ON JACOBI AND POINCARÉ SHAPE TRANSITIONS IN ROTATING NUCLEI* **

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A recently developed Fourier shape parametrisation has been used to evaluate the potential energy surfaces of rotating nuclei including, in particular, the non-axiality degree of freedom. Our analysis has been performed in a 4D deformation space, but the effect of two additional deformation degrees of freedom of higher multipolarity has been taken into account. The calculations were performed using the Lublin–Strasbourg Drop model (LSD), but without taking microscopic correction into account. No sign of a Poincaré shape transition has been observed.

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1. Introduction

An efficient and low-dimensional description of shapes of rotating and fissioning nuclei is one of the most difficult tasks nuclear physicists have been confronted with since the first paper of Bohr and Wheeler on nuclear fission theory [1]. It is rather well-known that the classical expansion of the nuclear surface in terms of spherical harmonics as proposed by Lord Rayleigh in the 19^{th} century, is not rapidly convergent (confer *e.g.* [2, 3]), and that one needs at least 14 first terms of that expansion in order to obtain an accurate profile of the liquid-drop fission barrier from its ground state, through the saddle up to the scission point. A reasonably good description of the fission barrier is obtained using the Funny-Hills (FH) parametrisation developed by Brack *et al.* [4] and its extended version known as the Modified Funny-Hill (MFH) shapes developed in the Lublin–Strasbourg collaboration [5]. Due to the limited class of nuclear shapes produced by both

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these parametrizations, it is, however, practically impossible to evaluate the accuracy of the energy of fissioning nuclei obtained in this way. The effect of rotation makes the picture even more complicated since the set of deformation parameters should be then able to describe the Maclaurin, Jacobi and Poincaré shape transition.

2. Theoretical model

In a majority of papers devoted to Jacobi and Poincaré transitions, one describes the shapes of deformed nuclei using the Rayleigh expansion. It has, however, been shown in Ref. [2] that in the above expansion, one needs to include all terms up to $\lambda = 14$ in order to describe properly the shape of the fission barrier from its saddle point up to vicinity of the scission configuration. A similar observation was also made by Mazurek *et al.* in a paper devoted to the nuclear Jacobi and Poincaré transitions [3]. An alternative way of describing nuclear shapes in cylindrical coordinates (ρ, φ, z) is given by the rapidly converging Fourier expansion of the square of the distance $\rho_{\rm s}$ from the z-axis to the surface [6]

$$\frac{\rho_{\rm s}^2(z,\varphi)}{R_0^2} = \sum_{n=1}^{\infty} \{a_{2n} \cos[(2n-1)u] + a_{2n+1} \sin(2nu)\} \frac{1-\eta^2}{1+\eta^2+2\eta\cos(2\varphi)}\,, \quad (1)$$

where $u = \frac{\pi}{2}\frac{z-z_{\rm sh}}{z_0}$, $-z_0 + z_{\rm sh} \leq z \leq z_0 + z_{\rm sh}$, and $z_0 = R_0 c$. The shift coordinate $z_{\rm sh}$ ensures that the centre of mass is located at the origin of the coordinate system, while the volume conservation condition gives the following relation between the elongation coordinate c of the nucleus and the Fourier expansion coefficients $c = \pi/(a_2 - a_4/3 + a_6/5 - \ldots)/3$.

In Eq. (1), one has made the simplifying assumption that the cross section perpendicular to z-axis has the form of an ellipsoid with half-axis a and b, having the same surface area as in the axial symmetric case, *i.e.* $\rho^2 = a\,b$. The non-axial deformation parameter η is defined as

$$\eta = \frac{b-a}{a+b}\,. \quad (2)$$

The LD fission path goes towards smaller a_2 and larger negative values of a_4, which is somewhat strange. One, therefore, introduces physically more intuitive collective coordinates which ensure, in addition, an optimal presentation of the potential energy landscape

$$\begin{aligned} q_2 &= a_2^{(0)}/a_2 - a_2/a_2^{(0)}\,, \qquad q_3 = a_3\,, \qquad q_4 = a_4 + \sqrt{(q_2/9)^2 + \left(a_4^{(0)}\right)^2} \\ q_5 &= a_5 - a_3(q_2-2)/10\,, \quad q_6 = a_6 - \sqrt{(q_2/100)^2 + \left(a_6^{(0)}\right)^2}\,, \end{aligned} \quad (3)$$

where $a_n^{(0)}$ is the value of the coordinate a_n for the spherical shape: $a_2 = 1.03205$, $a_4 = -0.03822$, and $a_6 = 0.00826$.

