Vol. 48 (2017) *ACTA PHYSICA POLONICA B* No 3

GCM+GOA ELECTROMAGNETIC MULTIPOLE TRANSITION OPERATORS AND SYMMETRIES OF GENERATING FUNCTIONS*

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(Received December 14, 2016)

An idea of symmetry-dependent form of the electromagnetic transition operators is presented by making use of the Generator Coordinate Method and the Gaussian Overlap Approximation. Using this approximation, it turns out that the form of electromagnetic transition operators acting in the collective nuclear space can be helpful in recognition of symmetries in nuclear spectra.

DOI:10.5506/APhysPolB.48.281

1. Introduction

One of the most popular methods of constructing nuclear collective models is the prescription given by Bohr and his collaborators many years ago [1–3]. However, in this case, a construction of quantum observables is not always unique, even the choice of space of quantum states is to some degree arbitrary. The electromagnetic multipole transition operators are a good example of these difficulties. In fact, there are not many papers devoted to this problem. Usually, the collective electromagnetic transition operators

* Presented at the Zakopane Conference on Nuclear Physics "Extremes of the Nuclear Landscape", Zakopane, Poland, August 28–September 4, 2016.

are obtained on a phenomenological basis. Their forms are rather a kind of guess based on expected transformation properties of these operators with a number of phenomenological coefficients which allow to fit the theoretical transition probabilities to the experimental data.

The phenomenological collective electric transition operators based on the generalized collective model are a typical example of this procedure [4].

The more fundamental way is to construct the collective transition operators from their transformation properties using the so-called prime factors [5]. However, the full set of required (in practice) prime factors is unknown. In addition, the formulae contain many arbitrary unknown scalar functions.

In this paper, we show an example of derivation of collective electromagnetic multipole transition operators making use of the Generator Coordinate Method with the generalized Gaussian Overlap Approximation [6, 7].

These results are discussed in the context of structure of the GCM collective space and symmetries of generating functions. Using an example of two types of axial symmetry SO(2) and D_∞, we analyse a kind of "selection rules" implied by the structure of constructed collective transition operators.

These "selection rules" can be used as an additional tool to the standard symmetry selection rules based on the Kronecker products for irreducible representations [8].

2. Generator Coordinate Method with Gaussian Approximation for non-Hermitian operators

The extended GCM+GOA method is designed for Hermitian operators [7]. In principle, GCM+GOA method allows to transform the Hermitian operators, like Hamiltonians, acting on the many-body nucleon state space, into the collective state space determined by the collective variables $q = (q^1, q^2, \ldots, q^s)$ parametrizing the trial function

$$|\Psi(q)\rangle = \int \mathrm{d}q\, f(q)|q\rangle\,. \tag{1}$$

Using this trial function and assuming the Gaussian Overlap Approximation, any matrix element of the Hermitian operator $\hat{A}$ can be expressed in terms of the collective space

$$\langle\Psi_2|\hat{A}|\Psi_1\rangle \approx \int \mathrm{d}q\sqrt{|g|}\,\phi_2(q)^\star\left(\hat{V}_A + \hat{F}_A + \hat{T}_A\right)\phi_1(q)\,, \tag{2}$$

where $\hat{A} \to \hat{\mathcal{A}} \approx \hat{V}_A + \hat{F}_A + \hat{T}_A$, and the collective functions are of the form of

$$\phi_k(q) = \int \mathrm{d}q' f_k\left(q'\right)\, \mathcal{N}^{1/2}\left(q, q'\right)\,. \tag{3}$$

Here, $\mathcal{N}^{1/2}(q,q')$ denotes the square root of the overlap of the generator function, see [7].

The 0-order $(\hat{V})$, 1-order $(\hat{F})$ and 2-order $(\hat{T})$ realizations of the operator $\hat{\mathcal{A}}$ are given by some combinations of the covariant derivatives of the overlap functions and can be found, *e.g.*, in [7].

As it was mentioned above, the transition operators are not-Hermitian operators. To use the GCM+GOA method, one needs to rewrite the transition operators as the combination of two Hermitian operators [9]

$$\hat{M}(\xi;lm) = \hat{M}_{+}(\xi;lm) + i\hat{M}_{-}(\xi;lm)\,, \tag{4}$$

where

$$\begin{aligned}
\hat{M}_{+}(\xi;lm) &= \frac{1}{2}\left(\hat{M}(\xi;lm) + \hat{M}(\xi;lm)^{\dagger}\right) \\
&= \frac{1}{2}\left(\hat{M}(\xi;lm) + (-1)^{m}\hat{M}(\xi;l,-m)\right)\,, \\
\hat{M}_{-}(\xi,lm) &= \frac{-i}{2}\left(\hat{M}(\xi;lm) - \hat{M}(\xi;lm)^{\dagger}\right) \\
&= \frac{-i}{2}\left(\hat{M}(\xi;lm) + (-1)^{m+1}\hat{M}(\xi;l,-m)\right)\,.
\end{aligned} \tag{5}$$

The label $\xi = E, M$ denotes either electric or magnetic transitions, respectively. The symbol l denotes the multipolarity and $m = -l, -l+1, \ldots, l$ labels SO(3) tensor components of the transition tensor operator.

