

MODIFICATION OF $\Delta(1232)$ IN THE NEUTRINO NUCLEUS REACTION

SWAPAN DAS

Nuclear Physics Division, Bhabha Atomic Research Centre
Mumbai-400085, India

and

Homi Bhabha National Institute
Anushakti Nagar, Mumbai-400094, India

(Received January 10, 2017; accepted April 24, 2017)

The remarkable observations in the pion-induced and charge-exchange reactions on a nucleus are the significant shift and broadening of the $\Delta(1232)$ -peak relative to free pion-nucleon scattering. For the forward going massless leptons, the weak process (ν_l, l) on a nucleon, according to Adler's partially conserved axial current theorem, is connected to the pion-nucleon scattering. This mechanism is also applicable to the nucleus. Therefore, the modification of the Δ -peak in a nucleus can be seen in the (ν_l, l) reaction since it is connected to the pion-nucleus scattering. To investigate this issue quantitatively, the double differential cross sections of the forward going ejectile l energy distribution in the (ν_l, l) reaction are calculated in the Δ -excitation region for both proton and nucleus. The measured pion-nucleon and pion-nucleus scattering cross sections in the quoted energy region are used in these reactions as an input. Since the (ν_l, l) reaction is connected to the pion-induced scattering, the features of the previous reaction is shown analogous to those of the latter.

DOI:10.5506/APhysPolB.48.729

1. Introduction

In the inclusive pion-nucleus scattering, the sizeable broadening and shift of the $\Delta(1232)$ -peak have been found in the pion-energy distribution spectrum relative to the pion scattering on the free nucleon [1, 2]. The origin of the quoted modification of Δ -isobar has been understood by studying the exclusive and semi-exclusive channels of the pion-nucleus scattering, such as elastic and inelastic pion production, pion absorption, *etc.*, [3]. The coherent multiple scattering of pion, mediated by the delta-hole propagation [4, 5], in the nucleus leads to the large shift and broadening of the peak of

the cross section in the elastic pion–nucleus scattering. In fact, the inelastic and reaction channels of the pion–nucleus scattering are also equally significant throughout the Δ -excitation region [3]. The Δ -resonance produced in the pion–nucleus scattering strongly couples to the quasi-free ($\pi, \pi N$) and two-body absorption (π, NN) channels which also modify the hadronic parameters of the Δ -isobar in the nucleus. These channels, except πN , do not occur for proton.

The modification of the $\Delta(1232)$ -peak has been also seen in the measured ejectile energy distribution spectrum in the inclusive (${}^3\text{He}, t$) reaction on the nucleus relative to proton [6]. This phenomenon is also found in other inclusive reactions, *e.g.*, (p, n) [7]. However, the position of the Δ -peak in this reaction (unlike that in the pion–nucleus scattering) does not show much dependence on the target mass number. To understand the origin of Δ -peak shift in the inclusive reactions, various models (describing the Δ -dynamics in the nucleus) were proposed [8].

To resolve the above issue, the exclusive/semi-exclusive experiments on the charge exchange reactions (*i.e.*, (p, n) [9] and (${}^3\text{He}, t$) [10]) were done in the $\Delta(1232)$ -resonance region. In these experiments, the ejectile energy/momentum was measured with the charged particles (*i.e.*, π^+ , π^+p , pp , *etc.*) detected in coincidence. According to these measurements, the shift of Δ -peak in the nucleus (seen in the inclusive measurements) arises because of the pp emission and coherent pion production which do not exist for proton. The latter is the key ingredient for the Δ -peak shift in the nucleus. The results of the (semi)exclusive measurements illustrate one-to-one correspondence with those of the pion–nucleus scattering [11].

