SYMMETRIES IN MIRROR ENERGY DIFFERENCES

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The emergence of regularity and symmetry in the Mirror Energy Differences (MED) have been investigated in the present work. The MED has been calculated between mirror pair using the energies of their excited states generated by the valence protons correlation and valence neutrons correlation, respectively. The similarity in the MED *i.e.* positive (increasing) trends of the $A = 47$ and $A = 49$ mirror pair nuclei was known a test of valence symmetry in mirror nuclei. The analogous similarity in the MED *i.e.* negative (decreasing) trends in mirror pair nuclei with Z or N two units more or less than the closed shell, has been proposed as further evidence of the particle-hole symmetry in MED.

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1. Introduction

Owing to the charge independence of nuclear interaction, the same spectra are expected for a pair of mirror nuclei (obtained by interchanging the protons and the neutrons). However, small differences in the excitation energy between the same states in the mirror nuclei have been observed. These small differences are called Mirror Energy Difference (MED). For a pair of mirror nuclei, the MED was defined [1] as the difference in excitation energy as a function of spin

$$\mathrm{MED}_{I,T} = E^*_{I,T,T_z=-T} - E^*_{I,T,T_z=T} \tag{1}$$

In Eq. (1), the subscripts I, T and T_z are the nuclear spin, isospin quantum number and projection of the isospin quantum number, respectively. The '$*$' indicates excited states. The projection of the isospin quantum number is defined as $T_z = (N - Z)/2$.

Basically, Eq. (1) represents differences in the energy of excited states between the high-Z minus low-Z mirror nuclei. The ground states are normalised to zero excitation energy, therefore, the Coulomb effect almost cancels out. However, the MED can be reliably interpreted in terms of structural phenomena such as changes in the Coulomb energy due to the spatial correlations of valence protons, and/or changes in radius/deformation as a function of spin. This stimulates us to calculate the MED values between mirror pair generated by the valence protons correlation minus valence neutrons correlation, respectively. For example, in $^{35}_{18}\mathrm{Ar}_{17}$, the low spins are generated by the breaking of proton pair, while in $^{35}_{17}\mathrm{Cl}_{18}$ by neutron pair *i.e.*, (^{35}Ar–^{35}Cl). The blocking due to the unpaired nucleon (or/and closed shell) favours the correlation/alignment of a pair of the other type. The MED can be defined as

$$\mathrm{MED}_{I\left(Z_{\mathrm{e,f}}-N_{\mathrm{e,f}}\right)} = E^*_{I,Z_{\mathrm{e,f}}} - E^*_{I,N_{\mathrm{e,f}}}\,, \tag{2}$$

where $E^*_{I,Z_{\mathrm{e,f}}}$ and $E^*_{I,N_{\mathrm{e,f}}}$ are the energy of excited states due to protons correlation and neutrons correlation, respectively, and subscripts $Z_{\mathrm{e,f}}$ and $N_{\mathrm{e,f}}$ are for protons and neutrons, respectively, e,f indicates either even number or first breaking of pair.

The plots of MED (calculated by using definition of MED given by Eqs. (1) or (2)) *vs.* spins show different patterns. The MED values with angular momentum for several mirror pair upto low spins (or before the first band crossings) have been examined by considering the MED, Eq. (2). Some empirical similarities/symmetries in MED trends have been observed. The cases of mirror nuclei having either number of protons or neutrons equal to two units away (less or more) from the closed shell [2–5] are special, as they show the negative (decreasing) trends of MED. On the other hand, in Ref. [6], several cases have been discussed, where MED *vs.* angular momentum plots show the positive (increasing) trends. In the present work, we show a possible qualitative (empirical) interpretation of the positive/negative MED trends. In addition, some symmetries which are expected due to different MED trends have been discussed. Primarily results have been mentioned earlier in Ref. [7], now we present a comprehensive picture of the outcomes.

