EMISSIVITY AND MEAN-FREE PATHS OF NEUTRINOS IN NEUTRON STAR MATTER VIA MODIFIED URCA PROCESSES*

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The modified Urca processes are the most effective weak interactions that produce neutrinos in neutron star medium. Emissivity and mean-free path of electron neutrinos arising from the neutron branch of the modified Urca process are calculated supposing the neutron star matter is a beta stable one. The effect of strong nuclear interactions on the rate of this weak interaction is taken into account through the pair nuclear density-dependent correlation functions adapted from the lowest-order constrained variational (LOCV) method using the AV18 two-body potential.

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1. Introduction

A neutron star is the remnant of a supernova explosion happening in the center of a large massive star due to its gravitational collapse. The temperature in the center of a newborn neutron star becomes less than 1 MeV just a few minutes after its birth. However, the huge number of neutrinos it emits, carries away so much energy that the temperature falls to around 0.1 keV within a few years. One of the most important processes that emits neutrinos in neutron star matter is Urca process. Lattimer *et al.* [1] showed that the most effective process which produces neutrinos is a direct Urca reaction

$$n \to p + e + \overline{\nu}_e \,, \qquad p + e \to n + \nu_e \,. \tag{1}$$

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Considering the conservation laws, we find out that direct Urca process is forbidden in most typical neutron stars. Adding a spectator nucleon can satisfy the necessary conservation laws

$$n+n \to n+p+e+\overline{\nu}_e \,, \qquad n+p+e \to n+n+\nu_e \,. \tag{2}$$

These reactions together are called the neutron branch of the Modified Urca (hereafter Murca) process and have the same rate in the beta stable nuclear matter.

Mean-free path and emissivity of neutrinos from neutron branch of Murca process have been calculated by a number of authors. Reference [2] is a review on this topic. In the present paper, we calculate both mean-free path and emissivity of non-degenerate (electron) neutrinos via neutron branch of Murca process. To approximate the effect of strong nucleon–nucleon interactions, we use density-dependent pair correlation functions extracted from the Lowest Order Constrained Variational (LOCV) method for beta stable matter [3]. The LOCV method is a pure variational and fully self-consistent technique with state and density-dependent correlation functions. This method does not include any free parameters except those coming from interaction. With respect to the above considerations, Section 2 is devoted to the mean-free path and emissivity calculations, then in Section 3, results are presented.

2. Emissivity and mean-free path calculations

The neutron branch of Murca process is a charged-current weak nuclear reaction in which nucleons also interact via strong nuclear forces. We denote spins and momenta of involved particles at this reaction as Refs. [4, 5]. The neutrino emissivity — the amount of energy carried away from unit of volume per second — is obtained by the expression [2]

$$\begin{aligned} Q \;=\; & 2\int \frac{(2\pi)^4}{(2\pi)^{18}} \left\langle |M_{\mathrm{i}\to\mathrm{f}}|^2 \right\rangle \varepsilon_\nu n_1 n_2 (1-n_{1'})\,(1-n_p)\,(1-n_e) \\ & \times \delta\left(E_{\mathrm{f}}-E_{\mathrm{i}}\right) \delta^3\left(P_{\mathrm{f}}-P_{\mathrm{i}}\right) \mathrm{d}^3P_1\, \mathrm{d}^3P_2\, \mathrm{d}^3P_1'\, \mathrm{d}^3P_p\, \mathrm{d}^3P_e\, \mathrm{d}^3P_\nu \end{aligned} \tag{3}$$

in which ns are the Fermi–Dirac distribution functions of fermions, Ps are momentum of different spices, ε_ν is neutrino's energy and $\langle |M_{\mathrm{i}\to\mathrm{f}}|^2\rangle$ is spin-averaged squared invariant amplitude of beta reaction which can be derived from evaluating transition rate.

