APPROXIMATELY EQUAL INTERVAL ENERGY SPACING PROPERTIES OF THE ENERGY BANDS IN LARGE DEFORMED RARE-EARTH AND ACTINIDE NUCLEI

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The Bohr hypothesis shows that there are simple rotational energies in the definition of the energies of ground state bands of the even–even rare earths and actinides. These energy levels are arranged as 0^+, 2^+, 4^+, …A sequence of the ratios of energies of the higher excited states to the 2^+ state gives rise to a number of integers which clarifies equal interval energy spacing in the formation of the levels in the ground state bands. Such an approximately equal interval energy spacing may also be traced for higher order bands of these nuclei and not only for even–even but also for a number of odd–even, even–odd and odd–odd nuclei. This work attempts to underline the peculiarities in the spectra of rare-earth and actinide elements. Analyses also indicate a physical effect in $K^\pi = 2^+$ band of ^{232}Th element.

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1. Introduction

The energy levels of nuclei, especially the even–even nuclei, have well-known signatures of systematic structure and approximately equal energy spacing of energy levels. There have been, of course, some studies of their energy relationship over almost all of the nuclear chart. In the last few decades, many scientists have proposed some techniques to solve this problem within the framework of different models, such as the Interacting Boson Model (IBM) [1–12], one of the mostly used to describe the approximately equal energy spacing properties of energy levels, the Variable Moment of Inertia (VMI) model [13, 14] and other — given in the related literature [15–19]. Most of these studies concern the ground state energy bands of the even–even nuclei for the range of Sr to Pb and the ratio of energy levels near doubly magic nuclei. The existence of collective energy level bands of rotational and vibrational types of deformed heavy elements, such as the rare earths and actinides, can now easily be identified from the nuclear energy data [20] having rotational energy spectrum due to their collective motions [21–23].

These energy spectra are generally defined by the total angular momentum I, parity and the quantum number K which is the projection of the angular momentum on the intrinsic coordinate axis of the nuclei. K can take values $K = I, I-1, \ldots, -I$ and it is a conserved quantum number for the nuclei having axial symmetry and it defines the angular momentum of the nuclei depending on the attached coordinate system, and has a definite and constant value for a specified intrinsic state of any deformed nuclei. Intrinsic energy E_K of slowly rotating nuclei depends on K quantum number. Intrinsic energy forms the base of the rotational energy band and the energy eigenvalue of such a band, and can be defined as

$$E(I,K) = E_K = E_{\mathrm{rot}}(I)\,, \tag{1}$$

where $E_{\mathrm{rot}}(I)$ is the rotational energy. For $K = 0$, E_{rot} is given as in the Bohr hypothesis [21, 22]

$$E_{\mathrm{rot}}(I) = \frac{\hbar^2}{2J} I(I+1)\,, \tag{2}$$

where J is the moment of inertia. General representation of the rotational energy for $K = 0$ can be written as

$$E_{\mathrm{rot}}(I) = AI(I+1) + BI^2(I+1)^2 + CI^3(I+1)^3 \ldots\,, \tag{3}$$

where $A, B, C, \ldots$ are constants.

2. Even–even nuclei

The ground state energy bands have zero quantum number ($K = 0$) together with even parity and even nuclear angular momentum. Their energy values can be defined by even numbers starting from zero, $0^+, 2^+, 4^+, 6^+, \ldots$ One can obtain an energy ratio, $E(I)/E(2^+)$, to define the energy eigenvalues if one uses the Bohr hypothesis, Eq. (2), such that,

$$R(I) = \frac{E(I^+)}{E(2^+)} = \frac{I(I+1)}{6}\,. \tag{4}$$

In this equation, $E(2^+)$ represents the energy of the first excited state and can be set as the unit energy for the even–even nuclei. Using Eq. (4), one can write

$$R\left(2^+\right) : R\left(4^+\right) : R\left(6^+\right) : R\left(8^+\right) \ldots = 1 : 3\tfrac{1}{3} : 7 : 12 \ldots \tag{5}$$

However, the real values for the above ratio, Eq. (5), are a little different and they are getting smaller and smaller starting from $I = 8$ due to the deviation of the rotational energy from $I(I+1)$ resulted from the change in the moment of inertia and the dependencies of the energies on some other factors. This study will only concentrate on the approximately equal energy spacing of the energy bands, not on their nature nor other details. The energy eigenvalues of the ground state bands are given by Eq. (3), more precisely, for the deformed heavy even–even nuclei and their values can be extracted from experimental nuclear data [20].