2. Discussion

2.1. Positive and negative trends of MED

The main objective of the present article is the study of symmetries in MED. The sign and order of MED are useful in extracting out the valuable information [8] regarding the nuclear structure of excited levels. The experimental data from various published works [6, 9–20] have been used. Figure 1 shows an example of the MED values trends by using Eqs. (1) and (2). Some similarities have appeared in $+/-2$ closed shell mirror pair systems, such as negative (decreasing) trend (Fig. 1 (d)) and positive (increasing) trend (Fig. 1 (c)) in the other mirror pair systems, when MED was plotted by using Eq. (2). However, no such symmetries arise, when MED was calculated using Eq. (1). It can be clearly noticed from Fig. 1 (a) that MED values trend obtained by using Eq. (1) will be different for $A = 47$ (positive) and $A = 49$ (negative) mirror pair systems although, both are positive when plotted against Eq. (2). Similarly, the MED values trend (Fig. 1 (b)) by using Eq. (1) will be different for $A = 53$ (positive) and $A = 59$ (negative), mirror pair systems. The similar trends are expected to exhibit some symmetries.

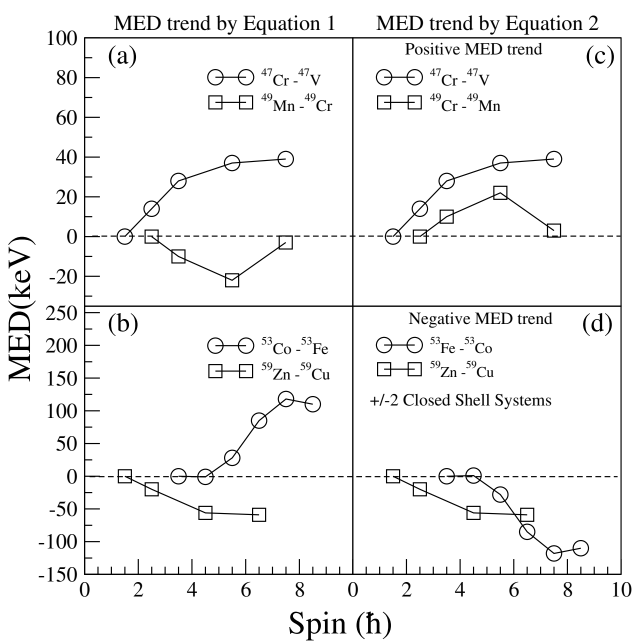

Fig. 1. A comparison of the MED trend plotted against Eqs. (1) and (2). The plots (c), (d) of MED against Eq. (2) exhibit some similarities, such as negative MED trend for $+/-2$ closed shell mirror pair systems and positive MED trend for others. However, the plots (a), (b) against Eq. (1) for the same systems exhibit different trends (no similarity). Experimental data were taken from [6, 9, 10, 12].

2.2. Valence symmetry in MED trends

In Ref. [6], a study of MED had been carried out using Eq. (1) for the trends of MED as a function of nuclear spin in the $A = 47$ and $A = 49$ mirror pairs, and the obtained symmetry in MED was proposed as an example of the valence symmetry. As mentioned above, Eq. (2) has been utilised for plotting the MED trends in the present study. Figure 1 (c) shows similar MED trends for $A = 47$ and $A = 49$ mirror pairs. It can be noticed that these mirror pairs show the positive trend of the MED and, therefore, it can be proposed that the positive trends of the MED in others mirror pairs are also examples of valence symmetry.

2.3. Particle-hole symmetry in MED trends

Generally, if the number of valence nucleons lie below the middle of a major shell, they are considered as particles; otherwise, they are taken to be holes. The particles and holes behave in similar manner near the closed shells *i.e.*, particle–hole symmetry [21]. The particles and holes spectra are connected by a symmetry relation $E_I(\text{particles}) \equiv E_I(\text{holes})$ [22–24].

To investigate the particle–hole symmetry in MED, such systems were chosen, where the low-spin states generated by the correlation of Z-particles/holes. In such a scenario, the Z-particles/holes will be even number, and the neutrons will be either closed shell or odd number. The MED as a function of spin using Eq. (2) for such possible systems, $A = 35, 38, 42, 43, 53, 54$ and 59 [15–20] is shown in Fig. 2. These mirror pairs have Z or N equal to two units away from the closed shell. The MED pattern shown in Figs. 2 (a), 2 (b) are due to the correlation of holes and particles, respectively, for odd-A nuclei. The MED patterns shown in Figs. 2 (c), 2 (d) are due to correlation of holes and particles respectively, for even–even nuclei.

