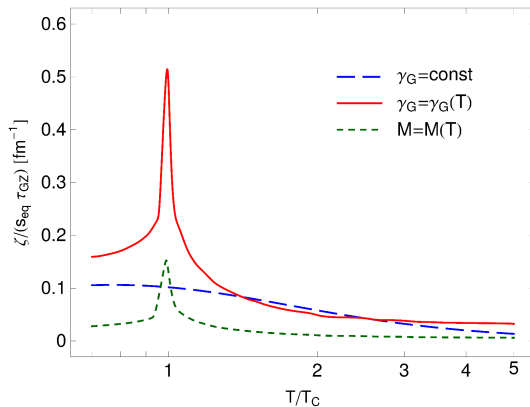


Fig. 8. (Colour on-line) Temperature dependence of the bulk viscosity obtained from Eq. (50) with the Gribov parameter fitted to the lattice data (solid/red line) and with the constant value $\gamma_G = 0.7$ GeV (long-dashed/blue line). The short-dashed/green line shows, for comparison, the bulk viscosity obtained from Eq. (51) with the effective mass $M(T)$ fitted to the lattice results.

It is interesting to observe that in the high temperature limit, where the Gribov parameter becomes proportional to temperature, $\gamma_G = \alpha T$ with $\alpha = \text{const}$, the bulk viscosity (50) vanishes. Similar property has the bulk viscosity (51) if $M/T = \text{const}$.

The same method may be used to calculate the shear viscosity from the relation $\eta = 1/2(P_{\perp} - P_{\parallel})/\partial_{\mu}u^{\mu}$ [67]. In this case, we may keep only anisotropic parts of δf , which leads to the expression

$$\eta = \frac{g\tau_{GZ}}{2T} \int \frac{dw d^2k_{\perp}}{(2\pi)^3\tau} \left(\frac{w^2}{\tau^2} - \frac{k_{\perp}^2}{2} \right) \left(1 - \frac{\gamma_G^4}{(w^2/\tau^2 + k_{\perp}^2)^2} \right)^2 \frac{w^2}{E^2\tau^2} f_{GZ} (1 + f_{GZ}) , \quad (52)$$

or, after changing the variables, to

$$\eta = \frac{g\gamma_G^5 \tau_{\text{GZ}}}{30\pi^2 T} \int_0^\infty dy \frac{(y^4 - 1)^2}{y^4 + 1} f_{\text{GZ}} (1 + f_{\text{GZ}}) . \quad (53)$$

The corresponding formula for the dispersion relation (1) is [37]

$$\eta_M = \frac{g_0 M^5}{30\pi^2} \frac{\tau_{\text{rel}}}{T} \int_0^\infty dy \frac{y^6}{y^2 + 1} f(1 + f) , \quad (54)$$

where, again, $f = \left[\exp(M\sqrt{y^2 + 1}/T) - 1 \right]^{-1}$. The results showing the temperature dependence of the shear viscosity are shown in Fig. 9. For three considered cases, we find a similar monotonic behaviour. We note that we do not obtain the increase of the ratio η/s for $T \rightarrow 0$. This feature is attributed to the chiral properties of pions at low temperatures [68], which are missing in our approach.

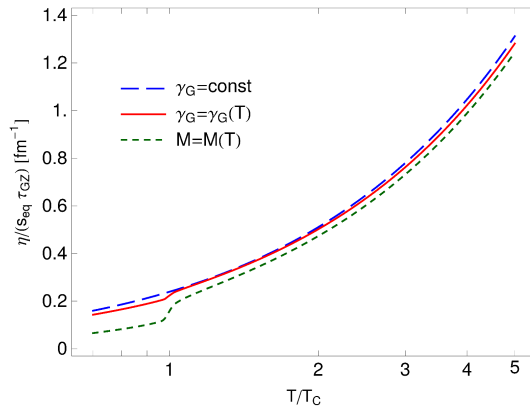


Fig. 9. (Colour on-line) Temperature dependence of the shear viscosity obtained from Eq. (53) with the Gribov parameter fitted to the lattice data (solid/red line) and with the constant value $\gamma_G = 0.7$ GeV (long-dashed/blue line). The short-dashed/green line shows, for comparison, the shear viscosity obtained from Eq. (54) with the effective mass $M(T)$ fitted from the lattice results.

4. Conclusions

In this work, we have calculated the bulk and shear viscosity of the Gribov–Zwanziger plasma in the case where the Gribov parameter depends on temperature. In this way, we have generalised the results obtained earlier

in Refs. [21, 22]. The temperature dependence of the Gribov parameter has been determined from the pure glue lattice data on the entropy density. The overall thermodynamic consistency has been achieved by the use of the temperature-dependent bag pressure.

