LIQUID-GAS PHASE TRANSITION IN HOT ASYMMETRIC MATTER WITH NL3*

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The liquid-gas phase transition of asymmetric nuclear matter is studied within the relativistic mean-field model with nonlinear isoscalar–isovector coupling. The density dependence of symmetry energy constrained from the measured neutron skin thickness of finite nuclei has also been used in the investigation. It is found that several features of liquid-gas phase transition, such as, liquid-gas coexistence region, critical values of pressure and isospin asymmetry and maximal isospin asymmetry increase with the softness of density dependence of symmetry energy. It is also found that the critical temperature is higher for softer symmetry energy.

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1. Introduction

The study of nuclear matter under extreme conditions such as large isospin asymmetry and high temperature have been of great interest for a number of years [1–9]. The intermediate energy heavy-ion collisions using neutron-rich radioactive beams indicate the features of theoretically predicted liquid-gas phase (LGP), in which the hot and compressed nucleus expands and breaks into several mass fragments (high-density liquid phase) and nucleons and light particles (low-density gas phase).

The earlier theoretical studies related to the thermodynamic properties of LGP transition were focused on symmetric nuclear matter, which is a one-component system [10–12]. On the other hand, the asymmetric nuclear matter is a two-component system, characterized by the baryon number and the third component of isospin, I_3. One of the main ingredients to study LGP transition in asymmetric nuclear matter is the density dependence of nuclear symmetry energy, $E_{\mathrm{sym}}(\rho)$. Unfortunately, patterns of behavior of symmetry energy predicted by various models are extremely diverse [13] except at nuclear saturation density, $\rho_0 \approx 0.16\ \mathrm{fm}^{-3}$, where, $E_{\mathrm{sym}}(\rho_0, T = 0) = 32 \pm 4$ MeV, which has been well-constrained.

Recently, some progress has been made to constrain symmetry energy around normal matter density from the experimental isoscaling data of intermediate heavy-ion collisions [14], isospin diffusion data [15, 16] and from the study of neutron skin thickness of several nuclei [17, 18]. Very recently, neutron skin data for ^{208}Pb from parity-violation experiment has been reported [19]. It is, therefore, very important to study the LGP transition in hot asymmetric nuclear matter with the equation of state (EOS) that has been constrained by these new experimental findings. Such theoretical investigations are expected to provide a basic understanding for future experiments with more neutron-rich radioactive ion beams.

In this paper, our aim is to understand the effect of well-constrained symmetry energy on the thermodynamic properties of LGP transition of hot asymmetric nuclear matter within the framework of the relativistic mean-field (RMF) model [20–22].

The paper is organized as follows: the RMF model with isoscalar–isovector coupling (Λ_v) has been introduced and the basic features of LGP transition have been described in Section 2. The numerical results for hot asymmetric nuclear have been presented and discussed in Section 3. The summary and conclusions are presented in the last section.

2. Formulation

The interaction Lagrangian density in the nonlinear RMF model is given by [18, 23]

$$\begin{aligned}\mathcal{L} \;=\; & \bar{\psi}\left[g_s\phi - \left(g_v V_\mu + \frac{g_\rho}{2}\tau\cdot\boldsymbol{b}_\mu + \frac{e}{2}\left(1+\tau_3\right)A_\mu\right)\gamma^\mu\right]\psi - \frac{\kappa}{3!}\left(g_s\phi\right)^3 \\ & -\frac{\lambda}{4!}\left(g_s\phi\right)^4 + \frac{\zeta}{4!}g_v^4\left(V_\mu V^\mu\right)^2 + \Lambda_v\left(g_\rho^2\boldsymbol{b}_\mu\cdot\boldsymbol{b}^\mu\right)\left(g_v^2 V_\mu V^\mu\right)\,. \qquad (1)\end{aligned}$$

Here, ψ is the isospin doublet nucleon field, interacting by the exchange of isoscalar–scalar σ (ϕ), isoscalar–vector ω (V^μ), isovector–vector ρ ($\boldsymbol{b}^\mu$), and the photon field (A^μ), respectively. The EOS of symmetric nuclear around the saturation density ρ_0 softens due to the nonlinear couplings κ and λ of σ meson, while the high density part is softens by the self-interaction coupling ζ of the ω meson.