It can be noticed that these patterns show negative trends for low spin. Moreover, a large change in the MED can be noticed for mirror pair system having N or Z two units lower than the closed shell (Figs. 2 (a), 2 (c)) as compared to mirror pair systems having Z or N are two units higher than the closed shell (Figs. 2 (b), 2 (d)). In some cases, a positive MED value is observed only for the first excited state, as shown in Fig. 2. This consistent anomaly is shown with "a^*" for the first excited state in Fig. 2. The full description of this anomaly had been given in Refs. [10, 25], although the origin of it is still unresolved. We are specifying few points which have relevance in our study without going into details. In Ref. [10], two-proton Coulomb matrix elements (CME) was used in conjunction with the shell-model wave functions to determine the Coulomb energy contribution for each level. Another solution was proposed by Zuker in Ref. [25] by considering an isospin non-conserving term of the nuclear interaction.

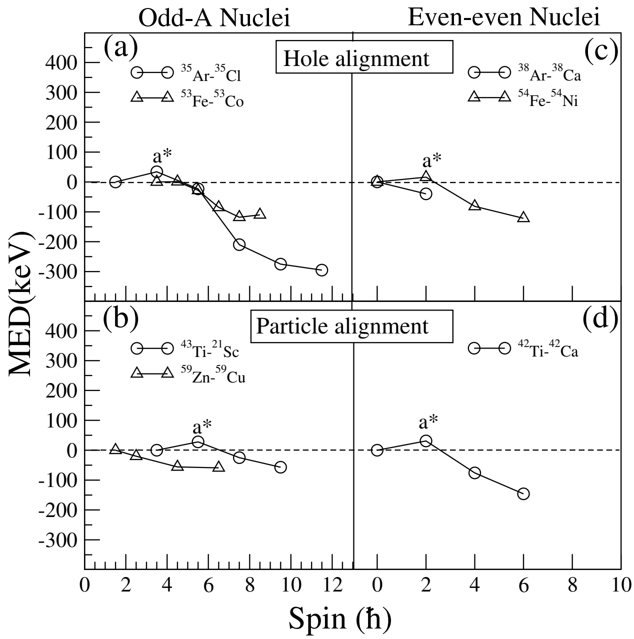

Fig. 2. (a), (b) Mirror energy differences *vs.* spins for odd-A nuclei $A = 35, 43, 53, 59$ [10, 12–14] and (c), (d) for even–even nuclei $A = 38, 42, 54$ [15–20]. (a), (c) due to holes correlation and (b), (d) due to particles correlation.

2.4. Empirical interpretation of positive and negative trends

Although the differences in excitation energy between excited states of mirror nuclei depends on several factors [1, 25–30], such as the difference of proton and neutron masses, isospin breaking terms of the nuclear interaction, the multipole Coulomb term and alignment, the radial term, single-particle corrections, the change in Coulomb interaction and change in shape parameter have major contribution in two different trends. The first one is responsible for the positive (increasing) MED trend, while the second is responsible for negative (decreasing) MED trend.

From Eq. (2), the MED values (Fig. 1 (c)) must have only the positive (increasing) trends, if we consider that the Coulomb field is mainly responsible for excitation energy differences between mirror nuclei. The ground states are normalised to zero excitation energy, therefore, the Coulomb effect almost cancels out. However, protons correlations/alignments affect the Coulomb energy, which, in turn, affects the energy of excited states. The Coulomb interaction between two protons coupled in time-reversed orbits is larger than for any other coupling, as the spatial overlap of their orbits is maximum. In particular, when a pair of protons aligns to the maximum value $(2j-1)$ in a single j shell, the Coulomb interaction between them reaches its minimum value as their spatial separation is largest [6, 9] in this configuration. As the Coulomb interaction is repulsive between protons, the effect of this correlation/alignment results in an increase in the excitation energy of the nuclear state. On the other hand, correlation/alignment of neutrons does not have any effect on the excitation energy.

From Eq. (2), the MED values (Fig. 1 (d)) must show only the negative (decreasing)trend, if we consider that the shape parameter is mainly responsible for the excitation energy differences between mirror nuclei.