We would like to involve the effect of nucleons strong interactions. We do this task by writing the state of the initial and final correlated nucleons as

$$\psi_{\mathrm{i}}^N = V^{-1}\!\int \mathrm{d}^3x_1 \mathrm{d}^3x_2 e^{i(p_1\cdot x_1+p_2\cdot x_2)} \sum_{s_a s_b} f^{nn}_{s_a s_1, s_b s_2}(x_1-x_2)\psi^\dagger_{s_a}(x_1)\psi^\dagger_{s_b}(x_2)|0\rangle \,,$$

$$\psi_{\rm f}^N = V^{-1}\int \mathrm{d}^3x'\mathrm{d}^3y' e^{i\left(p_1'\cdot x'+p_p\cdot y'\right)} \sum_{s_a s_b} f^{np}_{s_a s_1', s_b s_p}\left(x'-y'\right)\psi^\dagger_{s_a}(x')\phi^\dagger_{s_b}(y')|0\rangle \quad (4)$$

in which f functions are the pair nucleon correlation functions, $\psi^\dagger$ and $\phi^\dagger$ are neutron and proton creator operators, respectively. We approximate f^{nn} and f^{np} as

$$\begin{aligned} f^{nn}_{s_a s_a', s_b s_b'}(r) &= f^{nn}(n)\delta_{s_a s_a'}\delta_{s_b s_b'}\,, \\ f^{np}_{s_a s, s_b s'}(r) &= f^{np}_{\rm c}(r)\delta_{s_a s}\delta_{s_b s'} + f^{np}_{\rm t}(r)[s_{np}(r)]_{s_a s, s_b s'}\,, \end{aligned} \quad (5)$$

where $f^{np}_{\rm c}(r)$ and $f^{np}_{\rm t}(r)$ are the central and tensor part of the correlation function and $s_{np}(\widehat{r}) = 3\sigma_n \widehat{r}\ \sigma_p \widehat{r} - \sigma_n \sigma_n$ is the usual tensor operator in which $\sigma_{\rm i}$s are Pauli matrices. Main contribution of nn interaction is supposed to be in the $^1S_0$ channel and in the $^1S_0$ as well as $^3S_1$–$^3D_1$ channels for np interaction. We are also allowed to consider triangle approximation among three neutrons involved in the interaction $p_1' \simeq p_1 + p_2$, because all the particle's momenta are near their Fermi momentum and one can neglect Fermi momenta of proton, electron and neutrino in comparison with neutrons' one. Considering all these allowed approximations throughout long and tedious calculation, we obtain the following relation for the invariant transition amplitude [5]

$$\left\langle |M_{\rm i\to f}|^2\right\rangle = 3G^2\left(1+3f_{\rm A}^2\right)\mathcal{R}\,, \quad (6)$$

where G is Fermi weak coupling constant, $f_{\rm A}$ is axial vector renormalization factor and $\mathcal{R}$ is defined as

$$\mathcal{R} = \left(F_{\rm c}^2 + 12.67F_{\rm t}^2\right)\,. \quad (7)$$

The dimensionless factor carries the effect of nucleon–nucleon interaction through correlation functions as [5]

$$\begin{aligned} F_{\rm c} &= 4\pi k_{\rm F}^3 \int_0^\infty \mathrm{d}r\, r^2 j_0\left(k_{\rm F} r\right)\left[f^{nn}(r) f^{np}_{\rm c}(r) - 1\right]\,, \\ F_{\rm t} &= 4\pi k_{\rm F}^3 \int_0^\infty \mathrm{d}r\, r^2 j_2\left(k_{\rm F} r\right) f^{nn}(r) f^{np}_{\rm t}(r)\,, \end{aligned} \quad (8)$$

where $j_l(x)$ is the spherical Bessel function of the order of l. Putting the obtained relation for the transition amplitude in Eq. (3) and substituting the values of constants yields the expression for the electron neutrino emissivity from the neutron branch of the Murca process as

$$Q(n,T) = 3.9 \times 10^{15} \left(\frac{n_0}{n} y_e^{\frac{2}{3}} y_p^{\frac{1}{3}} \mathcal{R} \left(\frac{m_n^*}{m_n} \right)^3 \frac{m_p^*}{m_p} \right) \left(\frac{T}{T_0} \right)^8 , \tag{9}$$

where T is temperature, $T_0 = 3 \times 10^8$ K, $m_i^*(i = n,p)$ is the effective mass calculated in corresponding Fermi level, $y_e = n_e/n$ and $y_p = n_p/n$ are the electron and proton fractions, respectively. n_0 is the saturation density of symmetric nuclear matter. For a simple model of *npe* matter, by neglecting the presence of muons, we have $y_e = y_p$.