Denoting $R(4^+)/R(2^+) = r$ and using the measured energy values [20], one gets an approximated equation

$$R\left(2^+\right) : R\left(4^+\right) : R\left(6^+\right) : R\left(8^+\right) \ldots \approx 1 : r : 2r : 3r \ldots \tag{6}$$

instead of Eq. (5). If one considers the experimental data, the last expression clearly defines the "equal intervals" and illustrates the approximately equal interval energy spacing in the bands. If one rewrites Eq. (6) starting from the ratio $R(4^+)$, then gets the ratio of integers as

$$R\left(2^+\right) : R\left(4^+\right) : R\left(6^+\right) : R\left(8^+\right) \ldots \approx r : 2r : 3r \ldots = 1 : 2 : 3 : \ldots \tag{7}$$

An example of such an approximately equal interval energy spacing in energy bands is seen using the ratio $R(I) = E(I)/E(2^+)$ and starting from $I = 4$, what is presented in figure 1 for the ground state bands in the rare earths and actinides.

One may choose an energy unit $E_1(I_1^\pi) - E_0(I_0^\pi)$ and consider the related ratios to define the mentioned approximately equal interval energy spacing for the excited bands of the even–even nuclei. Here, $E_0(I_0^\pi)$ is the energy of the lowest state having I_0 spin and π parity, and $E_1(I_1^\pi)$ is the energy of the first excited state above the $E_0(I_0^\pi)$ having suitable I_1 and π quantum numbers. In the calculations of the energy ratio for excited bands, instead of Eq. (4), one could use, in general,

$$R_n = \frac{E_n\left(I_n^\pi\right) - E_0\left(I_0^\pi\right)}{E_1\left(I_1^\pi\right) - E_0\left(I_0^\pi\right)}\,, \tag{8}$$

where $E_n(I_n^\pi)$ is the energy of the $n^{\rm th}$ excited state above the $E_0(I_0^\pi)$ having suitable I_n and π quantum numbers. In that case, using the experimental

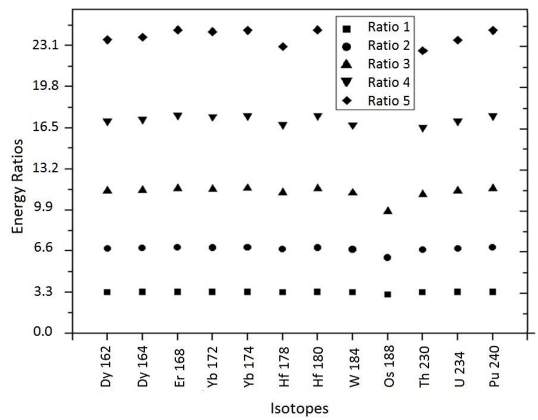

Fig. 1. Ratios for the ground state band of the even–even nuclei in the actinide and rare-earth region.

nuclear data [20], one gets

$$R_2 : R_3 : R_4 \ldots \approx r : 2r : 3r : \ldots (n-1)r \ldots , \tag{9}$$

where

$$r = \frac{E_2\left(I_2^\pi\right) - E_0\left(I_0^\pi\right)}{E_1\left(I_1^\pi\right) - E_0\left(I_0^\pi\right)} . \tag{10}$$

The level energies in the γ and β excited state bands have collective nature in the deformed even–even nuclei. β bands are defined by the collective vibrations having quantum number $K = 0$ since they preserve the symmetry axis. Energy levels of these bands are formed as $0^+, 2^+, 4^+, 6^+, \ldots$ and 0^+ is the first level that is formed by $\hbar\omega_\beta$ due to frequency of the vibration ω_β and the rotational spectra might be based on this first level by using Eq. (3). The γ bands defined by the collective vibrations corresponding to quantum number $K = 2$ also preserve the symmetry axis of the nuclei, hence the energy levels are arranged as $I = 2^+, 3^+, 4^+, 5^+, \ldots$ and plotted in figure 1.