Before discussing the negative MED trend, we will give attention to the regularities from energy systematics of even–even nuclei. The closed shell nuclei are considered as spherical and rather hard [31]. The nucleus becomes softer as nucleons are added or removed from the closed shell [31]. The following regularities of quadrupole deformation (β) are observed:

1. The relative change in the deformation between the closed shell (β_0) and two units away from the closed shell ($\beta_{\pm 2}$) nuclei are larger than the relative change in the deformation between nuclei with two units away from the closed shell ($\beta_{\pm 2}$) and nuclei with four units away from the closed shell ($\beta_{\pm 4}$) *i.e.*, $(\beta_{\pm 2}-\beta_0) > (\beta_{\pm 4}-\beta_{\pm 2})$. For example, the quadrupole deformation for the closed shell $^{56}_{28}\mathrm{Ni}_{28}$, $^{54}_{26}\mathrm{Fe}_{28}$ and $^{52}_{24}\mathrm{Cr}_{28}$ nuclei are $\beta_0 = 0.130(3)$, $\beta_{-2} = 0.203(3)$, and $\beta_{-4} = 0.212(4)$, respectively [32]. In this case, the $(\beta_{-2}-\beta_0)$ will be 0.073(3) and $(\beta_{-4}-\beta_{-2}) = 0.009(3)$, *i.e.*, $(\beta_{-2}-\beta_0) > (\beta_{-4}-\beta_{-2})$.

2. The energy of the first excited 2^+ state of $^{48}_{20}\mathrm{Ca}_{18}$ (2213.13(10) keV) is higher than of ${}_{18}\mathrm{Ar}_{20}$ (2167.64(5) keV) [32]. However, the alignments of holes (protons) reduces the Coulomb energy in $^{38}_{18}\mathrm{Ar}_{20}$, thereby implying a higher energy of 2^+ state of $^{38}_{18}\mathrm{Ar}_{20}$ as compared to $^{38}_{20}\mathrm{Ca}_{18}$, in contrast to what is observed. This apparent contradiction may be resolved if deformation of these nuclei is taken into account, where $^{38}_{18}\mathrm{Ar}_{20}$, $\beta = 0.161(4)$, has a larger deformation than $^{38}_{20}\mathrm{Ca}_{18}$, $\beta = 0.121(14)$.

3. The nuclear deformation at equal number of valence protons and neutrons is different. The nuclei with Z-valence particles/holes are more deformed than nuclei with equal number of N-valence particles/holes. For example, for two valence protons $^{42}_{22}\mathrm{Ti}_{20}$, $\beta = 0.290(4)$ and two valence neutrons $^{42}_{20}\mathrm{Ca}_{22}$, $\beta = 0.245(4)$ [32]. Similarly, a large change in deformation is expected due to particles/holes alignments in nuclei having particles/holes as protons with two units away from closed shell, as compared to nuclei with neutron two units away from closed shell.

From the above regularities, it is inferred that the MED of the mirror nuclei with Z or N two units away from the closed shell is sensitive mainly to the changes in shape/size due to particles/holes correlations/alignments. The effect of the change of deformation in the MED was also introduced in Refs. [6, 33, 34] and calculated within the Liquid Drop Model. From the above regularities and the calculations made in Refs. [6, 33, 34], one can assume/claim that the nuclei are different in deformation and the changes in their deformation as a function of angular momentum are also different. In this scenario, the trend of MED *vs.* spin has been found to be extremely sensitive to macroscopic effect, as shown in Figs. 1 (d) and 2. From the above regularities, a large change in the MED of mirror pairs (see Figs. 2 (a), 2 (c)) with Z or N equal to two units less than the closed shell has been observed as compared with Z or N equal to two units more than the closed shell (see Figs. 2 (b), 2 (d)).

3. Conclusion

The Mirror Energy Differences with angular momentum trends were examined for several mirror pair upto the low spins (or first band crossing). The positive (increasing) and the negative (decreasing) MED trends have been observed. The MED values between mirror pair were calculated using their excited states energies generated by the valence protons correlations and valence neutrons correlations, respectively. The negative trend of MED in mass $A = 35, 53, 38$ (with Z or N equal to two units smaller from the closed shell) has been compared with the MED in mass $A = 43, 59, 42$ (with Z or N equal to two units larger from the closed shell). It has been proposed to exhibit the further evidence of the particle–hole symmetry in the MED because the particles and holes behave in the identical manner near closed shells *i.e.* particle–hole symmetry. In these cases, the changes in nuclear shape parameters as a function of spin play a crucial role. The similar positive MED values trends of $A = 47$ and $A = 49$ mirror pair were exhibited as an example of the valence symmetry. In this scenario, the reduction in the Coulomb energy term due to the spatial correlations of pairs of valence protons plays a crucial role.

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