

# JET MASS FLUCTUATIONS AND FRAGMENTATION FUNCTIONS\*

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The effect of jet mass fluctuations on the fragmentation process is examined in the framework of a statistical hadronisation model. In this model, the fragmentation scale  $Q^2$  is taken to be the virtuality of the leading parton, and jet mass fluctuations are accounted for through this quantity. The scale evolution of the model is treated in the  $\phi^3$  theory with leading-order splitting function and one-loop coupling.

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## 1. Introduction

Energetic quarks and gluons produced in high-energy collisions create highly collimated bunches (jets) of hadrons. As this hadronisation (fragmentation) process cannot be handled by perturbation theory, usually phenomenological models or empirical formulas are used for the description of the distributions (fragmentation functions — FFs) of certain types of hadrons in jets initiated by a certain type of quark or gluon (parton). When the square of the total four-momentum of the jet (the mass  $M_{\text{jet}}^2 = P_{\text{jet}}^2$ ) is much smaller than the energy of the jet, the “leading” parton can be regarded as on-shell, and hadron distributions can be calculated as the convolution of hard cross sections and FFs (factorisation theorem [1, 2]).

However, there are cases when  $M_{\text{jet}}$  is not negligible compared to  $P_{\text{jet}}^0$ . For example [3], the masses of jets fluctuate according to

$$\rho(M) \sim \frac{\ln^b(M/\mu_0)}{M^c} \quad (1)$$

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in proton–proton ( $pp$ ) collisions at  $\sqrt{s} = 7$  TeV [4] and jet transverse momenta  $P_T^{\text{jet}} \in [200\text{--}600]$  GeV/ $c$ . The masses of such jets are typically of the order of 60–100 GeV/ $c^2$ . According to [3, 5], in the case of such “fat” jets, it is reasonable to parametrise FFs by the variable  $\tilde{x} = 2P_\mu^{\text{jet}} p_h^\mu / M_{\text{jet}}^2$  ( $p_h^\mu$  being the four-momentum of the hadron) and use  $\tilde{Q} = M_{\text{jet}}$  as the fragmentation scale instead of  $x = p_h^0 / P_{\text{jet}}^0$  and  $Q = \theta_c P_{\text{jet}}^0$  ( $\theta_c$  being the jet opening angle), which are most often used in the literature (*e.g.* [6–13]). On the one hand, looking at the schematic picture of a jet (Fig. 1, left), we only have two four-momenta  $P_\mu^{\text{jet}}$  and  $p_h^\mu$  (in the spin-averaged case) to construct scalars from. As most created hadrons are pions, we may neglect the mass of the hadron  $p_h^2 = m_\pi^2 \approx 0$ , and we are left with  $P_{\text{jet}}^2 = M_{\text{jet}}^2$  and  $P_{\text{jet}}^\mu p_h^\mu$ . Thus, it is reasonable to use  $\tilde{x}$  as the dimensionless variable, and  $M_{\text{jet}}$  as the fragmentation scale. On the other hand, the width of the phase space of hadrons inside the jet (Fig. 1, right), allowed by energy-momentum conservation, is equal to  $M_{\text{jet}}$ , rather than  $\theta_c P_{\text{jet}}^0$ .

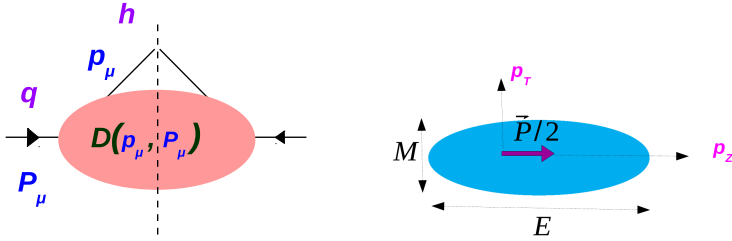


Fig. 1. Left: sub-graph of a jet with incoming initial parton  $q$  of momentum  $P_\mu$  and outgoing hadron  $h$  of momentum  $p_\mu$ . Right: the phase-space ellipsoid (with the centre  $\mathbf{P}/2$ , longer axis  $2a = E$  and smaller axis  $2b = M$ ), available for hadrons in a jet of momentum  $P_\mu = (E, \mathbf{P})$ . In the limit of  $|\mathbf{P}| \rightarrow E$ , the ellipsoid shrinks and Eq. (2) becomes a one-dimensional distribution of  $\tilde{x} = p^0/E = x$ .

If, however, we use  $M_{\text{jet}}$  as fragmentation scale, we need to take into account its fluctuations when fitting experimental data. In Sec. 2, I briefly summarise the statistical fragmentation model and its scale evolution discussed in detail in [3, 5]. Furthermore, I obtain the jet-mass averaged FF and compare it to experimental data measured in  $pp$  collisions at  $\sqrt{s} = 7$  TeV.