In figure 2, the ratio R_n is presented for β bands and, as it is seen, they also clearly show the equal interval energy spacing of the energy levels for different nuclei in the rare-earth and actinides region. This shows that the equal interval energy spacing of the energy levels is valid for the excited states having odd-spin and negative parity quantum numbers [24].

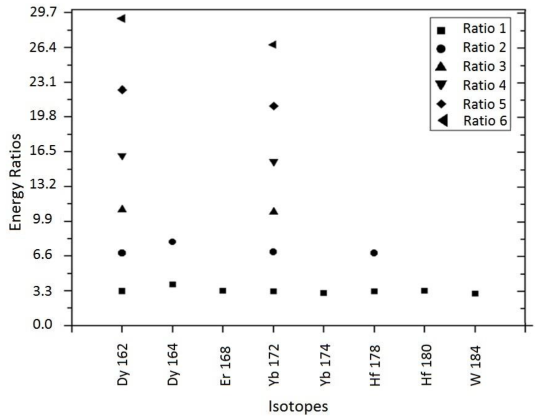

Fig. 2. Ratios for the excited state bands (β^- bands) of the even–even nuclei in the rare-earth region.

3. Nuclei having odd mass number

The observed energy levels are identified by the strong coupling of the last nucleon (proton or neutron) to the core having axial symmetry in the nuclei having odd mass number (A–odd; odd–even, even–odd). The spin for a definite K-quantum number can be determined by

$$I = K + \Omega\,, \qquad I = K+1, K+2, K+3, \dots\,, \tag{11}$$

where Ω is the projection of the angular momentum on the symmetry axis. The separation between the rotational energy bands can be calculated using the general representation [25]. For the total energy of any K-quantum number including the odd-A nuclei, it is given as

$$E(K,I) = E_K + AI(I+1) + BI^2(I+1)^2 + \dots$$
$$\begin{cases} (-1)^{I+\frac{1}{2}}\left(I+\frac{1}{2}\right)\left[A_1 + B_1 I(I+1) + \dots\right], & K = \frac{1}{2}\,,\\ (-1)^{I+1}(I+1)\left[A_2 + B_2 I(I+1) + \dots\right], & K = 1\,,\\ (-1)^{I+\frac{3}{2}}\left(I-\frac{1}{2}\right)\left(I+\frac{1}{2}\right)\left(I+\frac{3}{2}\right)\left[A_3 + B_3 I(I+1) + \dots\right], & K = \frac{3}{2}\,,\\ (-1)^{I+1}(I-1)(I+1)(I+2)\left[A_4 + B_4 I(I+1) + \dots\right], & K = 2\,, \end{cases} \tag{12}$$

where E_K is the beginning of the band, the constants A, B are the same as the constants given in Eq. (3). The parameters A_1, A_2, A_3 and A_4 are generally expressed in the form of A_{2K} and represent the perturbation due to the Coriolis interactions of the order of $2K$. Similarly, B_1, B_2, B_3 and B_4 are represented as B_{2K} and are suitable for A_{2K} (have smaller values than A_{2K}) but have higher degrees.

In most cases, it is enough to use the first two terms containing the parameters A and B to define the rotational bands of nuclei. As an example, one can use $A = 13,\ 77$ keV and $B = -6,\ 6$ eV to define the $K^{\pi} = 7/2^{+}$ ground state band of the ${}^{177}_{71}$Lu, but these two parameters are not enough to identify the $K^{\pi} = 9/2^{+}$ excited state band of ${}^{177}_{72}$Hf. Therefore, in such a situation, the band may be affected by the Coriolis interaction when the nucleus has large angular momentum. In such a case, the rotational energy in such a nuclei will be modified due to the strong Coriolis interaction of a single neutron moving on an orbit having quantum numbers [624 9/2] with a total angular momentum of $j = 13/2$. The motion of such a particle on an orbit can be defined by its intrinsic configuration and is represented by the asymptotic quantum numbers $[Nn_3\Lambda\Omega]$. The nuclear potential of heavy-deformed nuclei can be also imitated and compared to anisotropic oscillator potential [26] due to the huge shape deformations. The parameters used to define the orbits of a particle (proton or neutron) moving in such a potential are the total oscillation quantum number N, the number of the vibrating quants parallel to the potential symmetry axis n_3, the projection of the particle orbital momentum on the symmetry axis Λ, and the projection of the total angular momentum on the symmetry axis $\Omega = K$.

