

# EFFECT OF EMISSION OF EXTRA LEPTON PAIR FOR PRECISE MEASUREMENT OF $W$ -BOSON MASS\*

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In the paper, we present results for the final state emissions of lepton pairs in decays of heavy-intermediate states, principally of  $Z$  boson, but of some importance for the  $W$  decays as well. The presented semi-analytical calculation and PHOTOS MC program are in a numerical agreement to better than 5% of pair effects. Suggestions for the future works are given.

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## 1. Introduction

One of the purposes of the LHC experiments is to improve precision of the  $W$ -boson mass measurement. Precision measurements of the  $W$ -boson mass rely on a precise reconstruction of momenta for the final state leptons [1] and on comparison of  $W$  production and decay with those of  $Z$  in the LHC and, in particular, LHC detector conditions. The QED effects of the final state radiation play an important role in such experimental studies [2]. The final state bremsstrahlung is included in all simulation chains and should be studied together with the detector response to leptons.

In the present paper, we will concentrate on the effects related to additional pair emissions in decays of heavy bosons, mainly  $Z$ . These effects should be included starting from the second order of QED, *i.e.* from the  $\mathcal{O}(\alpha^2)$  corrections. The typical Feynman diagrams for pair corrections of the final states are shown in Fig. 1.

Precise calculations include radiative corrections, which are usually calculated with a help of MC generators. The experimental data are compared to expectations from the MC simulation. A convenient way for after-burner

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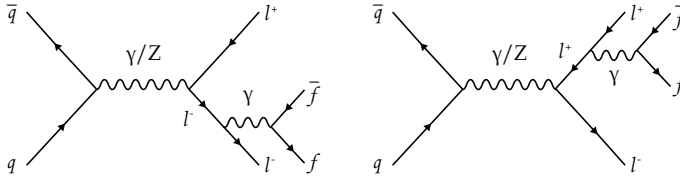


Fig. 1. Real pair correction.

type algorithms is to use a hard process MC generator (like PYTHIA [3]) to generate series of events  $pp \rightarrow Z/\gamma^* \rightarrow$  lepton pair and then to modify some of two-lepton final states with appropriate probability to be four-lepton states.

The goal of present paper is to derive analytically the correction to the  $Z$ -boson decay due to the emission of extra lepton pair and to compare this prediction with the previous one which is made by the PHOTOS MC generator [4].

## 2. Description of the factorization scheme

In what follows, we implement the amplitude of emission of extra lepton pair from a final state, *i.e.*  $Z \rightarrow \ell^+ \ell^- + (f \bar{f})$ , by a factor  $\widetilde{B}_f$  which multiplies the Born amplitude. The cross section of this process, using the notation from Ref. [5], writes

$$\sigma = \int d\Omega F |M_B|^2, \quad (1)$$

where  $F$  is a factorized term<sup>1</sup> for pair emission as used in Ref. [5] and  $\Omega$  is a four-body phase space,  $|M_B|^2$  is square of Born level matrix element. The factorized term for pair emission  $F$  is

$$F = \left(\frac{\alpha}{\pi}\right)^2 \frac{1}{\pi^2} \left( \frac{2p - aq}{aq^2 - 2pq} - \frac{2p' - aq}{aq^2 - 2p'q} \right)_\mu \left( \frac{2p - aq}{aq^2 - 2pq} - \frac{2p' - aq}{aq^2 - 2p'q} \right)_\nu \times \frac{4q_1^\mu q_2^\nu - q^2 g^{\mu\nu}}{2q^4}, \quad (2)$$

where  $p^\mu$ ,  $(p')^\mu$  denote the 4-momentums of outgoing leptons, and  $q_1^\mu$ ,  $q_2^\mu$  — of additional emitted leptons;  $q^\mu = q_1^\mu + q_2^\mu$ ;  $q_1^2 = q_2^2 = \mu^2$ ;  $p^2 = (p')^2 = m^2$ . For the purpose of our calculations, the parameter  $a$  (for more details, see Ref. [5]) is set to be 0. We will return to the question of normalization of (2) at the end of the section. The phase space  $\Omega$  is given by

<sup>1</sup> One should note that such factorization is performed in soft pair approximation, *i.e.* momentums of leptons before and after the emission differ slightly.

$$\Omega = \int \frac{d^3q_1}{2(q_1)_0(2\pi)^3} \frac{d^3q_2}{2(q_2)_0(2\pi)^3} \frac{d^3p}{2p_0(2\pi)^3} \frac{d^3p'}{2p'_0(2\pi)^3} (2\pi)^4 \times \delta^4(R - p - p' - q_1 - q_2), \tag{3}$$

where  $R$  is the 4-momentum of ingoing  $Z$  boson.

