INFLUENCE OF DIPOLE DEFORMATIONS ON ELECTRIC TRANSITIONS IN ^{156}Gd NUCLEUS*

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The aim of this work is to show the importance of the center-of-mass motion generated by the octupole modes and connected with these induced dipole deformations on ^{156}Gd nucleus in its ground-state configuration. The investigation is performed within a quadrupole–octupole collective approach in the presence of rotational motion.

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1. Introduction

The octupole deformations of the nucleus induces the change of the center-of-mass position. Therefore, the quadrupole and octupole collective vibrations and rotations considered in the current model result with a kind of periodic translational motion of the nucleus as a whole. The kinetic energy of such a mode is not explicitly taken into account in the Hamiltonian.

We examine here, on average, an effect of the center-of-mass motion for the $B(\mathrm{E1})$ and $B(\mathrm{E2})$ reduced probabilities, by considering the dipole $\alpha_{1\rho}$, $\rho = \{-1, 0, +1\}$ deformations, induced by coupling of the quadrupole $\alpha_{2\mu}, \mu = \{-2, 0, 2\}$ and octupole $\alpha_{3\nu}, \nu = \{-3, -2, \ldots, 3\}$ variables describing the nuclear surface in the spherical coordinates

$$R(\vartheta,\varphi) = R_0 c(\alpha)\Bigg[1 + \alpha_{10}Y_{10}(\vartheta,\varphi) + \alpha_{20}Y_{20}(\vartheta,\varphi) + \alpha_{30}Y_{30}(\vartheta,\varphi)$$
$$+2\alpha_{11}\mathrm{Re}(Y_{11}(\vartheta,\varphi)) + 2\alpha_{22}\mathrm{Re}\left(Y_{22}(\vartheta,\varphi)\right) + 2\sum_{\mu=1}^{3}\alpha_{3\mu}\mathrm{Re}(Y_{3\mu}(\vartheta,\varphi))\Bigg]. \quad (1)$$

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Till now, dipole deformation parameters are known to be mainly responsible for the center-of-mass motion. Nevertheless, as shown in [1], their large enough values are capable to significantly change the shape of nuclear body, affecting the total nuclear potential energy surface as well.

2. Collective Hamiltonian in intrinsic frame

An usual way of defining a vibrational–rotational collective models accounts on choosing the collective Hamiltonian with respect to the laboratory frame using the laboratory variables, *e.g.* $\alpha^{\rm lab}_{\lambda\mu}$ and then, transforming it to the intrinsic coordinate system.

The procedure according to this prescription was studied *e.g.* in [2] which results with a consistent formula for quadrupole–octupole collective Hamiltonian in the intrinsic frame. Such Hamiltonian, due to its complexity, occurs to be difficult to apply in practice.

Contrarily to the above idea, we build the vibrational–rotational Hamiltonian already in the intrinsic frame applying a standard *adiabatic approximation* which allows for the separation of the vibrational and rotational modes. Such separation is generally possible due to significantly different (by 2–3 orders of magnitude) energy scales of both collective modes.

We propose even more simplified approach in which quadrupole and octupole vibrational motions are totally decoupled in the kinetic energy term. This approximation seems to be reliable since the off-diagonal components of the mass tensor, responsible for such coupling, are about one order of magnitude smaller than the smallest value of the diagonal, quadrupole or octupole mass-tensor components in the neighborhood of the equilibrium configuration.

Neglecting this coupling, we determine two independent mass tensors: first, for pure quadrupole motion admitting that the octupole deformation is fixed to zero and the second, corresponding to the octupole motion, for which the quadrupole configuration corresponds to the ground state.

Now, let us define the collective Hamiltonian in question. Within the above approximation, a realistic, quantized quadrupole–octupole-vibrational collective Hamiltonian with varying mass parameters and moments of inertia may be written as

$$\hat{\mathcal{H}}_{\rm coll}(\alpha_2,\alpha_3,\Omega) = \frac{-\hbar^2}{2}\left\{\frac{1}{\sqrt{|B_2|}}\sum_{\nu\nu'=0}^{2}\frac{\partial}{\partial\alpha_{2\nu}}\sqrt{|B_2|}\left[B_2^{-1}\right]^{\nu\nu'}\frac{\partial}{\partial\alpha_{2\nu'}}\right.$$
$$\left.+\frac{1}{\sqrt{|B_3|}}\sum_{\mu\mu'=0}^{3}\frac{\partial}{\partial\alpha_{3\mu}}\sqrt{|B_3|}\left[B_3^{-1}\right]^{\mu\mu'}\frac{\partial}{\partial\alpha_{3\mu'}}\right\}+\hat{H}_{\rm rot}(\Omega)+\hat{V}(\alpha_2,\alpha_3)\,, \quad (2)$$