To calculate the kinetic coefficients, we have introduced the kinetic model based on the relaxation time approximation. We have shown that this approach is consistent with the basic energy-momentum conservation law. The final results for the viscosities are proportional to the relaxation time, however, otherwise, they depend only on the temperature of the system, in particular, via $\gamma_G(T)$ and $B(T)$. The evidence for a peaked structure of the bulk viscosity in the phase transition region has been found. Compared to more standard calculations with a temperature-dependent mass, the use of the Gribov dispersion relation yields a much broader enhancement.

We thank Konrad Tywoniuk and Nan Su for clarifying discussions. V.B. and W.F. were supported by the Polish National Science Center (NCN) grant No. DEC-2012/06/A/ST2/00390. R.R. was supported by the Polish National Science Center (NCN) grant No. DEC-2012/07/D/ST2/02125.

REFERENCES

- [1] V. Goloviznin, H. Satz, *Z. Phys. C* **57**, 671 (1993).
- [2] A. Peshier, B. Kampfer, O.P. Pavlenko, G. Soff, *Phys. Lett. B* **337**, 235 (1994).
- [3] M. Bluhm, B. Kampfer, *Phys. Rev. D* **77**, 034004 (2008).
- [4] F.G. Gardim, F.M. Steffens, *Nucl. Phys. A* **825**, 222 (2009).
- [5] F. Giacosa, *Phys. Rev. D* **83**, 114002 (2011).
- [6] M.I. Gorenstein, S.-N. Yang, *Phys. Rev. D* **52**, 5206 (1995).
- [7] B. Vanderheyden, G. Baym, *J. Statist. Phys.* **93**, 843 (1998).
- [8] J.P. Blaizot, E. Iancu, A. Rebhan, *Phys. Rev. Lett.* **83**, 2906 (1999).
- [9] A. Chodos *et al.*, *Phys. Rev. D* **9**, 3471 (1974).
- [10] J. Baacke, *Acta Phys. Pol. B* **8**, 625 (1977).
- [11] E.V. Shuryak, *Phys. Rep.* **61**, 71 (1980).
- [12] J. Cleymans, R.V. Gavai, E. Suhonen, *Phys. Rep.* **130**, 217 (1986).
- [13] V.N. Gribov, *Nucl. Phys. B* **139**, 1 (1978).
- [14] D. Zwanziger, *Phys. Rev. Lett.* **94**, 182301 (2005).
- [15] D. Zwanziger, *Phys. Rev. D* **76**, 125014 (2007).
- [16] K. Fukushima, N. Su, *Phys. Rev. D* **88**, 076008 (2013).
- [17] F.E. Canfora, *Eur. Phys. J. C* **75**, 326 (2015).