The collective representation $\hat{Q}(\xi;lm)$ of $\hat{M}(\xi;lm)$ within GCM+GOA approximation can be thus written as follows [9]:

$$\hat{M}(\xi;lm) \to \hat{Q}(\xi;lm) \approx \left(\hat{V}_{+} + i\hat{V}_{-}\right) + \left(\hat{F}_{+} + i\hat{F}_{-}\right) + \left(\hat{T}_{+} + i\hat{T}_{-}\right)\,. \tag{6}$$

3. Nuclear shape symmetry and the collective transition operators

The generating function of the GCM method is usually chosen by means of some microscopic calculations to reproduce approximate configurations of nucleons in a nucleus. Obviously, if this nucleus has a shape symmetry, this property should be reproduced with the same symmetry of the generating function. In such cases, the collective transition operators can have special features implied by these symmetries.

In this section, to present the predictive power of shape symmetry-dependent collective transition operators, we consider two simple cases of axially symmetric nuclear shapes which correspond to two different symmetry groups: D_{∞} and SO(2).

The first group D_∞ has a reacher structure because it consists of not only all rotations $R_z(\mathit{\Omega}_3)$ around the z-axis but also the rotations about the angle π around the infinite number of axes perpendicular to the z-axis. This means that generators of this group are the rotations $R_z(\mathit{\Omega}_3)$ and, for example, the rotation C_{2y} around the y-axis about the angle π.

The D_∞-symmetric nuclear shapes are the axially symmetric shapes which are identical on both ends of the body along the z-axis like the regular ellipsoid. It excludes the octupole shapes, *e.g.*, pear-like shapes.

The second axial symmetry group is the group $\mathrm{SO}(2) \subset \mathrm{D}_\infty$ generated only by the rotations $R_z(\mathit{\Omega}_3)$ and which allows, *e.g.*, for octupole pear-like nuclear shapes.

There is a question, if the collective transition operators obtained for these two similar but different shape symmetries allow to distinguish between both cases.

To answer the question, let us consider the generating functions consisting of the rotation operator $\hat{R}(\mathit{\Omega})$ parameterized by the Euler angles $\mathit{\Omega} = (\mathit{\Omega}_1, \mathit{\Omega}_2, \mathit{\Omega}_3)$ and the intrinsic generating function describing vibrating nuclear shape with the appropriate symmetry

$$|\mathit{\Omega}\beta\rangle = \hat{R}(\mathit{\Omega})|\beta\rangle\,. \tag{7}$$

In the following, for simplicity, we are using only one vibrational collective variable β, *i.e.*, we consider 3 rotational degrees of freedom $\mathit{\Omega}$ and one vibrational β. However, because of the axial symmetry of the intrinsic generating function, the third Euler angle $\mathit{\Omega}_3$ is irrelevant and should be excluded from the collective manifold.

In the first case of D_∞-invariant distribution of nucleons, the intrinsic generating function has to fulfil two conditions:

$$\hat{R}(0,0,\mathit{\Omega}_3)|\beta\rangle = |\beta\rangle\,, \qquad \hat{C}_{2y}|\beta\rangle = |\beta\rangle\,. \tag{8}$$

Effectively, the generating functions do not depend on $\mathit{\Omega}_3$.

The metric tensor of this three-dimensional collective manifold is diagonal, where the rotation and vibration parts can be written in the following form, $k, k' = 1, 2$:

$$\begin{aligned} g_{\mathit{\Omega}_1\mathit{\Omega}_1} &= \sin^2\mathit{\Omega}_2\langle\beta|\left(\hat{J}_y\right)^2|\beta\rangle\,, \qquad g_{\mathit{\Omega}_2\mathit{\Omega}_2} = \langle\beta|\left(\hat{J}_y\right)^2|\beta\rangle\,,\\ g_{\beta\beta} &= \langle\beta|\frac{\overleftarrow{\partial}}{\partial\beta}\frac{\overrightarrow{\partial}}{\partial\beta}|\beta\rangle\,. \end{aligned} \tag{9}$$

Here, the operators $\hat{J}_k$ are components of the total angular momentum operator in the nucleon space.

In the following, we consider the collective transition operators expanded up to the first non-vanishing term (either $\hat{V}$ or $\hat{F}$ or $\hat{T}$).