The rotational energy spectra for suitable bands are also defined by their intrinsic configurations, other than the parameters A, and B defining the rotational energy spectra for the mentioned nuclei. The plots presented in figures 3 and 4 are the ratios calculated by Eq. (8). They are also in agreement with the expectations of Eq. (9) and show the approximately equal interval energy spacing of the energy levels for the even–odd and odd–even rare earth and actinides nuclei.

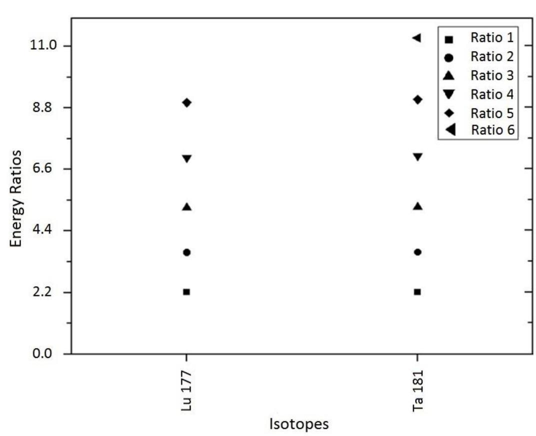

Fig. 3. Ratios for the ground state bands of the odd–even rare-earth nuclei.

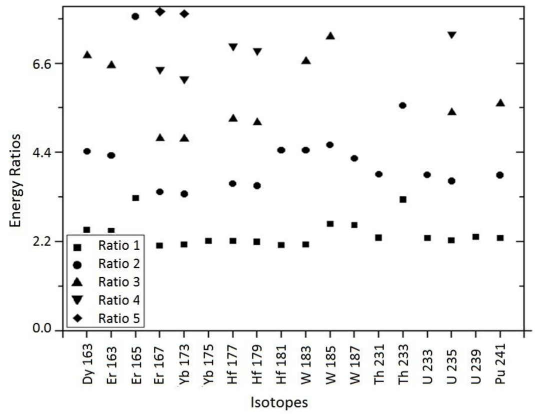

Fig. 4. Ratios for the ground state bands of the even–odd rare-earth and actinide nuclei.

4. Odd–odd nuclei

The energy bands of odd–odd nuclei are, in most cases, defined by their rotational band structures and the intrinsic states are identified by the orbits of the single neutron or proton. The projection of the total angular momentum of the single neutron or proton on the symmetry axis Ω_p and Ω_n, respectively, may form two bands for $K = |\Omega_p + \Omega_n|$ and $K = |\Omega_p - \Omega_n|$. In the case of $\Omega_p = \Omega_n = 1/2$, $K = 0$ and $K = 1$, bands are formed and the Coriolis interaction may be strong and, due to such an interaction, a deviation in the energy given by Eq. (12) is seen. The configuration of the single neutron or proton may be defined, which gives different energy bands, such as the configurations $[633\,7/2]_n$ and $[523\,7/2]_p$ resulted from $K^\pi = 0^-$

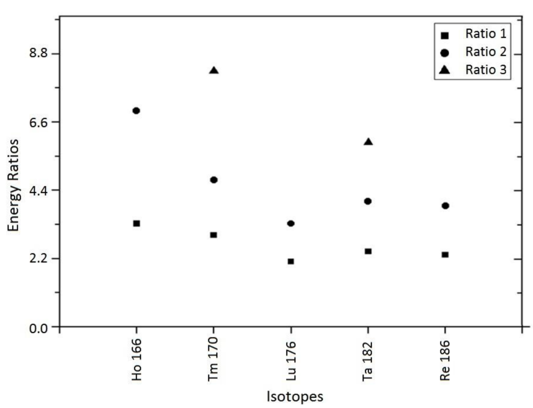

Fig. 5. Ratios for the ground state bands of the odd–odd rare-earth nuclei.

ground state and two $K^\pi = 0^- - K^\pi = 7^-$ excited states in ${}^{166}_{67}$Ho nucleus. Approximately equal interval energy spacing of these odd–odd nuclei are shown in figure 5.