## 2. A statistical fragmentation model

Although the scale evolution of FFs can be obtained using perturbation theory, a non-perturbative input is needed, namely the form of FFs at an initial scale  $Q_0$ . I use the statistical fragmentation model [3, 5] for this purpose. In this model, the microcanonical ensemble is used (as in many cases in the literature [3, 14–23]) to account for the finiteness of the energy and

multiplicity of jets. Besides, negative-binomial hadron multiplicity distributions are also taken into account, thus the single particle distribution inside jets of fix four-momentum  $P_{\text{jet}}^\mu$  becomes

$$D(x, Q_0^2) = A_0 \left[ 1 + \frac{q_0 - 1}{\tau_0} x \right]^{-1/(q_0 - 1)} - B_0 \quad (2)$$

with  $q_0$  and  $\tau_0$  being parameters,  $A_0$  is fixed by the normalisation condition, and  $B_0 = A_0[1 + \frac{q_0 - 1}{\tau_0}]^{-1/(q_0 - 1)}$  to ensure that  $D(x = 1, Q_0^2) = 0$  at the boundary of the phase space ( $x = 1$ ).

For simplicity, I treat the scale evolution in the  $\phi^3$  theory, where the DGLAP equation reads

$$\partial_t D(x, t) = g^2 \int_x^1 \frac{dz}{z} D\left(\frac{x}{z}, t\right) \Pi(z) \quad (3)$$

with the leading-order (LO) splitting function

$$\Pi(x) = x(1 - x) - \frac{1}{12}\delta(1 - x), \quad (4)$$

scale variable  $t = \ln(Q^2/\Lambda^2)$  and  $\Lambda$  being the scale, where  $g^2(t) = 1/(\beta_0 t)$ , the 1-loop coupling of the  $\phi^3$  theory diverges; and  $\beta_0$  being the first coefficient of the beta function of the  $\phi^3$  theory. The solution is

$$D(x, t) = \int_x^1 \frac{dz}{z} g(z, t) D\left(\frac{x}{z}, t_0\right) \quad (5)$$

with the kernel

$$\begin{aligned} g(x, t) \sim & \delta(x - 1) + \sum_{k=1}^{\infty} \frac{b^k(t)}{k!(k-1)!} \sum_{j=0}^{k-1} \frac{(k-1+j)!}{j!(k-1-j)!} \\ & \times x \ln^{k-1-j} \left[ \frac{1}{x} \right] \left[ (-1)^j + (-1)^k x \right] \end{aligned} \quad (6)$$

and the statistical fragmentation function given in Eq. (2) to serve as an initial function at the starting scale  $t_0 = \ln(Q_0^2/\Lambda^2)$ .

In order to compare this result to experimental data, we need to calculate the jet-mass averaged FF

$$\frac{dN}{dz} = \int dM \rho(M) D \left[ z, \ln \left( \frac{M^2}{\Lambda^2} \right) \right], \quad (7)$$

as in the case of available experimental data sets the jet mass is not fixed. There is only one published data-set pair, in the case of which the kinematical properties of jets used when making the mass distribution and the fragmentation function coincides. This is the case of jets with transverse momenta  $P_T^{\text{jet}} \in [400, 500]$  GeV/ $c$  in  $pp$  collisions at  $\sqrt{s} = 7$  TeV [4, 24]. As Fig. 2 shows, in the case of this data set, smooth description of the measured FF can be achieved with Eq. (7). The parameters of the mass distribution were obtained in [3] by fitting Eq. (1) to data in [4].

This result nicely supports the idea of using the jet mass as the fragmentation scale. This way, however, it would be advantageous to have experimental data on fragmentation functions inside jets of fixed mass instead of the fixed energy or transverse momentum.

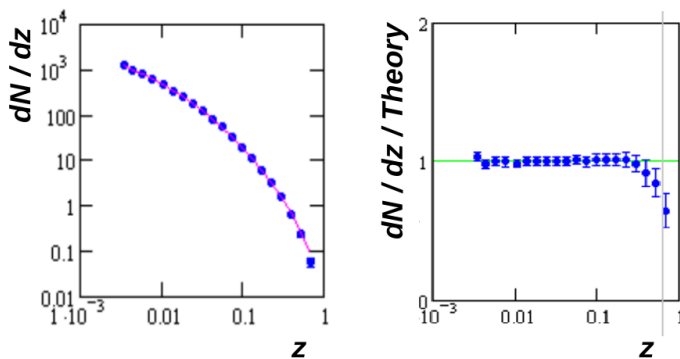


Fig. 2. Left: comparison of the jet-mass averaged FF Eq. (7) and measured FF inside jets of  $P_T^{\text{jet}} \in [400, 500]$  GeV/ $c$  in  $pp$  collisions at  $\sqrt{s} = 7$  TeV. The parameters of the initial fragmentation function Eq. (2) at scale  $Q_0 = 1$  GeV are  $q_0 = 1.275$ ,  $\tau_0 = 0.02$ ,  $\beta_0 = 0.1$ ,  $\Lambda = 0.2$ . Parameters of the mass distribution Eq. (1) are  $b = 70$ ,  $c = 18$  and  $\mu_0 = 1.4$  GeV. Right: data over theory plot.

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