The symmetry energy for Eq. (1) is given by

$$E_{\mathrm{sym}}(\rho) = \frac{k_{\mathrm{F}}^2}{6E_{\mathrm{F}}^*} + \frac{g_\rho^2}{12\pi^2}\frac{k_{\mathrm{F}}^3}{m_\rho^{*2}}\,, \qquad (2)$$

where $E_{\rm F}^* = \sqrt{k_{\rm F}^2 + m^{*2}}$, $k_{\rm F}$ and $m^* = m - g_s\phi_0$ are the Fermi energy, Fermi momentum and effective mass of the nucleon, respectively. The effective ρ meson mass is $m_\rho^{*2} = m_\rho^2 + 2g_\rho^2(\Lambda_v g_v^2 V_0^2)$. The Λ_v coupling modifies the density dependent of symmetry energy by m_ρ^{*2} without affecting the saturation properties.

In the present work, we use the Lagrangian parameter set NL3* [24] with different Λ_v couplings. The original NL3* parameter set was obtained by fitting the model parameters to certain ground state properties of the finite nuclei [24]. There, the Λ_v and ζ couplings were taken to be zero. For our present calculations, we use the extended NL3* model [18, 23, 25] with isovector coupling Λ_v, which is then varied along with g_ρ to generate various $E_{\rm sym}(\rho)$.

Since only the average value of $E_{\rm sym}(\rho, T=0)$ is constrained by binding energy at the full saturation density which corresponds to $k_{\rm F} = 1.30$ fm^{-1} [25, 26], we have obtained all the combinations of Λ_v and g_ρ by adjusting to a constant $E_{\rm sym}(\bar\rho, T=0) = 26.345$ MeV for NL3*, at an average density $\bar\rho$ which corresponds to $k_{\rm F} = 1.15$ fm^{-1}. In this range of Λ_v, the values of the binding energy per particle for ^{208}Pb are found to be within the uncertainty of the measured value of $B/A = 7.87 \pm 0.02$ MeV, and the proton density distribution is also practically unaltered. We then use $\Lambda_v = 0.00$ and 0.03 to explore the effect of $E_{\rm sym}(\rho)$ on the LGP transition in hot asymmetric nuclear matter. We choose these two particular values of Λ_v because the resulting values of the neutron skin of ^{208}Pb turn out to be 0.29 fm and 0.20 fm respectively, which are well within the limits of the recently measured neutron skin data [19]. Further, for $\Lambda_v = 0.00$ and 0.03, the resulting symmetry energies, their slope parameter L and $K_{\rm asym}$ are in a reasonable agreement with the other experimental results, such as, neutron skins of several nuclei as well as the isoscaling and isospin diffusion data [15–17]. The values of Λ_v and g_ρ couplings along with the slope parameter L and $K_{\rm asym}$ are listed in Table I.

TABLE I

The isoscalar–isovector couplings Λ_v and g_ρ^2 along with slope parameter L and $K_{\rm asym} \approx K_{\rm sym} - 6L$ for NL3*.

Λ_v	g_ρ^2	L [MeV]	$K_{\rm asym}$ [MeV]
0.00	83.72	122.64	−630.24
0.01	96.06	88.84	−586.45
0.02	112.69	67.82	−461.04
0.03	136.26	54.29	−325.94

The energy density $\mathcal{E}$ which is obtained by the thermodynamic potential Ω [3] at finite temperature and density is given by

$$\mathcal{E} = \frac{2}{(2\pi)^3}\sum_{q=n,p}\int \mathrm{d}^3k E^*(k)\left([n_q(k)]_+ + [n_q(k)]_-\right) + \frac{m_s^2\phi^2}{2} + \frac{\kappa}{3!}(g_s\phi)^3 + \frac{\lambda}{4!}(g_s\phi)^4 + \frac{m_v^2V_0^2}{2} + \frac{\zeta}{8}(g_vV_0)^4 + \frac{m_\rho^2b_0^2}{2} + 3\Lambda_v (g_vV_0)^2(g_\rho b_0)^2 \,. \quad (3)$$