where α_2 and α_3 describe symbolically the subspaces of the quadrupole and octupole variables, $B_2(\alpha_2)$, $B_3(\alpha_3)$ denote the quadrupole and octupole mi-

croscopic mass tensor, respectively, and $|B_2| = \det B_2(\alpha_2)$, $|B_3| = \det B_3(\alpha_3)$. Those symmetric mass tensors are calculated using the *cranking* approximation [3]. Since the experimental ground-state energy spectrum as well as the sequence of B(E2)s in ^{156}Gd nucleus indicate that the ground state of this nucleus is stiff against the quadrupole deformation, the coupling of rotational and vibrational modes seems to be weak. In the first approximation, the absence of the corresponding term in Hamiltonian (2) is therefore justified. In this work, an effective approximation leading to the collective potential in the six-dimensional collective space of $\{\alpha_2, \alpha_3\}$ variables is the macroscopic–microscopic model. This model, for a reasonable choice of single-particle potential, pairing interaction and the smooth liquid-drop-type energy formula, is able to give reliable estimates of the potential energy landscapes $\hat{V}(\alpha_2, \alpha_3)$. In what follows, one applies the Woods–Saxon potential [4] with the so-called *universal* set of parameters [5], refitted to the newest single-particle data [6]. The leading liquid-drop energy term is developed here by the Lublin–Strasbourg Drop formula (LSD) [7]. The shell energy correction is calculated via traditional Strutinsky approach [8]. Eventually, the pairing energy correction is given by the particle number projected BCS approach [9].

As already mentioned, since the energy scales of the vibrational and rotational modes are significantly different, they are assumed to be fully decoupled. The resulting rotational term $\hat{H}_{\rm rot}(\Omega)$ depends, therefore, only on the Euler angles and the static shape of the nucleus corresponding here to the ground state. Since the rotational part of the Hamiltonian has also to be scalar with respect to the symmetrization group $\bar{G}_{\rm s}$ [10], it is sensible to construct it out of the irreducible tensors of $\bar{G}_{\rm s}$ symmetry group [11]. Such tensors with respect to the intrinsic SO(3) group are built of the spherical components of the angular momentum operators defined in the body-fixed frame.

The basis in which the Hamiltonian is diagonalized contains, in both the quadrupole and octupole parts, 0-, 1-, 2-, 3-phonon harmonic-oscillator one-dimensional solutions of positive or negative parity, while in the rotational part-Wigner functions of Euler angles, see *e.g.* [12].

3. Center-of-mass motion and electric transition operators

The center-of-mass vector $\vec{r}_{\rm CM} = \vec{r}_{\rm CM}(\alpha_{1\mu}, \alpha_{20}, \alpha_{22}, \{\alpha_{3\nu}\})$ of a nucleus of total mass M is defined as

$$\vec{r}_{\rm CM} = \frac{1}{M} \int\limits_V \vec{r}\rho(\vec{r}) {\rm d}^3\vec{r}, \tag{3}$$

where $\rho(\vec{r})$ is nuclear density distribution. The $\vec{r}_{\rm CM}$ is the combination of four order polynomials in $\alpha_{1\mu}$ with the coefficients depending on the

remaining quadrupole and octupole deformations. These polynomials are solved with respect to $\alpha_{1\mu}$ variables by the condition $\vec{r}_{\rm CM} = 0$

$$\alpha_{1\mu} = \alpha_{1\mu}(\vec{r}_{\rm CM} = 0, \alpha_{20}, \alpha_{22}, \{\alpha_{3\nu}\})\,. \tag{4}$$

Obtained in such a way the so-called induced $\alpha_{1\mu}$s as a function of $\alpha_{2\nu}$s and $\alpha_{3\mu}$s, inserted into the definition of the nuclear surface (1), ensure that the nuclear surface is defined in the center-of-mass frame. The above consideration points also out that used here quadrupole and octupole $\alpha_{\lambda\mu}$ deformations are the only independent variables of this approach.