In order to calculate the mean-free path of neutrinos, a similar integral of the form of Eq. (3) is considered except there is no integration on the energy and momentum of neutrinos [4]

$$\begin{aligned} \lambda^{-1}(\varepsilon_\nu) \; &= \; 2 \int \frac{(2\pi)^4}{(2\pi)^{18}} \left\langle |M_{\mathrm{i}\to\mathrm{f}}|^2 \right\rangle \varepsilon_\nu n_1 n_2 (1-n_{1'})(1-n_p)(1-n_e) \\ &\times \delta\left(E_\mathrm{f} - E_\mathrm{i}\right) \delta^3\left(P_\mathrm{f} - P_\mathrm{i}\right) \mathrm{d}^3 P_1 \, \mathrm{d}^3 P_2 \, \mathrm{d}^3 P_1' \, \mathrm{d}^3 P_p \, \mathrm{d}^3 P_e \, . \end{aligned} \tag{10}$$

By symmetry considerations, one can conclude that the rate of reactions and as a result the invariant transition amplitude is the same as what we obtained for the emissivity. Finally, we can find the mean-free path as a function of neutrino energy, ε_ν

$$\lambda^{-1}(\varepsilon_\nu) = 3.74 \times 10^{-7} \frac{n_0}{n} \frac{y_e}{0.01} \mathcal{R} \left(\frac{m_n^*}{m_n} \right)^3 \frac{m_p^*}{m_p} g(x_\nu) T_{10}^4 \; km^{-1} \, , \tag{11}$$

where $T_{10} = T/10^{10}$ and

$$g(x_\nu) = 2 \left(1 + \frac{10}{9\pi^2} x_\nu^2 + \frac{1}{9\pi^4} x_\nu^4 \right) \Big/ \left(1 + e^{-x_\nu} \right) \tag{12}$$

in which $x_\nu = \frac{\varepsilon_\nu}{K_\mathrm{B} T}$ is the dimensionless energy of neutrinos.

3. Results

We could find an expression for neutrino emissivity and mean-free path dependence on the dimensionless factor $\mathcal{R}$ which includes the effects of strong interactions among correlated nucleons in the super-dense medium of neutron star. In the present paper, to calculate this important factor, we used the density-dependent correlation functions extracted from the LOCV method [3] applied for the beta stable *npe* matter with AV18 two-body potential proposed among nucleons. Since this factor has the crucial role in our estimation of emissivity and mean-free path, we have plotted its total value and its central and tensor part in the left panel of Fig. 1. As it is shown, the total value of $\mathcal{R}$ and the central component get closer at higher densities. It is due to the increasing cancellation from $j_2(k_{\mathrm{F}}r)$ in the integrand of Eq. (8) as the density increases. However, the f_{t}^{np} extracted from LOCV method also weakens by increasing the density. The overall effect is that the role of tensor character weakens as the density increases. More details about this factor are discussed in Ref. [5].