The distribution function for nucleons and anti-nucleons (denoted by the subscript $\pm$), as usual [3, 27], is taken to be

$$[n_q(k)]_\pm = \frac{1}{\exp\left[(E^*(k)\mp\nu_q)/T\right]+1}\,, \qquad (q=n,p) \quad (4)$$

where, $E^*(k) = \sqrt{k^2+m^{*2}}$ and $\nu_q = \mu_q - g_vV_0 \pm g_\rho b_0/2$ are the effective energy and effective chemical potential of the nucleons, respectively. The chemical potentials can be determined from the conserved baryon and isospin densities

$$\rho = \frac{2}{(2\pi)^3}\int \mathrm{d}^3k\,[G_p(k)+G_n(k)]\,, \quad (5)$$

$$\rho_3 = \frac{2}{(2\pi)^3}\int \mathrm{d}^3k\,[G_p(k)-G_n(k)]\,, \quad (6)$$

where $G_q(k) = [n_q(k)]_+ - [n_q(k)]_-$, $(q=n,p)$.

The asymmetric nuclear matter is stable against LGP if its free energy F is lower than the coexisting liquid (L) and gas (G) phases *i.e.*, $F(T,\rho) < (1-\chi)F^{\mathrm{L}}(T,\rho^{\mathrm{L}}) + \chi F^{\mathrm{G}}(T,\rho^{\mathrm{G}})$ with the density $\rho = (1-\chi)\rho^{\mathrm{L}} + \chi\rho^{\mathrm{G}}$, where $0<\chi<1$ and $\chi = V^{\mathrm{G}}/V$ is the volume fraction of the total volume occupied by the gas phase. The stability condition implies the following inequalities [3]:

$$\rho\left(\frac{\partial P}{\partial\rho}\right)_{T,\alpha} > 0\,, \quad (7)$$

$$\left(\frac{\partial\mu_p}{\partial\alpha}\right)_{T,P} < 0 \qquad \text{or} \qquad \left(\frac{\partial\mu_n}{\partial\alpha}\right)_{T,P} > 0\,, \quad (8)$$

where $\alpha = (\rho_n - \rho_p)/\rho$ is the isospin asymmetry parameter.

Equation (7) indicates mechanical stability which means that at positive isothermal compressibility, a system remains stable at all the densities. Equation (8) stems from chemical instability which shows that some energy is required to change the concentration in a stable system while maintaining pressure and temperature fixed. A system with two phases is energetically favorable, if any one of these conditions *i.e.*, Eqs. (7), (8) gets violated. The two phase coexistence is governed by Gibbs's criteria

$$\mu_q\left(T,\rho^{\mathrm{L}}\right) = \mu_q\left(T,\rho^{\mathrm{G}}\right) \qquad (q=n,p)\,, \tag{9}$$

$$P\left(T,\rho^{\mathrm{L}}\right) = P\left(T,\rho^{\mathrm{G}}\right)\,. \tag{10}$$

3. Results and discussion

The density dependence of symmetry energy, $E_{\mathrm{sym}}(\rho,T=0)$, at different values of isoscalar–isovector couplings Λ_v for the parameter set NL3* has been presented in Fig. 1. It is found that a stiffer symmetry energy at supranormal densities ($\rho \geq \rho_0$) leads to a softer energy dependence at subsaturation densities. At $\Lambda_v = 0.00$, the density dependence of symmetry energy is almost linear with increasing density. On the other hand, for the finite values of Λ_v coupling, the symmetry energy increases faster up to $2\rho_0$ and then it increases slowly (almost saturated) with increasing ρ. This behavior of symmetry energy with increasing ρ can be understood from Eq. (2), since for $\Lambda_v = 0.00$, $E_{\mathrm{sym}}(\rho) \propto k_{\mathrm{F}}^3 \propto \rho$. Similarly, for the finite values of Λ_v coupling, the repulsive potential $U_v = g_v V_0$ contributes to m_ρ^* and this makes symmetry energy softer at high densities. In the inset of Fig. 1, we have shown the density dependence of symmetry energy for $\Lambda_v = 0.00$ and 0.03 up to the saturation density, ρ_0.