After integration over angles, we bring (1) to the form of

$$\sigma = \frac{1}{(2\pi)^6} \int \left[ \frac{1}{(2\pi)^2} \frac{\lambda^{\frac{1}{2}}\left(1, \frac{m^2}{s}, \frac{m^2}{s}\right)}{8} |M_B|^2 d\cos\theta_p d\varphi_p \right] \times \widetilde{B}_f, \tag{4}$$

where  $\theta_p, \phi_p$  define orientation of  $p$  in a rest frame of  $Z$ ,  $s = R^2$  is square of invariant mass of lepton pair before emission of additional pair,  $\lambda$ -function is defined in the standard way

$$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc. \tag{5}$$

The factor, which represents real correction due to emission of additional lepton pair, writes

$$\begin{aligned} \widetilde{B}_f = & -\frac{2}{3s} \left(\frac{\alpha}{\pi}\right)^2 \lambda^{-\frac{1}{2}}\left(1, \frac{m^2}{s}, \frac{m^2}{s}\right) \int_{4m^2}^{(\sqrt{s}-2\mu)^2} dM_{LL}^2 \int_{4\mu^2}^{(\sqrt{s}-M_{LL})^2} \frac{dM_{ll}^2}{M_{ll}^2} \\ & \times \sqrt{1 - \frac{4\mu^2}{M_{ll}^2}} \left(1 + \frac{2\mu^2}{M_{ll}^2}\right) \left[ \frac{m^2 \sqrt{1 - \frac{4m^2}{M_{LL}^2}} \lambda^{\frac{1}{2}}(s, M_{LL}^2, M_{ll}^2)}{M_{ll}^2 M_{LL}^2 + \frac{m^2}{M_{LL}^2} \lambda(s, M_{LL}^2, M_{ll}^2)} \right. \\ & \left. + \frac{M_{LL}^2 - 2m^2}{s - M_{ll}^2 - M_{LL}^2} \ln \frac{s - M_{ll}^2 - M_{LL}^2 - \sqrt{1 - \frac{4m^2}{M_{LL}^2}} \lambda^{\frac{1}{2}}(s, M_{LL}^2, M_{ll}^2)}{s - M_{ll}^2 - M_{LL}^2 + \sqrt{1 - \frac{4m^2}{M_{LL}^2}} \lambda^{\frac{1}{2}}(s, M_{LL}^2, M_{ll}^2)} \right], \tag{6} \end{aligned}$$

where  $M_{LL}$  is invariant mass of lepton pair after emission,  $M_{ll}$  is invariant mass of additional lepton pair. The expression in square brackets in (4) contains a square of the Born level matrix element and 2-body phase space (see Ref. [6], for example). In our semi-analytical calculation, this expression corresponds to the number of events obtained by PYTHIA for each interval of invariant masses of lepton pair.

We should stress that in soft limit, *i.e.*  $M_{ll} \rightarrow 0$  and  $M_{LL} \sim \sqrt{s}$ , the factor  $\widetilde{B}_f$  given by (6) coincides with such a factor which is given by formula (5) of Ref. [5] for the case of emission of soft lepton pair from the initial state.

Now, we return to the question of normalization of (2). The factor  $\frac{1}{(2\pi)^6}$  in front of (4) should be omitted. This factor is a consequence of prefactor in expression for  $F$  in Eq. (2), where factorized part is mixed with terms which come from 2-body phase space. It is considered in Ref. [5].

### 3. Numerical results

With formula (6), we take integrals numerically and compare results with output of PHOTOS MC generator. For simulation by PHOTOS and for semi-analytical calculation, we first generate the sample of events from PYTHIA with initialization as given in Fig. 2. In order to complete results for PHOTOS, its algorithm is simply applied on events generated by PYTHIA. For calculation with formulae (6), we move events that are generated by PYTHIA, to every possible bin with probabilities obtained from formula (6).

```

WeakSingleBoson:ffbar2gmZ = on
23:onMode = off
23:onIfAny = 11
23:mMin = 10.0
23:mMax = 200.0
HadronLevel:Hadronize = off
SpaceShower:QEDshowerByL = off
SpaceShower:QEDshowerByQ = off
PartonLevel:ISR = off
PartonLevel:FSR = off
Beams:idA = 2212
Beams:idB = 2212
Beams:eCM = 14000.0

```

Fig. 2. Initialization parameters for PYTHIA.