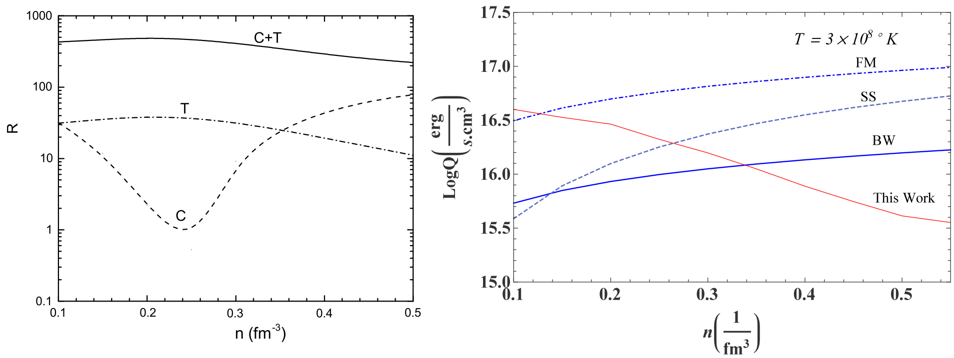

Fig. 1. Left: Dimensionless correlation factor *versus* baryon density. Right: Emissivity of electron neutrinos *versus* baryon density. Ther result of Ref. [6] (FM), Ref. [7] (SS) and Ref. [8] (BW) are presented for comparison.

In order to adapt the numerical value of the emissivity from Eq. (9), we need, in addition to $\mathcal{R}$, the effective masses and proton and electron fractions. We have applied the values $\frac{m_p^*}{m_p} = \frac{m_n^*}{m_n} = 0.8$ as used by Ref. [4], and $y_p = y_e$ is obtained from LOCV beta stable calculations at different nuclear density. The right panel of Fig. 1 illustrates the differences between various methods used for calculating the emissivity from neutron branch of Murca process. As it is shown, including the density-dependent pair correlation functions yields a different trend for emissivity. Despite of all previous calculations, we see that the emissivity decreases by increasing the baryon density.

Calculating the mean-free path of neutrinos is important from the point of view of investigating the transparency distance of neutron star matter to neutrinos. Equation (11) shows that the mean-free path is a function of energy and density. In the left panel of Fig. 2, we have shown the mean-free path of neutrinos *versus* their energy at two different densities. We have presented the results of Ref. [4] for comparison. Haensel and Jerzak in Ref. [4] used the pair correlation functions of symmetric nuclear matter at nuclear saturation density for the whole range of densities. It is seen that in both cases by increasing the neutrino's energy, the mean-free path is decreased but in our case, by increasing the density, the mean-free path increases, while in the Ref. [4] case it decreases. To illustrate the behavior of mean-free path with respect to density, we fix the energy of neutrinos and plot the mean-free path *versus* density. As it is seen in the right panel of Fig. 2, the mean-free path of neutrinos increases at higher densities. It is consistent with the results of Fig. 1. By increasing the baryon density, the correlation factor decreases and hence reduces the rate of Murca reactions. So it will be reasonable to have lower values of emissivity and higher mean-free paths. These results are important for investigating the transparency distance of neutrinos and can also have sensible effects on the temperature evolution of neutron stars.

In conclusion, using a microscopical many-body method and density-dependent correlation functions can improve our knowledge about neutrino's emissivity and mean-free path in high density neutron star matter.

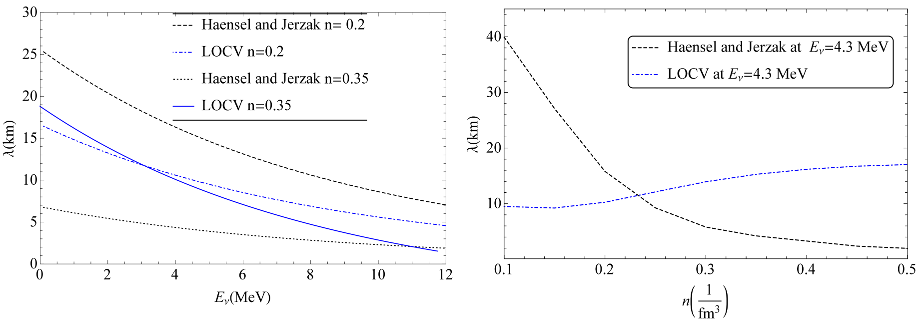

Fig. 2. Left: Mean-free path of neutrinos at two different densities at $T = 5 \times 10^{10}$ *versus* neutrino energy. Results of Ref. [4] are presented for comparison. Right: Mean-free path of neutrinos *versus* baryon densities at $T = 5 \times 10^{10}$. Results of Ref. [4] are presented for comparison.

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