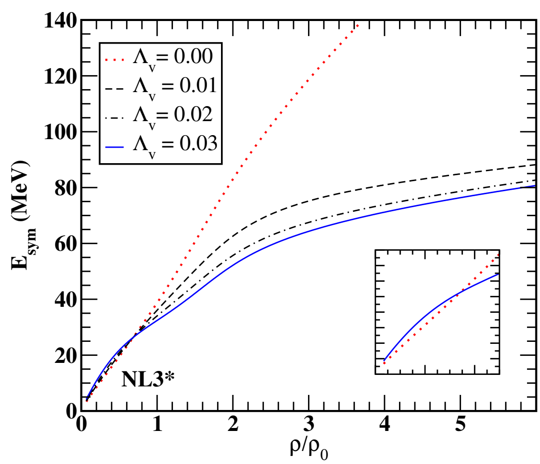

Fig. 1. Density dependence of symmetry energy for NL3* with different couplings of Λ_v. The inset shows the variation for $\Lambda_v = 0.00$ and 0.03 respectively for NL3* up to the nuclear saturation density.

The pressure as a function of ρ with different values of α at fixed temperature $T = 10$ MeV in the original NL3* model ($\Lambda_v = 0.00$) and NL3* with $\Lambda_v = 0.03$ has been presented in the upper and the lower panel of Fig. 2, respectively. As seen from Fig. 2, for both $\Lambda_v = 0.00$ and 0.03, the pressure (dotted curves) decreases with increasing ρ below a critical value of α which indicates a negative incompressibility and, thereby, a mechanically unstable system. The stable LGP (two phase) configuration is obtained by the Maxwell construction (solid lines) at each value of ρ. This feature is analogous to the intermediate energy heavy-ion collisions [28, 29] when the hot matter in liquid phase (high density) expands it and enters in the coexistence LGP where the pressure decreases for the two-component asymmetric matter at a fixed $\alpha \neq 0$ and, finally, the system leaves the coexistence region and vaporizes into the gas (low density) phase. For symmetric nuclear matter ($\alpha = 0$), the pressure remains constant at all the densities. As far as the effect of symmetry energy on the isotherms is concerned, it is found that NL3* with $\Lambda_v = 0.03$ enforces the onset of pure liquid phase to a higher

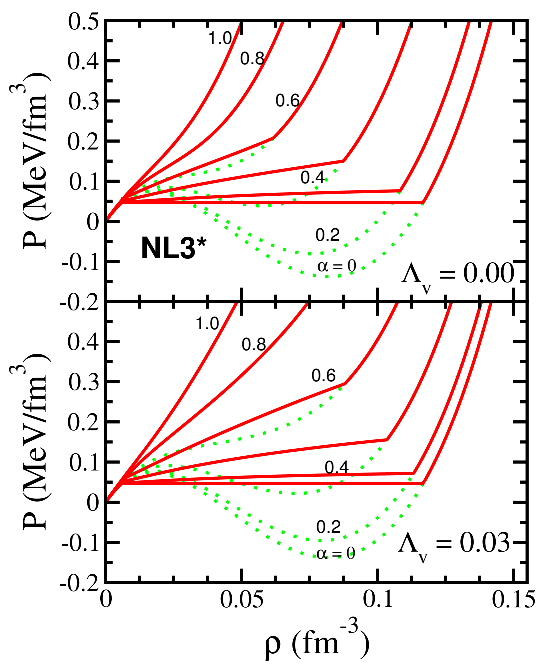

Fig. 2. The pressure of asymmetric nuclear matter for various isospin asymmetry parameter α with respect to density ρ at temperature $T = 10$ MeV for NL3* with $\Lambda_v = 0.00$ and 0.03 are shown in the upper and the lower panel, respectively. The solid curves refer to the stable matter, while dotted curves refer to the unstable single phase matter.

density compared to the original NL3* ($\Lambda_v = 0.00$). This suggests a wider coexistence region and higher value of critical pressure P_c (above which the mixed LGP vanishes) for NL3* with $\Lambda_v = 0.03$ for each α. The detailed discussion is presented below.