In contrast to those found in the pion-induced and charge-exchange reactions, the shift of the $\Delta(1232)$ -peak in the nucleus is not seen in the inclusive γ -induced [12] and (e, e') reactions [13]. In these reactions, the Δ -excitation occurs due to the transverse $\gamma N \Delta$ coupling but its decay, *i.e.*, $\Delta \rightarrow \pi^+p$, proceeds because of the longitudinal $\pi N \Delta$ coupling. Since these couplings are orthogonal to each other, the coherent pion production is inhibited in the photo- and electro-induced nuclear reactions [14]. The broadening of the Δ -peak, seen in these reactions, arises because of the Fermi motion of nucleon and opening up additional channels, like $N \Delta \rightarrow NN$ [4, 15]. In fact, universality exists for the integrated cross section of the inclusive (e, e') reaction because it is the same for all nuclei [13].

The modification of $\Delta(1232)$ in the nucleus has been taken into account for a large variety of neutrino–nucleus reactions such as inclusive scattering, incoherent and coherent pion production [16]. It must be mentioned that the cross section of the (ν_l, l) reaction under appropriate constraint can be connected to that of the pion-induced scattering using Adler’s partially conserved axial current (PCAC) theorem [17]. According to this theorem,

the (ν_l, l) reaction can be interpreted in terms of the pion-induced reaction at the target, provided the massless lepton l is moving parallel to the incoming massless ν_l . The effect of the finite mass of lepton l in the (ν_l, l) reaction, as discussed by Adler [17], can be taken into account by incorporating the lepton mass correction factor C_{lm} in the cross section

$$C_{\text{lm}} = \left[1 - \frac{m_l^2 q_0}{2(m_\pi^2 E_l + m_l^2 q_0)} \right]^2, \quad (1)$$

with $q_0 = E_{\nu_l} - E_l$. m_l and m_π are masses of the lepton l and pion, respectively. As illustrated above, the (ν_l, l) reaction on a nucleus can be connected to the pion-nucleus scattering. Therefore, the distinct Δ modification in nucleus in the (ν_l, l) reaction can occur because that is seen in the pion-nucleus scattering.

To disentangle the above issue, the double differential cross section $\frac{d\sigma}{dE_{e^-} d\Omega_{e^-}}$ of the forward going ejectile energy E_{e^-} distribution in the (ν_{e^-}, e^-) reaction, using Adler's PCAC theorem [17], is expressed in terms of the cross section of the pion-induced reaction. To calculate $\frac{d\sigma}{dE_{e^-} d\Omega_{e^-}}$ of the previous reaction in the Δ -excitation region, the energy-dependent measured cross section of the latter reaction in this energy region has been used as an input. Two aspects of the considered reaction, stated above, should be emphasized. (i) The (ν_{e^-}, e^-) reaction is preferred since the masses of the leptons in the Δ -excitation region are negligibly small. It could be more appropriate (compare to other flavors of neutrino) for the use of the Adler theorem. The forward going electron e^- is considered. In such case, the outgoing lepton e^- moves parallel to the incoming neutrino ν_{e^-} . This is the requirement of the Adler theorem. (ii) The measured cross section of the pion-induced reaction is used as an input. The advantage of it is that all kinds of mechanism for this reaction are taken into account.

2. Formalism

The matrix element describing the weak process $\nu_{e^-} + A \rightarrow e^- + X$, using the phenomenological $(V - A)$ theory for weak interaction [18], can be written as

$$\mathcal{M}_{\text{fi}} = \frac{G_W}{\sqrt{2}} l_\mu \langle X | (V^\mu - A^\mu) | A \rangle, \quad (2)$$

where $G_W \simeq 1.15 \times 10^{-5} \text{ GeV}^{-2}$ is the empirical value of Fermi coupling constant and l_μ denotes the leptonic weak current, *i.e.*, $\bar{u}_e \gamma_\mu (1 - \gamma_5) u_{\nu_e}$, which can be written as $l_\mu = \mathcal{M}(e\nu_e) q_\mu$ for the forward going massless leptons. $\langle |\mathcal{M}(e\nu_e)|^2 \rangle$ (which appears in the cross section) is expressed afterwards.