The impact of the dipole deformations on the properties of atomic nuclei are not often discussed in the literature. For example, in Refs. [1, 13], the influence of $\alpha_{1\nu}$ on the potential energy surface of superdeformed configurations in thorium isotopes is discussed. As we can infer from Fig. 1, the total macroscopic–microscopic nuclear energy in vicinity of the ground state point is almost insensitive to the presence of dipole α_{10} deformation varied in substantially wide range, *i.e.* between -0.5 to 0.5. It means that, in a crude approximation, the major effect of this dipole deformation is the translation of the nucleus as a whole along OZ-axis without significant modification of the nuclear surface. The same can be said about the influence of non-axial α_{11} dipole deformation.

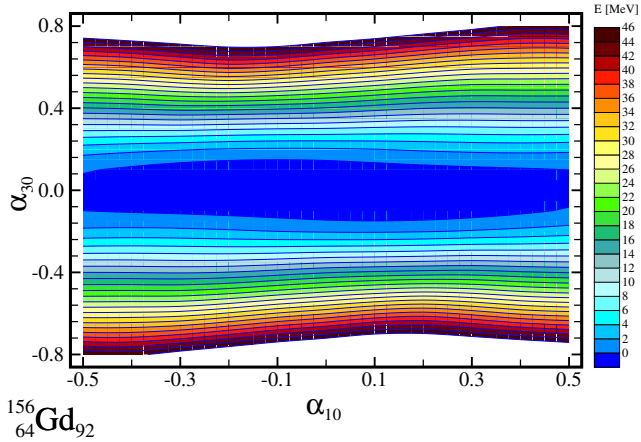

Fig. 1. Total nuclear energy as a function of axial octupole α_{30} and axial dipole α_{10} deformations for the ground-state configuration of ^{156}Gd nucleus. Other non-axial octupole deformations as well as non-axial dipole α_{11} are equal to zero.

The further increase of α_{10} values leads to visibly more and more compact nuclear shapes and, as a result, notable decrease of energy. The non-axial deformation, in turn, is discovered to be responsible for creating "necked" shapes. Such a feature may be interesting since even compact quadrupole–octupole shape with the presence of large values of α_{11} can split into two

pieces. Now, let us pass to the solutions of Eq. (4) on the six-dimensional grid of $\{\alpha_{20}, \alpha_{22}, \alpha_{3\mu}\}$ deformation parameters. Initially, for each point of this mesh, the vector of the center-of-mass shift $\vec{r}_{\mathrm{CM}}$ is determined. In the next step, this shift is expressed in terms of dipole deformations α_{10} and α_{11} which, as above mentioned, are mainly responsible for producing such shift.

The parameters α_{10} and α_{11} induced by couplings of, in general, all considered six degrees of freedom are displayed in Figs. 2 and 3. Nevertheless, more detailed studies of $\alpha_{\lambda\mu} \otimes \alpha_{\lambda'\mu'}$ tensor couplings reveal that parameter α_{10} is, in its leading part, induced by the second order couplings of octupole axial α_{30} with α_{20} and α_{32} with α_{22}. Similarly, α_{11} appears due to the couplings of α_{20} with α_{31} as well as α_{22} with α_{31} and/or α_{33}.

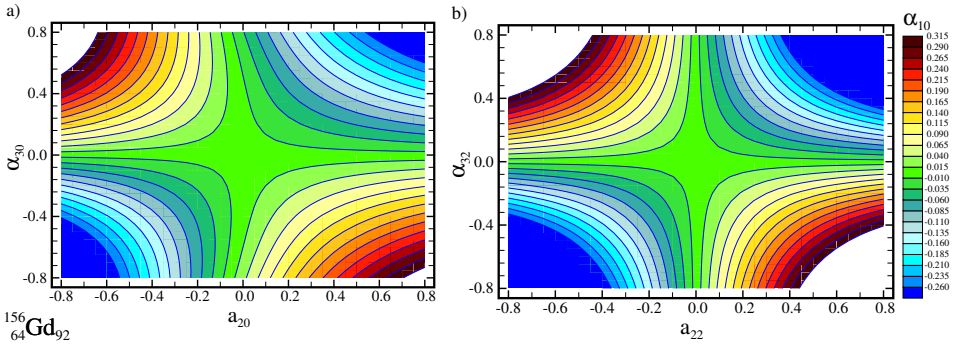

Fig. 2. Dependence of dipole axial α_{10} on the octupole and quadrupole planes: axial (α_{20},α_{30}) and non-axial (α_{22},α_{32}) deformation parameters.