For NL3* with $\Lambda_v = 0.00$ and 0.03, the chemical potentials for neutrons and protons as a function of α at a fixed temperature $T = 10$ MeV and pressure $P = 0.11$ MeV/fm^{-3} are shown in Fig. 3. The bare nucleon mass has been subtracted from the chemical potentials. For fixed pressure and Λ_v, the solutions of Gibbs conditions (Eqs. (9) and (10)) require equal pressures and chemical potentials for two phases with different concentrations. Such solutions can be found by means of the geometrical construction as shown in Fig. 3. For both the values of Λ_v, two different values of α define high-density liquid phase boundary with small $\alpha = \alpha_1(T, P)$ and low-density gas phase boundary with large $\alpha = \alpha_2(T, P)$ respectively.

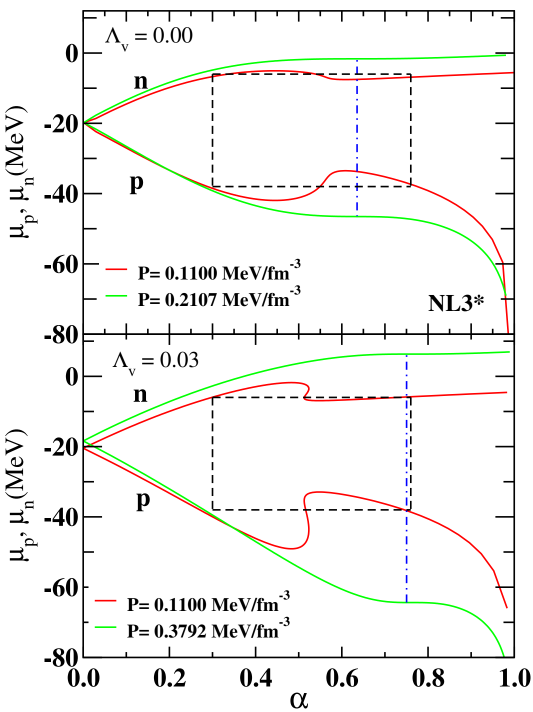

Fig. 3. The chemical potential isobars as a function of isospin asymmetry parameter α with $\Lambda_v = 0.00$ and 0.03 for NL3* at fixed temperature $T = 10$ MeV are shown in the upper and the lower panel, respectively.

In Fig. 3, we plot the chemical potentials for neutrons and protons as a function of α for two values of pressure $P = 0.11$ MeV/fm^{-3} and the critical pressure $P_{\rm c}$, respectively. We find that for $P = 0.11$ MeV/fm^{-3}, there is not much variation in phase boundaries $\alpha = \alpha_1(T, P)$ and $\alpha = \alpha_2(T, P)$ for $\Lambda_v = 0.00$ and 0.03. On the other hand, the critical value of $P_{\rm c}$ is different for $\Lambda_v = 0.00$ and 0.03. This observation indicates that the symmetry energy dependence of Λ_v in NL3* leads to different phase boundaries $\alpha = \alpha_1(T, P)$ and $\alpha = \alpha_2(T, P)$ between these two values of pressure for a given Λ_v. Hence, it is expected to predict different thermodynamic properties for LGP transition for different values of Λ_v.

The critical pressure $P_{\rm c}$ for LGP is the pressure beyond which the matter is stable, but below which second inequality (8) gets violated. The critical pressure $P_{\rm c}$ is determined by $(\partial\mu/\partial\alpha)_{T,P_{\rm c}} = \left(\partial^2\mu/\partial\alpha^2\right)_{T,P_{\rm c}} = 0$. In Fig. 3, we also show the chemical potential isobars at critical pressure (light gray/green solid lines, color online). The rectangle from Gibbs condition then collapses into a vertical line at $\alpha \equiv \alpha_{\rm c}$. The values of $P_{\rm c}$ and $\alpha_{\rm c}$ at a given temperature define the critical point which refers to the upper boundary of instability with respect to the pressure variation. The critical points $P_{\rm c}, \alpha_{\rm c}$ at $T = 10$ MeV for NL3* with $\Lambda_v = 0.00$ and 0.03 are $(0.2107, 0.6354)$ and $(0.3792, 0.7503)$, respectively. We find that an overall softer symmetry energy (larger Λ_v) which corresponds to the stiffer behavior at sub-saturation densities gives systematically larger critical pressure and an enhanced asymmetry in the system.