Henceforth, the superscript on the electron is dropped in some places. V^μ and A^μ are the hadronic vector and axial vector currents respectively. $|A\rangle$ is the initial state of the nucleon (or nucleus), whereas $|X\rangle$ represents the final state (except lepton l) of the reaction. Using the Adler theorem for massless leptons, \mathcal{M}_{fi} in Eq. (2) can be factorized as

$$\mathcal{M}_{\text{fi}} = -i \frac{G_W}{\sqrt{2}} \mathcal{M}(e\nu_e) f_\pi \mathcal{M}(\pi^a A \rightarrow X). \quad (3)$$

$f_\pi (= 0.13 \text{ GeV})$ is the pion weak decay constant. $\mathcal{M}(\pi^a A \rightarrow X)$ denotes the matrix element for the $\pi^a A \rightarrow X$ reaction. In fact, conserved vector current (CVC) and partially conserved axial current (PCAC) hypotheses are applied to get this equation.

The double differential cross section $\frac{d\sigma}{dE_e d\Omega_e}$ of the forward going lepton energy E_e distribution in the (ν_{e-}, e^-) reaction on proton or nucleus, using \mathcal{M}_{fi} in Eq. (3), can be expressed as

$$\frac{d\sigma}{dE_e d\Omega_e} = \frac{\pi^2}{(2\pi)^5} \frac{k_e k_\pi}{k_{\nu_e}} G_W^2 \langle |\mathcal{M}(e\nu_e)|^2 \rangle f_\pi^2 \sigma_t(\pi^a A \rightarrow X), \quad (4)$$

with $\langle |\mathcal{M}(e\nu_e)|^2 \rangle = \frac{16k_{\nu_e} k_e}{(E_{\nu_e} - E_e)^2} \cdot \sigma_t(\pi^a A \rightarrow X)$ is to be replaced by $2\sigma_t(\pi^+ A \rightarrow X)$ for π^+ scattering [19]. Since the electron mass correction factor, *i.e.*, C_{lm} in Eq. (1), is found very close to unity in the Δ -excitation region, the above equation can be used to calculate the cross sections of the $A(\nu_{e-}, e^-)X$ reaction in the quoted energy region.

3. Result and discussions

To calculate the double differential cross section $\frac{d\sigma}{dE_e d\Omega_e}$ for the inclusive (ν_{e-}, e^-) reaction on proton and nucleus, the measured total cross sections $\sigma_t(\pi A)$ of the π^+ proton and inclusive π^+ nucleus scattering are used in Eq. (4). $\sigma_t(\pi A)$ for proton in the Δ -excitation region is taken from Ref. [20], and those for the inclusive pion–nucleus scattering are taken from Ref. [2]. The calculated results at $E_{\nu_e} = 1 \text{ GeV}$ for the ejectile energy E_e distribution are presented in Fig. 1 for proton and ^{12}C , ^{27}Al , ^{56}Fe nuclei. The energy transfer, *i.e.*, $q_0 = E_{\nu_e} - E_e$ is shown by the upper scale of the horizontal axis. Because of the large broadening of Δ -resonance in the nuclei heavier than Fe, the distinct Δ -peak is not seen in the measured cross section $\sigma_t(\pi A)$ of those nuclei. Therefore, they are not considered in the present context. Figure 1 distinctly shows the modification of Δ -isobar in nuclei in the inclusive (ν_{e-}, e^-) reaction. The shift (towards higher ejectile energy) and broadening of the Δ -peak in ^{12}C nucleus relative to proton is significantly large (*i.e.*, $\sim 50\text{--}60 \text{ MeV}$) and those are found to increase with the size of the nucleus.