5. Equal interval energy spacing properties and energy band levels of ^{232}Th

The energy levels of ^{232}Th have been measured experimentally down to 5.162 keV and are known to be totally around 100. However, only 56 of them are identified in the $K^\pi = 4^+$ (two photon gamma bands) 2^+, 0^+ (ground state bands), 0^+ and 0^- bands [20]. In figures 6 and 7, approximately equal interval energy spacing properties of ground and excited state bands, other than $K^\pi = 4^+$, using Eq. (4) and Eq. (8) are shown respectively.

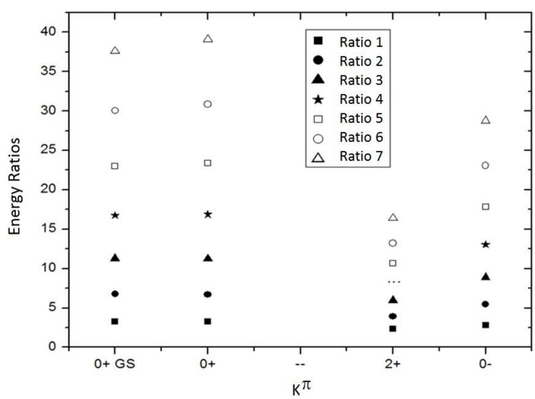

Fig. 6. Ratio values of $K^\pi = 0^+$(GS), 0^+, 2^+, 0^- bands for ^{232}Th nucleus.

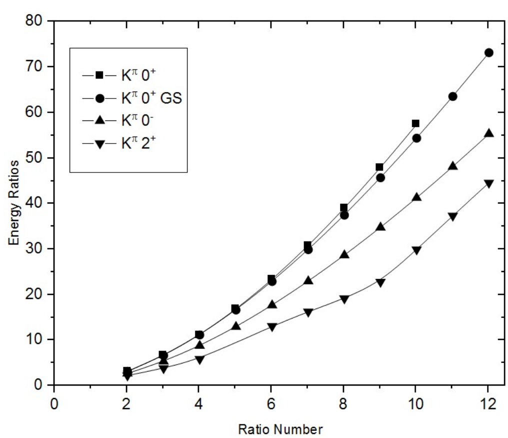

Fig. 7. $K^\pi = 0^+$(GS), 0^+, 2^+, 0^- bands and ratio values of ^{232}Th nucleus.

It can be easily seen that the equal interval energy spacing properties given by Eq. (7) and Eq. (9) are approximately verified. The r values for $K^\pi = 0^+$(GS) as given in Eq. (10) are the same, but the r values $K^\pi = 2^+$ and 0^+ are a little different, however, they approximately obey Eq. (9). The empty column at $K^\pi = 2^+$ at the fourth row corresponds to the experimental value which cannot been measured up to now.

6. Conclusion

In this study, an existence of an "approximately equal interval energy spacing", or namely, equal interval energy spacing in the energy band levels of heavy-deformed nuclei, especially in the rare earth and in the actinides region, is presented. This property of the energy spectra is very important since it allows to use the collective modes in the identification of the nuclear level density parameters [24, 27] in the applied nuclear physics [28]. The same approximately equal interval energy spacing properties have been observed and presented for the four energy band levels of ^{232}Th nucleus. The behavior of the ratio r *versus* ratio number, which is the number of the repetitions of the equal interval energy spacing of energy bands as defined in Eq. (7), illustrates physical effect at the tenth value of for $K^\pi = 2^+$ band which may result from the change in the moment of inertia. The mentioned equal interval energy spacing of energy bands property is also very important in the calculations of the entropy and the temperature of nuclei [29, 30].

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