All the pairs of solutions of Gibbs conditions, $\alpha_1(T, P)$ and $\alpha_2(T, P)$ describe a binodal surface which is shown in Fig. 4. The binodal surface is divided into two branches by the critical point (CP) and a point of equal

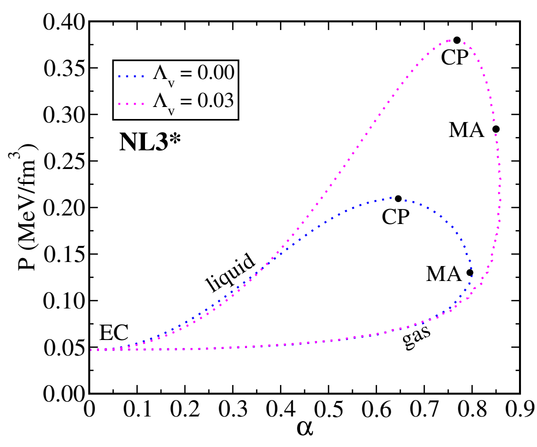

Fig. 4. The binodal surface for NL3* with $\Lambda_v = 0.00$ and 0.03 at temperature $T = 10$ MeV.

concentration (EC). The EC corresponds to symmetric nuclear matter which is independent of Λ_v. One branch corresponds to the high-density (liquid) phase and the other one corresponds to low-density (gas) phase, respectively. The liquid phase is less asymmetric compared to the gas phase. It is found that the critical point P_c, α_c depends on the density dependence of symmetry energy which corresponds to different values of Λ_v.

In Fig. 4, we also indicate the maximal isospin asymmetry (MA), $\alpha_{\rm MA}$ which means the minimal proton fraction of the system. Thus, more neutron-rich matter when compressed/expanded at fixed α will never encounter a co-existence phase. It is also found that $\alpha_{\rm MA}$ is quite sensitive to the symmetry energy.

It is quite evident that the studies such as the present work have a strong influence on experimentally observed isospin distillation phenomena [30]. In isospin distillation phenomena, gas is more neutron-rich compared to the more symmetric liquid phase and the magnitude of isospin distillation is more sensitive to the symmetry energy used. However, it is difficult to access it experimentally.

In Fig. 5, we show the critical temperature T_c as a function of α for NL3* with $\Lambda_v = 0.00$ and 0.03. The critical temperature T_c of LGP transition for symmetric nuclear matter ($\alpha = 0$) is 14.4 MeV. It has been found that T_c decreases rapidly for $\alpha_c \geq 0.4$. The figure also reveals that for larger Λ_v (softer density dependence of symmetry energy), the coexisting LGP have larger values of T_c.

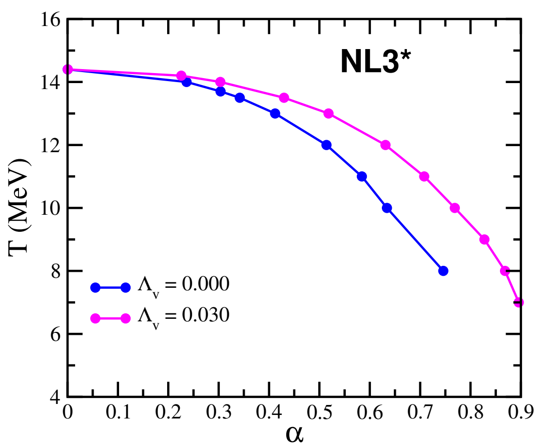

Fig. 5. The critical temperature T_c as a function of isospin asymmetry parameter α for NL3* with $\Lambda_v = 0.00$ and 0.03 are shown.