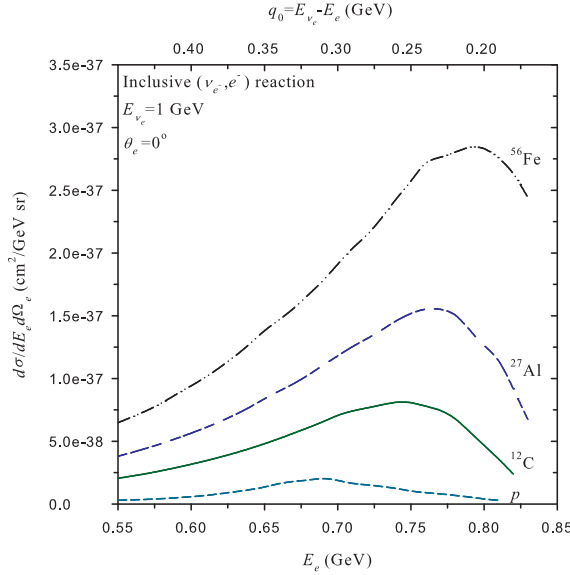


Fig. 1. (Color online) The shift and broadening of the Δ -peak in the inclusive (ν_{e-}, e^-) reaction on nuclei relative to proton. The modification of the peak increases with the size of the nucleus.

It should be mentioned that the (ν_{e-}, e^-) reaction on proton in the considered energy region proceeds as $\pi^+p \rightarrow \Delta^{++} \rightarrow \pi^+p$. The quasi-free proton knock-out in the (ν_{e-}, e^-) reaction on a nucleus is analogous to π^+p reaction. Apart from it, there are many other channels (*i.e.*, exclusive and semi-exclusive reactions) opened up for the nucleus which cannot occur for proton. The cross section of the inclusive reaction is the conglomeration of those of the exclusive and semi-exclusive reactions. Therefore, the origin of the Δ -peak shift, shown in Fig. 1, can be understood by studying the exclusive and semi-exclusive (ν_{e-}, e^-) reactions on the nucleus.

To disentangle the above issue, $\frac{d\sigma}{dE_e d\Omega_e}$ is calculated for the exclusive and semi-exclusive (ν_{e-}, e^-) reactions on the nucleus using the measured total cross sections of the exclusive and semi-exclusive pion-nucleus scattering in Eq. (4). The total cross section $\sigma_t(\pi A)$ of the pion-nucleus scattering in the $\Delta(1232)$ -resonance region is composed of that due to the exclusive and semi-exclusive channels [3]: $\sigma_t(\pi A) = \sigma_{t,el}(\pi A) + \sigma_{t,in}(\pi A) + \sigma_{t,ab}(\pi A) + \sigma_{t,cx}(\pi A)$. The partial cross sections (elaborated afterwards) in this equation are almost equal to each other in the peak region, except that of the charge exchange reaction, *i.e.*, $\sigma_{t,cx}(\pi A)$, which is much less compared to others [3]. Therefore, the latter is omitted in the following discussions.

The calculated forward going ejectile energy E_e distribution spectra in the exclusive and semi-exclusive (ν_{e-}, e^-) reactions on ^{12}C nucleus in the Δ -excitation region are shown in Fig. 2. $\frac{d\sigma}{dE_e d\Omega_e}$ calculated using the

measured elastic pion–nucleus scattering cross section $\sigma_{t,\text{el}}(\pi^{12}\text{C})$ [3] in Eq. (4) describes that for the elastic (coherent) pion production in the (ν_{e^-}, e^-) reaction on the quoted nucleus. It is presented by the dot-dashed curve in this figure. The inelastic pion–nucleus scattering consists of many final states, *e.g.*, πA^* , πpX *etc.* Amongst them, the quasi-free knock-out reaction in the nucleus (as mentioned earlier) has one-to-one correspondence with the pion–nucleon reaction. The cross section for the inelastic pion production in the quoted reaction (shown by the dot-dot-dashed curve) is calculated using the measured total inelastic pion–nucleus scattering cross section $\sigma_{t,\text{in}}(\pi^{12}\text{C})$ [3] in Eq. (4). The short-dashed curve represents $\frac{d\sigma}{dE_e d\Omega_e}$ calculated using the measured total pion absorption cross section, *i.e.*, $\sigma_{t,\text{ab}}(\pi^{12}\text{C})$ [3], in Eq. (4). In fact, this describes the cross section for $^{12}\text{C}(\nu_{e^-}, e^-)$ reaction where no pion is in the final state. It can be added that the dominant contribution to this reaction arises due to the two-nucleon emission in the final state, which proceeds because of the elementary reaction $N\Delta \rightarrow NN$ occurring in the nucleus, *i.e.*, two-particle–two-hole (2p–2h) excitation in the nucleus in the Δ -excitation region. Therefore, the quoted reaction can be described by the weak longitudinal response function (for the charge changing neutrino scattering) due to 2p–2h excitations in the nucleus in the considered energy region (*e.g.*, see the calculation due to Ruiz Simo *et al.*, [21]). For comparison, $\frac{d\sigma}{dE_e d\Omega_e}$ of the inclusive $^{12}\text{C}(\nu_{e^-}, e^-)$ reaction is shown by the solid line in Fig. 2. It is distinctly visible in this figure that the shift of