In Fig. 3, distribution of the number of events of  $Z$  production and decay as a function of invariant mass of four leptons is presented. Solid line represents data by PYTHIA×PHOTOS. Points correspond to semi-analytical evaluation.

In Fig. 4, the ratio of the two last distributions as a function of invariant mass of four leptons is presented.

Analyzing Figs. 3 and 4, we can conclude that PHOTOS is in agreement with analytical calculation fairly well. Numerical precision of agreement is better than 5% of the pair effect. Estimation is limited by the numerical calculation and CPU time. It can be improved rather easily.

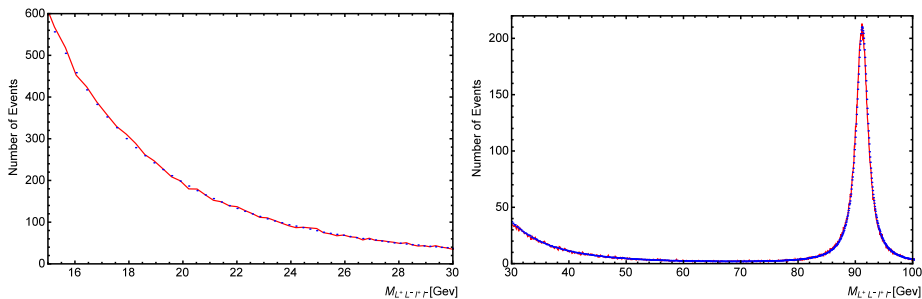


Fig. 3. Comparison of PHOTOS and semi-analytical calculations for the process  $pp \rightarrow Z \rightarrow e^+e^-(e^+e^-)$ . The solid line represents data by PYTHIA $\times$ PHOTOS. Points represent simulation by PYTHIA, modified with formula (6).

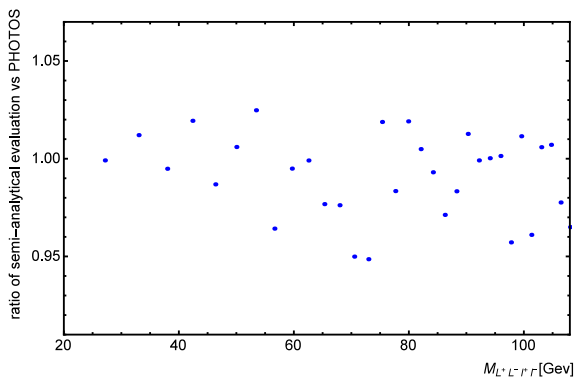


Fig. 4. Number of events from PYTHIA multiplied by a factor resulting from formula (6) divided by number of events from PYTHIA $\times$ PHOTOS.

### 4. Conclusions

Computation of distribution of the number of emissions of additional electron-positron pair in the process  $pp \rightarrow Z \rightarrow e^+e^-$  on the invariant mass of two electron-positron pairs by PHOTOS is in a good agreement with computation based on analytic formula. Our calculation is based on an extension of previous calculation [5], where soft approximation for emission of pairs was used. Since rigorous expression (3) for 4-body phase space is implemented in PHOTOS, it can be developed to operate full matrix element for additional lepton pair emission similarly to modifications that are used in Refs. [7, 8] for photon emission.

Semi-analytical and PHOTOS approaches allow to investigate processes with any type of emitting and emitted lepton pairs, *i.e.* processes  $pp \rightarrow Z \rightarrow e^+e^-(e^+e^-, \mu^+\mu^-)$  and  $pp \rightarrow Z \rightarrow \mu^+\mu^-(e^+e^-, \mu^+\mu^-)$ . Such investigation can be a further step of our research.

The presented test complements the PHOTOS comparison to results for 4-fermion final states obtained from KORALW Monte Carlo [9] for  $Z$ -boson decay, where extremely narrow width of intermediate  $Z$  boson is used to block emission of pairs from initial state, but preserve gauge invariance. Further tests are reported in Ref. [4].

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