4. Summary and conclusions

In summary, we investigate the effect of isoscalar–isovector interaction on LGP transition in hot asymmetric nuclear matter. For this purpose, the NL3* [24] parameter set with $\Lambda_v = 0.00$ and 0.03 has been used. The symmetry energy $E_{\rm sym}$ at zero temperature is well-constrained by the experimental isoscaling data of intermediate heavy-ion collisions [14], isospin diffusion data [15, 16], and the neutron skin thickness of several nuclei [17, 18] and also the recently measured neutron skin for ^{208}Pb from parity-violation experiment [19]. We found a considerable sensitivity of symmetry energy on the basic features of LGP. Thus, our present investigation indicates that the precise information on the density dependence of symmetry energy may be obtained from the analysis of observables related to LGP transition in future experiments with exotic beams.

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REFERENCES

[1] P. Bonche *et al.*, *Nucl. Phys. A* **436**, 265 (1985).
[2] J. Besprovany, S. Levit, *Phys. Lett. B* **217**, 1 (1989).
[3] H. Müller, B.D. Serot, *Phys. Rev. C* **52**, 2072 (1995).
[4] M. Baldo, L.S. Ferreira, *Phys. Rev. C* **59**, 682 (1999).
[5] W.L. Qian, R.K. Su, *Phys. Lett. B* **520**, 217 (2001).
[6] Ph. Chomaz, M. Colonna, J. Randrup, *Phys. Rep.* **389**, 263 (2004).
[7] V. Baran, M. Colonna, V. Greco, M.D. Toro, *Phys. Rep.* **410**, 335 (2005).
[8] J. Xu, L.W. Chen, B.A. Li, H.R. Ma, *Phys. Lett. B* **650**, 348 (2007).
[9] B.K. Sharma, S. Pal, *Phys. Rev. C* **81**, 064304 (2010).
[10] D.Q. Lamb, J.M. Lattimer, C.J. Pethick, D.G. Ravenhall, *Phys. Rev. Lett.* **41**, 1623 (1978).
[11] G.F. Bertsch, P.J. Siemens, *Phys. Lett. B* **126**, 9 (1983).
[12] S.J. Lee, A.Z. Mekjian, *Phys. Rev. C* **63**, 044605 (2001).
[13] P. Danielewicz, R. Lacey, W.G. Lynch, *Science* **298**, 1592 (2002).
[14] D.V. Shetty, S.J. Yennello, G.A. Souliotis, *Phys. Rev. C* **76**, 024606 (2007).
[15] L.W. Chen, C.M. Ko, B.A. Li, *Phys. Rev. Lett.* **94**, 032701 (2005).
[16] B.A. Li, L.W. Chen, C.M. Ko, *Phys. Rep.* **464**, 113 (2008).

[17] M. Centelles, X. Roca-Maza, X. Viñas, M. Warda, *Phys. Rev. Lett.* **102**, 122502 (2009).

[18] B.K. Sharma, S. Pal, *Phys. Lett. B* **682**, 23 (2009).

[19] S. Abrahamyan *et al.*, *Phys. Rev. Lett.* **108**, 112502 (2012).

[20] Y.K. Gambhir, P. Ring, A. Thimet, *Ann. Phys. (NY)* **198**, 132 (1990) and references cited therein.

[21] P. Ring, *Prog. Part. Nucl. Phys.* **37**, 193 (1996) and references cited therein.

[22] D. Vretenar, A.V. Afanasjev, G.A. Lalazissis, P. Ring, *Phys. Rep.* **409**, 101 (2005).

[23] B.G. Todd-Rutel, J. Piekarewicz, *Phys. Rev. Lett.* **95**, 122501 (2005).

[24] G.A. Lalazissis *et al.*, *Phys. Lett. B* **671**, 36 (2009).

[25] C.J. Horowitz, J. Piekarewicz, *Phys. Rev. C* **66**, 055803 (2002).

[26] C.J. Horowitz, J. Piekarewicz, *Phys. Rev. Lett.* **86**, 5647 (2001).

[27] P. Wang, *Phys. Rev. C* **61**, 054904 (2000).

[28] J.B. Elliot *et al.*, *Phys. Lett. B* **418**, 34 (1998).

[29] J. Pochodzalla *et al.*, *Phys. Rev. Lett.* **75**, 1040 (1995).

[30] H.S. Xu *et al.*, *Phys. Rev. Lett.* **85**, 716 (2000).