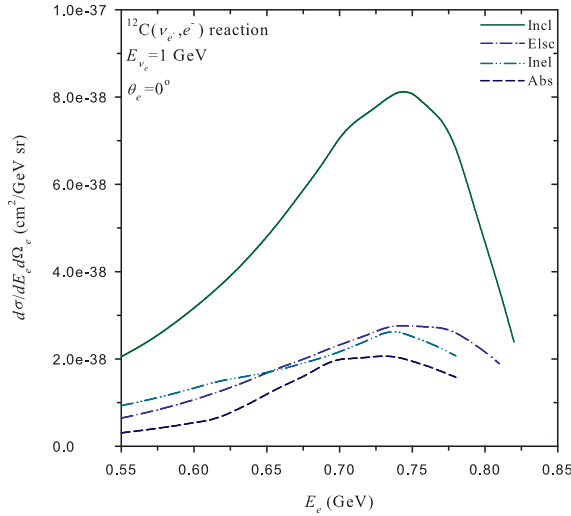


Fig. 2. (Color online) The cross sections of the inclusive and exclusive (ν_{e^-}, e^-) reactions on ^{12}C nucleus. The labels appearing in the figure represent various reactions: “Incl” (inclusive reaction), “Els” (elastic pion production), “Inel” (inelastic pion production) and “Abs” (no pion in the final state).

Δ -peak in the inclusive (ν_{e-}, e^-) reaction on the nucleus relative to proton (as shown in Fig. 1) arises because of the exclusive and semi-exclusive channels, *i.e.*, the pion production (both elastic and inelastic) and absorption, which are not possible for proton.

The calculated results in Figs. 1 and 2 are presented for the fixed neutrino beam energy. Since the energy of the incoming neutrino flux is not fixed in experiments, the energy-dependent neutrino flux averaged double differential cross section is measured. It can be expressed as

$$\frac{d\langle\sigma\rangle}{dq_0 d\Omega_e} = \frac{\int dE_{\nu_e} \Phi(E_{\nu_e}) \frac{d\sigma(E_{\nu_e})}{dq_0 d\Omega_e}}{\int dE_{\nu_e} \Phi(E_{\nu_e})}, \quad (5)$$

where $q_0 = E_{\nu_e} - E_e$ is the energy transfer to the nucleus. $\Phi(E_{\nu_e})$ describes the neutrino energy-dependent flux distribution. The forward energy transfer distribution spectra in the inclusive (ν_{e-}, e^-) reaction are calculated using $\Phi(E_{\nu_e})$ due to the MiniBooNE Collaboration (see Fig. 27 in Ref. [22]). The calculated spectra for proton and ^{12}C nucleus in the Δ -excitation region are presented in Fig. 3. This figure distinctly shows the modification of the Δ -peak in the nucleus, which corroborates those illustrated in Fig. 1 for the fixed neutrino beam energy.