The potential energy surfaces of deformed rotating nuclei were evaluated using the rotating Lublin–Strasbourg Drop (LSD) model [7]

$$\begin{aligned} M(Z,N;\mathrm{def}) = {} & ZM_{\mathrm{H}} + NM_{\mathrm{n}} - b_{\mathrm{elec}}\, Z^{2.39} \\ & + b_{\mathrm{vol}}\left(1-\kappa_{\mathrm{vol}} I^2\right) A + b_{\mathrm{surf}}\left(1-\kappa_{\mathrm{surf}} I^2\right) A^{2/3} B_{\mathrm{surf}}(\mathrm{def}) \\ & + b_{\mathrm{cur}}\left(1-\kappa_{\mathrm{cur}} I^2\right) A^{1/3} B_{\mathrm{cur}}(\mathrm{def}) + \frac{3}{5}\frac{e^2 Z^2}{r_0^{ch} A^{1/3}} B_{\mathrm{Coul}}(\mathrm{def}) \\ & - C_4 \frac{Z^2}{A} + E_{\mathrm{cong}}(Z,N) B_{\mathrm{cong}}(\mathrm{def}) + \frac{\hbar^2}{2\mathcal{J}_{\mathrm{rig}}(\mathrm{def})} L(L+1)\,, \quad (4) \end{aligned}$$

where the functions $B_{\mathrm{surf}}(\mathrm{def})$, $B_{\mathrm{cur}}(\mathrm{def})$ and $B_{\mathrm{Coul}}(\mathrm{def})$ describe respectively the nuclear surface and the Coulomb energy relative to the spherical shape. The change with deformation of the congruence energy is taken according to Ref. [8]. All parameters of the LSD model are the same as in Ref. [7]. The nuclear radius constant $r_0 = 1.2\,\mathrm{fm}$ was taken for calculating the rigid-body moment of inertia $\mathcal{J}_{\mathrm{rig}}$. Evaluating the rotation energy we have assumed in the following that rotation always takes place around an axis of largest moment of inertia.

3. Results

The calculations were performed for several nuclei from different mass regions. In what follows, we present the results for one light and one mid-mass nucleus, namely ^{46}Ti and ^{120}Cd having respectively the fissility parameter $x = 0.23$ and 0.39, smaller or nearly equal to the Businaro–Gallone value [9]. For these nuclei, the LD saddle point shows an octupole deformation when rotation is not present. According to previous investigations (see *e.g.* Refs. [3, 10], there is a certain chance to observe the Poincaré transition (*i.e.* pear-like stationary shapes) in such nuclei.

The macroscopic potential energy surfaces (PES) of a rotating ^{46}Ti nucleus is presented in Fig. 1. It is seen that already at spin $L = 20$ a visible Maclaurin transition appears: the rotating nucleus becomes oblate deformed ($q_2 = -0.29,\ \eta = 0$). Some kind of "relection symmetry" of this minimum is also visible around the point ($q_2 = 0.15,\ \eta = 0.10$) which, according to Eq. (1), describes simply the same shape of the nucleus but oriented differently in space. At $L = 28\,\hbar$, ^{46}Ti is still oblate deformed, while at $L = 30\,\hbar$ the Jacobi transition to a triaxial shape can be observed.