To sum up, in Fig. 2, we can observe that the contribution to the total induced dipole deformation α_{10} due to $\alpha_{22} \otimes \alpha_{32}$ coupling is stronger than the one received from axial deformations $\alpha_{20} \otimes \alpha_{30}$. If, as shown in Fig. 3, induced α_{11} parameter is considered, it is mainly contributed by the couplings of $\alpha_{22} \otimes \alpha_{33}$ and $\alpha_{20} \otimes \alpha_{31}$, while significantly weaker by $\alpha_{22} \otimes \alpha_{31}$ one. Clearly, in a chosen full collective state, real contribution of above quadrupole–octupole couplings is decided by the quadrupole–octupole phonon structure of the corresponding wave function.

3.1. Impact of dipole deformations on E1 transitions

Having solved the corresponding Schrödinger equation with Hamiltonian (2), we can calculate, as the next step, the inter-band dipole B(E1) transitions between the ground-state and the octupole, negative-parity bands. In the single-particle picture, it is commonly known that dipole transitions, contrary to the quadrupole B(E2)s, are strongly affected by mutual distance between centers of mass of proton (charge) and neutron mass distributions. In the presented collective approach, both those centers of mass are assumed

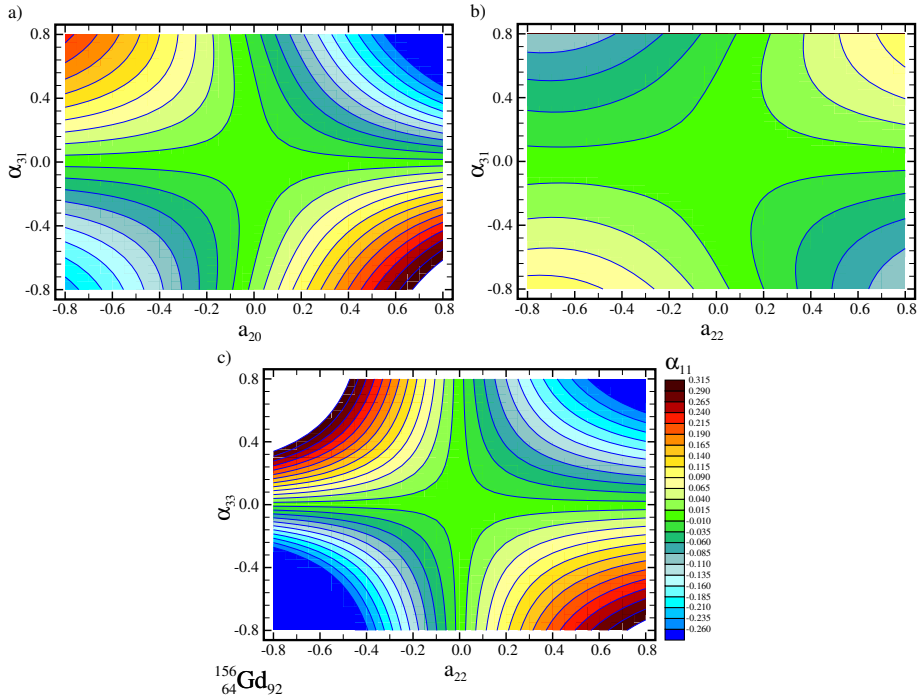

Fig. 3. Dependence of dipole non-axial α_{11} on the combination of the quadrupole and octupole (α_{20},α_{31}), (α_{22},α_{31}) and (α_{22},α_{33}) deformation parameters.

to stay in the same point, thus the center-of-mass shift of the total mass distribution with respect to the onset of the coordinate system seems to be the appropriate quantity. Such assumption emerges directly from the natural assumption that considered average nuclear surface $R(\vartheta,\varphi)$ is common for proton and neutron distributions.

According to the methodology outlined in Eqs. (3) and (4), the latter can be expressed in terms of induced $\alpha_{1\nu}$ deformations of the nuclear body.