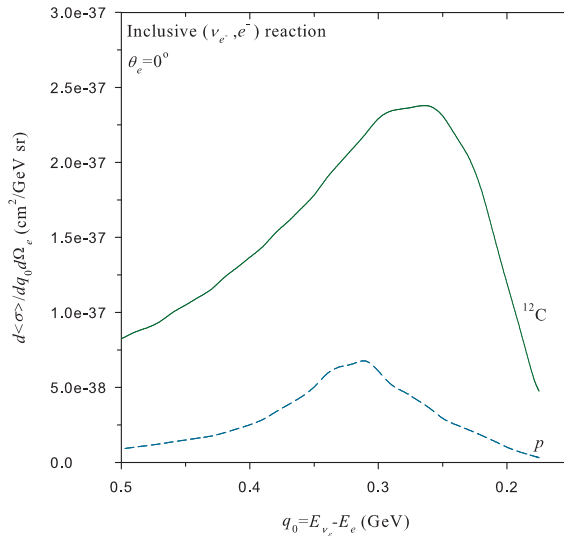


Fig. 3. (Color online) The flux averaged differential cross section for the forward going ejectile in the inclusive (ν_{e-}, e^-) reaction in the Δ -excitation region. $q_0 = E_{\nu_e} - E_e$ is the energy transfer to the nucleus. The modification of the Δ -peak is distinctly visible in the figure.

The modification of $\Delta(1232)$ in the nucleus is shown to occur in the (ν_{e-}, e^-) reaction. To look for that in the heavier (ν, l) reaction, the double differential cross section is calculated for the $(\nu_{\mu-}, \mu^-)$ reaction on nuclei. The results calculated at the fixed beam energy, *i.e.*, $E_{\nu_{\mu}} = 1$ GeV, for the forward going ejectile energy E_{μ} distribution is shown in Fig. 4. This figure shows that the modification of Δ in the $(\nu_{\mu-}, \mu^-)$ reaction is qualitatively similar to that in the (ν_{e-}, e^-) reaction presented in Fig. 1. The reduction in the cross section of the previous reaction, compared with that of the latter, occurs because of the finite mass correction factor, given in Eq. (1), for the lepton in the finite state.

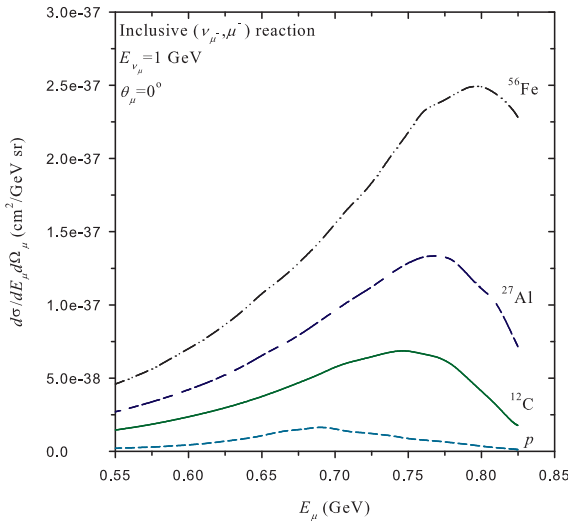


Fig. 4. (Color online) The same as those presented in Fig. 1 but for the $(\nu_{\mu-}, \mu^-)$ reaction. The modification of Δ -peak is qualitatively similar to that in Fig. 1.

As mentioned earlier, the energy-dependent neutrino flux averaged cross section can only be measured. If the flux distributions of ν_e and ν_{μ} neutrinos are different, the shape of the flux averaged cross sections of (ν_{e-}, e^-) and $(\nu_{\mu-}, \mu^-)$ reactions may not be similar. To explore it for the MiniBooNE flux distribution (see Fig. 27 in Ref. [22]), the $\nu_{\mu-}$ flux averaged differential cross section $\frac{d\langle\sigma\rangle}{dq_0 d\Omega_{\mu}}$ for the forward going μ^- in the $(\nu_{\mu-}, \mu^-)$ reaction is calculated following Eq. (5), and the calculated results are presented in Fig. 5. Figures 3 and 5 show that there is not much difference between the shape of the MiniBooNE flux averaged cross sections of (ν_{e-}, e^-) and $(\nu_{\mu-}, \mu^-)$ reactions, though the energy-dependent flux distributions of ν_{e-} and $\nu_{\mu-}$ neutrinos are quite different [22]. The shape of $\frac{d\langle\sigma\rangle}{dq_0 d\Omega_{\mu}}$ is narrower compared with that of

$\frac{d\langle\sigma\rangle}{dq_0 d\Omega_e}$, and the previous cross section is smaller than the latter. It arises, as stated earlier, because of the mass correction factor incorporated for the lepton in the final state.