Similar PESs for ^{46}Ti but for higher angular momenta $L = 40$, 46 and 48 $\hbar$ are presented in Fig. 2. It is visible in the top-left figure that at spin $L = 40\,\hbar$, the nucleus ^{46}Ti becomes prolate deformed ($q_2 = 1.8$) and axially symmetric ($\eta = 0$). The top-right figure shows, for the same angular momentum, the cross section of the PES along the (q_2, q_3) plane. The minimum

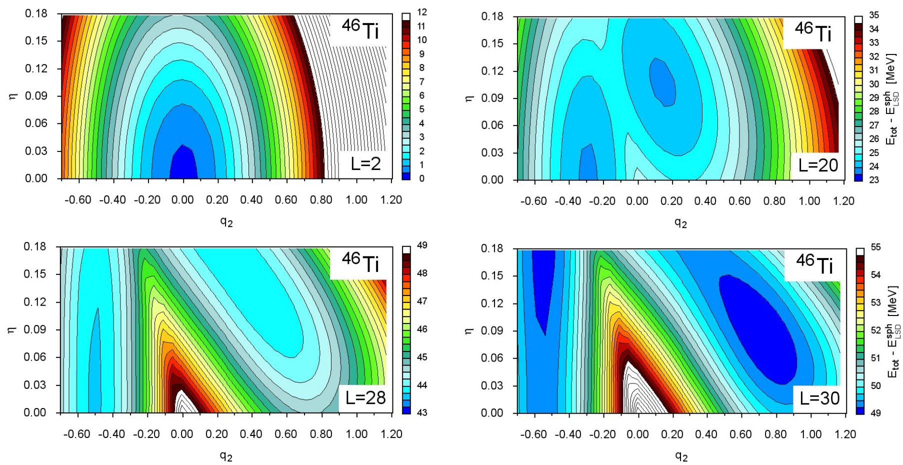

Fig. 1. Potential energy landscape of ^{46}Ti in the (q_2, η) plane for the low angular momenta $L = 2$, 20, 28, and 30 $\hbar$.

corresponds to $q_3 = 0$, *i.e.* no pear-like deformation is observed. Taking higher multipolarities q_4 and q_6 into account does not change this picture what is visible in the bottom-left figure in which the PES cross section is calculated at the $L = 40\,\hbar$ ground state ($q_2 = 1.8$, $q_3 = 0$, $\eta = 0$). At $L = 42\,\hbar$ (bottom-right figure), the fission barrier of ^{46}Ti becomes smaller and the nucleus finally becomes unstable against centrifugal fission for very high L values.

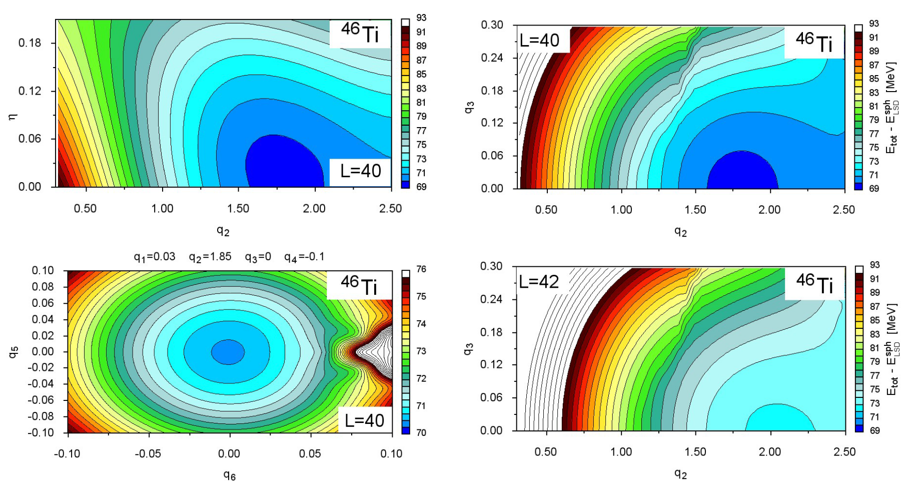

Fig. 2. The same as in Fig. 1 but for higher angular momenta.