Using the formulae from paper [9] for the lowest order approximation ($\hat{V}$) for D_∞-symmetric shapes, the multipole transition operator is derived as:

$$\begin{aligned}\hat{Q}^{(0)}(\xi;lm) &= {D^l_{m0}}^\star(\Omega_1,\Omega_2,0)\,\delta(l\in 2\mathbb{Z}) \\ &\times\Bigg\{\left(1+\frac{1}{2}\dim(X)\right)\langle\beta|\hat{M}(\xi;l0)|\beta\rangle + \\ &-\frac{1}{2}\frac{\langle\beta|\overleftarrow{\frac{\partial}{\partial\beta}}\hat{M}(\xi;l0)\overrightarrow{\frac{\partial}{\partial\beta}}|\beta\rangle}{\langle\beta|\overleftarrow{\frac{\partial}{\partial\beta}}\overrightarrow{\frac{\partial}{\partial\beta}}|\beta\rangle} - \frac{\langle\beta|\hat{J}_y\hat{M}(\xi;l0)\hat{J}_y|\beta\rangle}{\langle\beta|\left(\hat{J}_y\right)^2|\beta\rangle}\Bigg\}. \end{aligned} \tag{10}$$

It is important to notice that these operators **vanish for all odd multipolarities**.

In the case of odd multipolarities, *e.g.*, for the dipole transitions, the first non-vanishing operator is the term ($\hat{F}$)

$$\hat{Q}^{(1)}(\xi;1m) = +\sqrt{2}B_1(\beta)\hat{L}_{1m}(\Omega_1,\Omega_2,0)\,, \tag{11}$$

where

$$B_1(\beta) = \frac{-\mathrm{Im}\langle\beta|\hat{J}_y\hat{M}(\xi;11)|\beta\rangle}{\langle\beta|\hat{J}_y^2|\beta\rangle} \tag{12}$$

and $\hat{L}_{1m}(\Omega_1,\Omega_2,0)$ is the total collective angular momentum operator as a function of only two Euler angles Ω_1,Ω_2. The terms with the third Euler angle disappear.

The case when the intrinsic generating function is only SO(2)-invariant is very similar to previously considered symmetry D_∞. The collective manifold has exactly the same structure as in the previous case. However, the 0-order approximation of the collective transition **operator does not disappear for odd multipolarities**

$$\begin{aligned}\hat{Q}^{(0)}(\xi;lm) &= {D^l_{m0}}^\star(\Omega_1,\Omega_2,0)\Bigg\{\left(1+\frac{1}{2}\dim(X)\right)\langle\beta|\hat{M}(\xi;l0)|\beta\rangle + \\ &-\frac{1}{2}\frac{\langle\beta|\overleftarrow{\frac{\partial}{\partial\beta}}\hat{M}(\xi;l0)\overrightarrow{\frac{\partial}{\partial\beta}}|\beta\rangle}{\langle\beta|\overleftarrow{\frac{\partial}{\partial\beta}}\overrightarrow{\frac{\partial}{\partial\beta}}|\beta\rangle} - \frac{\langle\beta|\hat{J}_y\hat{M}(\xi;l0)\hat{J}_y|\beta\rangle}{\langle\beta|\left(\hat{J}_y\right)^2|\beta\rangle}\Bigg\}. \end{aligned} \tag{13}$$

This is the main and important difference between the both symmetries.

4. Conclusions

The axial symmetry SO(3) implies the Hamiltonian symmetry selection rules which in this case are governed by the conservation law of the third component of angular momenta. The lowest order GCM+GOA form of the transition operator is a set of deformation-dependent functions multiplied by the Wigner function corresponding to the appropriate angular momentum.

In the case of D_∞ symmetry, in addition to the standard Hamiltonian symmetry selection rules, for the transition matrix elements determined by the Kronecker products of irreducible representations (notation as in [10]): $\{A_1 = A_1 \times A_1;\ A_2 = A_1 \times A_2;\ A_1 = A_2 \times A_2;\ E_\mu = E_\mu \times A_1 = E_\mu \times A_2 = A_2 \times E_\mu;\ E_{\mu_1+\mu_2} + E_{\mu_1-\mu_2} = E_{\mu_1} \times E_{\mu_2},\ \mu_1 \neq \pm\mu_2;\ E_{\mu_1\pm\mu_2} + A_1 + A_2 = E_{\mu_1} \times E_{\mu_2},\ \mu_1 = \pm\mu_2\}$, one expects that due to properties of obtained electromagnetic transition operators, the dipol and octupole transitions are zero or small. The lowest non-zero transition operator is the product of a vibrational function and the angular momentum operator (the allowed transformations are only between states having the same angular momentum).

This example shows that the shape symmetry-dependent transition operators can be useful tool for searching of nuclear symmetries. This can be important for searching higher point symmetries like tetrahedral one, where the Hamiltonian symmetry selection rules do not give a clear criterion for discovering of this symmetry.

This work was partially supported by the Polish–French COPIN collaboration of the project 04-113 and the Bogoliubov–Infeld JINR program.

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