For the nucleus of A nucleons and Z protons and effective radius of proton distribution $R_0 \approx 1.2\,A^{1/3}$, the E1 transition operators expressed in the body-fixed frame, $\hat{Q}_{1\nu}$, are given in the form of the second order intrinsic electric multipole moment operators as [14]

$$\hat{Q}^{(\mathrm{intr})}_{1\nu} = \frac{3ZeR_0}{4\pi}\left\{\alpha_{1\nu} + \frac{\lambda+2}{2\sqrt{4\pi}}\sum_{\lambda_1=2,\lambda_2=2}^{3}\sqrt{\frac{(2\lambda_1+1)(2\lambda_2+1)}{3}} \times(\lambda_1 0\lambda_2 0|10)(\alpha_{\lambda_1}\otimes\alpha_{\lambda_2})_{1\nu}\right\}, \tag{5}$$

where λ_1, λ_2 are the multipolarities of the intermediate $\alpha_{\lambda_1\mu_1}$ and $\alpha_{\lambda_2\mu_2}$ tensors used in the tensor coupling. If one wishes to investigate the B(E1) probabilities without the center-of-mass shift effects, the induced value of the dipole deformation, $\alpha_{1\nu}$, in Eq. (5) should be put as zero.

The obtained inter-band dipole B(E1) probabilities, with and without the presence of the induced by the quadrupole–octupole coupling $\alpha_{1\nu}$ variables in the transition operator, are listed in Table I. The experimental data for ^{156}Gd are taken from Ref. [15].

TABLE I

The predicted and measured (Exp.) E1 reduced transition probabilities between positive- and negative-parity bands. Symbols $\alpha_{3\mu}$ specify the type of the band-head one-phonon excitations. Values in parentheses are obtained without the induced dipole $\alpha_{1\nu}$ deformation in the transition operator of Eq. (5).

Transition	Theory B(E1) [10^{-3} W.u.]				
$I_i^\pi \to I_j^\pi$	α_{30}	α_{31}	α_{32}	α_{33}	Exp.
$3^- \to 2_1^+$	58 (33)	37 (14)	0.36 (0.23)	0.006 (0.0056)	0.98(21)
$3^- \to 4_1^+$	81 (46)	51 (20)	0.51 (0.33)	0.0087 (0.0083)	0.77(16)
$5^- \to 4_1^+$	66 (38)	42 (17)	0.41 (0.27)	0.0072 (0.0071)	$0.85^{+0.19}_{-0.38}$
$5^- \to 6_1^+$	81 (46)	51 (21)	0.53 (0.33)	0.0081 (0.0078)	$0.64^{+0.14}_{-0.29}$

Comparing the above four negative-parity model bands, we can observe that the band based on the *tetrahedral-like* α_{32} excitation reproduces within the error bars the two of measured four experimental B(E1) values. The $B(\mathrm{E1}, 5^- \to 4_1^+)$ transition is slightly out of the confidence interval whereas the $B(\mathrm{E1}, 3^- \to 2_1^+)$ transition is too small by a factor of two. This interesting feature of the tetrahedral negative-parity states, which generates the closest to the experimental B(E1) probabilities clearly indicates that, in fact, all four octupole degrees of freedom can be relevant to understand the complexity of negative-parity bands. If, for example, the ground-state (final) vibrational band-head is described by the 0-phonon structure, the order of magnitude of the E1 transition matrix element is set mainly by the amplitudes of the α_{30} and α_{31} 1-phonon excitations in the initial negative-parity state. The presence of the other two, α_{32} and α_{33}, octupole excitations affects the value of this matrix element only insignificantly. As investigated in

the above mentioned tetrahedral state, the overall contribution of both the α_{30} and α_{31} modes does not exceed 10%. The other meaningful conclusion from Table I is that if the induced dipole deformations are not present in the transition operator, the corresponding B(E1) probabilities become, on average, twice as much too low for α_{30}-, α_{31}-, α_{32}-based one-phonon octupole model bands compared to values obtained with induced $\alpha_{1\nu}$ variables, see numbers in parentheses in Table I. Another, not presented here, study indicates that the band based on α_{33} structure lies around 1 MeV higher than the lowest one, thus it is out of the interest. Hence, the center-of-mass problem cannot be neglected when calculating the collective dipole E1 transitions. On the contrary, for the quadrupole intraband E2 transitions, this effect is investigated to be practically non-essential.

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