The majority of properties of the PESs of rotating ^{46}Ti observed in Figs. 1 and 2 remains valid for ^{120}Cd with the only difference that the magnitude of the angular momentum changes. In Fig. 3, the pronounce

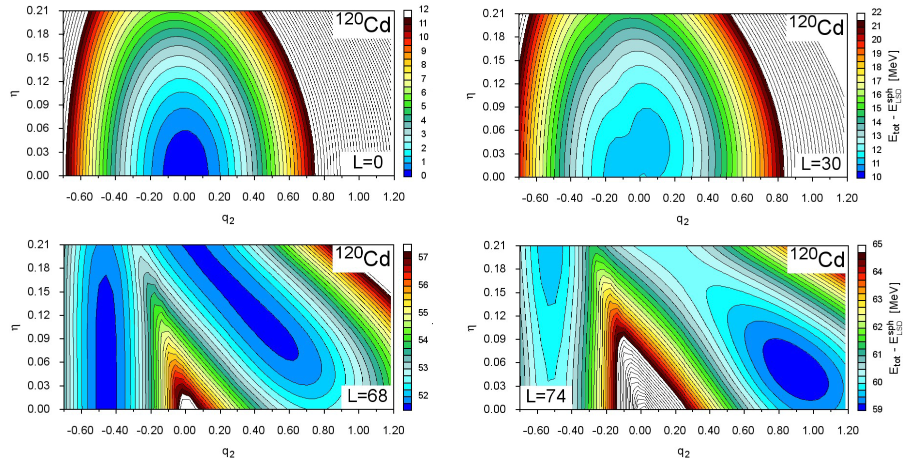

Fig. 3. Potential energy landscape of ^{120}Cd in the (q_2, η) plane for the low angular momenta $L = 0$, 30, 68, and 74 $\hbar$.

Maclaurin transition is visible for spins in the interval of $30\,\hbar \leq L \leq 68\,\hbar$, while at $L = 74\,\hbar$ (bottom-right figure), the Jacobi shape transition is observed. Around spin $L = 90\,\hbar$, the nucleus ^{120}Cd becomes axially symmetric and prolate what can be seen in Fig. 4. Similar to the case of ^{46}Ti, this minimum is stable against pear-like (q_3), as well as higher-multipolarity (q_5

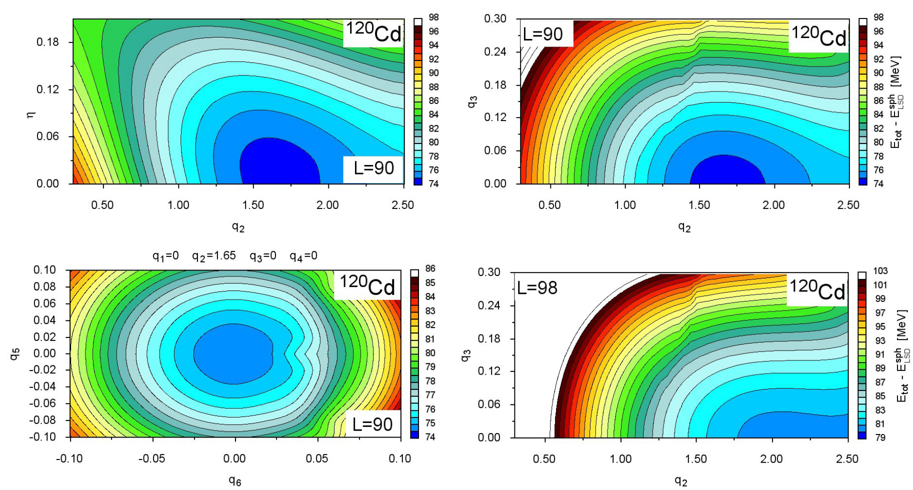

Fig. 4. The same as in Fig. 3 but for higher angular momenta.

and q_6) deformations q_6. The largest angular momentum at which ^{120}Cd survives is $L = 98\,\hbar$. For higher spins, the macroscopic fission barrier completely disappears.

4. Conclusions

The following conclusion can be drawn from our investigations:

— The new Fourier shape parametrization of deformed nuclei is very rapidly converging, that $n = 6$ is good for applications.

— A 4D space consisting of: q_2, q_3, q_4 and η deformation parameters is sufficient to describe Jacobi and Poincaré transitions in rotating nuclei. The role of higher-order multipolarity parameters is negligible.

— No evidence of a Poincaré transition is found in rotating nuclear liquid drops.

Since our investigation was preformed using the rotation liquid-drop model, the inclusion of shell effects may slightly modify this picture for certain nuclei.

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