- [18] N. Su, K. Tywoniuk, *Phys. Rev. Lett.* **114**, 161601 (2015).
- [19] A. Bandyopadhyay, N. Haque, M.G. Mustafa, M. Strickland, *Phys. Rev. D* **93**, 065004 (2016).
- [20] S. Borsanyi *et al.*, *J. High Energy Phys.* **1207**, 056 (2012).
- [21] W. Florkowski, R. Ryblewski, N. Su, K. Tywoniuk, *Acta Phys. Pol. B* **47**, 1833 (2016).
- [22] W. Florkowski, R. Ryblewski, N. Su, K. Tywoniuk, *Phys. Rev. C* **94**, 044904 (2016).
- [23] G. Torrieri, B. Tomasik, I. Mishustin, *Phys. Rev. C* **77**, 034903 (2008).
- [24] R.J. Fries, B. Müller, A. Schafer, *Phys. Rev. C* **78**, 034913 (2008).
- [25] G.S. Denicol, T. Kodama, T. Koide, P. Mota, *Phys. Rev. C* **80**, 064901 (2009).
- [26] A. Monnai, T. Hirano, *Phys. Rev. C* **80**, 054906 (2009).
- [27] P. Bożek, *Phys. Rev. C* **81**, 034909 (2010).
- [28] J. Noronha-Hostler *et al.*, *Phys. Rev. C* **88**, 044916 (2013).
- [29] P.B. Arnold, G.D. Moore, L.G. Yaffe, *J. High Energy Phys.* **0011**, 001 (2000).
- [30] P.B. Arnold, G.D. Moore, L.G. Yaffe, *J. High Energy Phys.* **0305**, 051 (2003).
- [31] P.B. Arnold, C. Dogan, G.D. Moore, *Phys. Rev. D* **74**, 085021 (2006).
- [32] G.D. Moore, O. Saremi, *J. High Energy Phys.* **0809**, 015 (2008).
- [33] H.B. Meyer, *Phys. Rev. Lett.* **100**, 162001 (2008).
- [34] H.B. Meyer, *Phys. Rev. D* **76**, 101701 (2007).
- [35] M. Haas, L. Fister, J.M. Pawłowski, *Phys. Rev. D* **90**, 091501 (2014).
- [36] N. Christiansen, M. Haas, J.M. Pawłowski, N. Strodthoff, *Phys. Rev. Lett.* **115**, 112002 (2015).
- [37] C. Sasaki, K. Redlich, *Phys. Rev. C* **79**, 055207 (2009).
- [38] M. Bluhm, B. Kampfer, K. Redlich, *Phys. Rev. C* **84**, 025201 (2011).
- [39] A.S. Khvorostukhin, V.D. Toneev, D.N. Voskresensky, *Phys. Rev. C* **83**, 035204 (2011).
- [40] S. Plumari, W.M. Alberico, V. Greco, C. Ratti, *Phys. Rev. D* **84**, 094004 (2011).
- [41] P. Chakraborty, J.I. Kapusta, *Phys. Rev. C* **83**, 014906 (2011).
- [42] V. Ozvenchuk *et al.*, *Phys. Rev. C* **87**, 064903 (2013).
- [43] J.-W. Chen, J. Wang, *Phys. Rev. C* **79**, 044913 (2009).
- [44] J. Noronha-Hostler, J. Noronha, C. Greiner, *Phys. Rev. Lett.* **103**, 172302 (2009).
- [45] G. Policastro, D.T. Son, A.O. Starinets, *Phys. Rev. Lett.* **87**, 081601 (2001).
- [46] P. Beninca, A. Buchel, A.O. Starinets, *Nucl. Phys. B* **733**, 160 (2006).
- [47] A. Buchel, *Phys. Rev. D* **72**, 106002 (2005).

- [48] S.I. Finazzo, R. Rougemont, H. Marrochio, J. Noronha, *J. High Energy Phys.* **1502**, 051 (2015).
- [49] D. Li, S. He, M. Huang, *J. High Energy Phys.* **1506**, 046 (2015).
- [50] K. Lichtenegger, D. Zwanziger, *Phys. Rev. D* **78**, 034038 (2008).
- [51] G. Boyd *et al.*, *Nucl. Phys. B* **469**, 419 (1996).
- [52] D. Kharzeev, K. Tuchin, *J. High Energy Phys.* **0809**, 093 (2008).
- [53] F. Karsch, D. Kharzeev, K. Tuchin, *Phys. Lett. B* **663**, 217 (2008).
- [54] P. Romatschke, *Phys. Rev. D* **85**, 065012 (2012).
- [55] M. Alqahtani, M. Nopoush, M. Strickland, *Phys. Rev. C* **92**, 054910 (2015).
- [56] V.V. Begun, M.I. Gorenstein, O.A. Mogilevsky, *Ukr. J. Phys.* **55**, 1049 (2010).
- [57] V.V. Begun, M.I. Gorenstein, O.A. Mogilevsky, *Int. J. Mod. Phys. E* **20**, 1805 (2011).
- [58] M. Chojnacki, W. Florkowski, *Acta Phys. Pol. B* **38**, 3249 (2007).
- [59] J.D. Bjorken, *Phys. Rev. D* **27**, 140 (1983).
- [60] A. Bialas, W. Czyz, *Phys. Rev. D* **30**, 2371 (1984).
- [61] A. Bialas, W. Czyz, A. Dyrek, W. Florkowski, *Nucl. Phys. B* **296**, 611 (1988).
- [62] W. Florkowski, R. Ryblewski, M. Strickland, *Phys. Rev. C* **88**, 024903 (2013).
- [63] P.L. Bhatnagar, E.P. Gross, M. Krook, *Phys. Rev.* **94**, 511 (1954).
- [64] G. Baym, *Phys. Lett. B* **138**, 18 (1984).
- [65] G. Baym, *Nucl. Phys. A* **418**, 525C (1984).
- [66] W. Florkowski, R. Ryblewski, M. Strickland, *Nucl. Phys. A* **916**, 249 (2013).
- [67] A. Muronga, *Phys. Rev. C* **69**, 034903 (2004).
- [68] L.P. Csernai, J. Kapusta, L.D. McLerran, *Phys. Rev. Lett.* **97**, 152303 (2006).