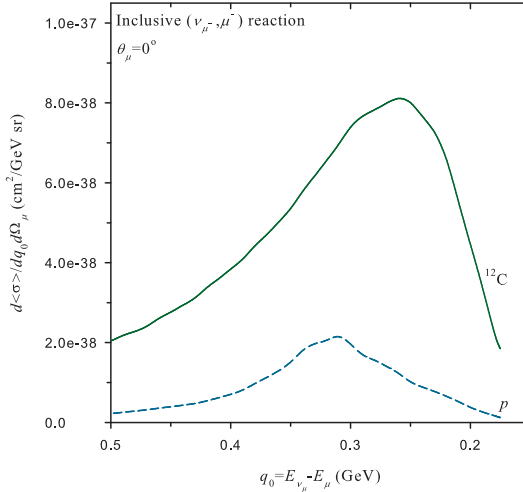


Fig. 5. (Color online) The same as those presented in Fig. 3 but for the (ν_{μ^-}, μ^-) reaction (see the text).

4. Conclusions

The double differential cross sections of the forward going lepton l energy distribution in the (ν_l, l) reactions on proton and nuclei have been calculated to look for the modification of $\Delta(1232)$ -isobar in the nucleus. Adler's PCAC theorem is used to connect the cross section of the (ν_l, l) reaction to that of the pion-induced reaction. The measured cross section of the latter is used (as an input) to avoid the uncertainties in its calculated cross section. The calculated results show that the distinct shift and broadening of the Δ -peak in the nucleus relative to proton in the inclusive (ν_l, l) reaction, and those in the nucleus increase with the size of nucleus. It appears because of the exclusive and semi-exclusive channels which occur in the nucleus. These channels do not exist for proton. The modification of the Δ -peak in the neutrino-nucleus reaction, studied using Adler's PCAC theorem, is similar to that seen in the hadron (pion)-nucleus reaction which occurs at the periphery of the nucleus. It is unlike to that occurs in the gamma- or electron-induced nuclear reaction, though the nuclear reaction based on the electro-weak interaction takes place throughout the volume of the nucleus.

The author takes the opportunity to thank an anonymous referee for giving valuable comments which helped to improve the quality of the paper.

REFERENCES

- [1] F. Binon *et al.*, *Nucl. Phys. B* **17**, 168 (1970); *Phys. Rev. Lett.* **35**, 145 (1975); N.D. Gabitzsch *et al.*, *Phys. Lett. B* **47**, 234 (1973).
- [2] A.S. Clough *et al.*, *Nucl. Phys. B* **76**, 15 (1974); A.S. Carroll *et al.*, *Phys. Rev. C* **14**, 635 (1976).
- [3] D. Ashery, *Nucl. Phys. A* **335**, 385 (1980); D. Ashery *et al.*, *Phys. Rev. C* **23**, 2173 (1981); D. Ashery, J.P. Schiffer, *Annu. Rev. Nucl. Part. Sci.* **36**, 207 (1986); S. Ciulli *et al.*, *Z. Phys. A* **302**, 45 (1981).
- [4] E. Oset, H. Toki, W. Weise, *Phys. Rep.* **83**, 281 (1982).
- [5] M. Hirata *et al.*, *Ann. Phys. (N.Y.)* **120**, 205 (1979); **108**, 116 (1977); L.S. Kisslinger, W.L. Wang, *Ann. Phys. (N.Y.)* **99**, 374 (1976).
- [6] D. Contardo *et al.*, *Phys. Lett. B* **168**, 331 (1986).
- [7] C.W. Bjork *et al.*, *Phys. Lett. B* **63**, 31 (1976); C.G. Cassapakis *et al.*, *Phys. Lett. B* **63**, 35 (1976); C. Glass *et al.*, *Phys. Rev. D* **15**, 36 (1977); D.A. Lind *et al.*, *Can. J. Phys.* **65**, 637 (1987); B.E. Bonner *et al.*, *Phys. Rev. C* **18**, 1418 (1978).
- [8] H. Esbensen, T.-S.H. Lee, *Phys. Rev. C* **32**, 1666 (1985); E. Oset, E. Shino, H. Toki, *Phys. Lett. B* **224**, 249 (1989); V.F. Dmitriev, *Phys. Lett. B* **226**, 219 (1989); J. Delorme, P.A.M. Guichon, *Phys. Lett. B* **263**, 157 (1991); T. Udagawa, P. Oltmanns, F. Osterfeld, S.W. Hong, *Phys. Rev. C* **49**, 3162 (1994).
- [9] J. Chiba *et al.*, *Phys. Rev. Lett.* **67**, 1982 (1991).
- [10] T. Hennino *et al.*, *Phys. Lett. B* **283**, 42 (1992); **303**, 236 (1993).
- [11] Swapan Das, *Phys. Rev. C* **66**, 014604 (2002); B.K. Jain, Swapan Das, *Phys. Lett. B* **386**, 33 (1996); B. K rfg n, F. Osterfeld, T. Udagawa, *Phys. Rev. C* **50**, 1637 (1994); B. K rfg n, P. Oltmanns, F. Osterfeld, T. Udagawa, *Phys. Rev. C* **55**, 1819 (1997).
- [12] J. Arends *et al.*, *Phys. Lett. B* **98**, 423 (1981).
- [13] J.S. O'Connell *et al.*, *Phys. Rev. Lett.* **53**, 1627 (1984); *Phys. Rev. C* **35**, 1063 (1987); R.M. Sealock *et al.*, *Phys. Rev. Lett.* **62**, 1350 (1989).
- [14] E. Oset, W. Weise, *Phys. Lett. B* **94**, 19 (1980); *Nucl. Phys. A* **368**, 375 (1981).
- [15] J.H. Koch, E.J. Moniz, N. Ohtsuka, *Ann. Phys. (N.Y.)* **154**, 99 (1984); J.H. Koch, N. Ohtsuka, *Nucl. Phys. A* **435**, 765 (1985).
- [16] S.K. Singh, M.J. Vicente-Vacas, E. Oset, *Phys. Lett. B* **416**, 23 (1998); T. Leitner, L. Alvarez-Ruso, U. Mosel, *Phys. Rev. C* **73**, 065502 (2006); M. Martini, M. Ericson, G. Chanfray, J. Marteau, *Phys. Rev. C* **80**, 065501 (2009); S.X. Nakamura *et al.*, *Phys. Rev. C* **81**, 035502 (2010); X. Zhang, B.D. Serot, *Phys. Rev. C* **86**, 035502 (2012).

- [17] S.L. Adler, *Phys. Rev.* **135**, B963 (1964) [Erratum *ibid.* **137**, AB4 (1965)].
- [18] W.S.C. Williams, *An Introduction to Elementary Particles*, Academic Press, New York 1971.
- [19] H. Kamano, S.X. Nakamura, T.-S.H. Lee, T. Sato, *Phys. Rev. D* **86**, 097503 (2012).
- [20] R.M. Barnett *et al.* [Particle Data Group], *Phys. Rev. D* **54**, 1 (1996); <http://pdg.lbl.gov/xsect/contents.html>
- [21] I. Ruiz Simo *et al.*, arXiv:1604.08423 [nucl-th].
- [22] A.A. Aguilar-Arevalo *et al.* [MiniBooNE Collaboration], *Phys. Rev. D* **79**, 072009 (2009).