

MINLO FOR MULTI-JET PROCESSES*

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In this paper, the recent progress in improving the Minlo procedure for generating inclusive event samples that are (N)NLO accurate in various jet multiplicities is discussed. As a proof of principle, a selection of the predictions for Higgs boson production in association with up to two jets is shown. The predictions are simultaneously accurate at NNLO for observables inclusive in H production, NLO accurate in H +jet production and NLO accurate in H +2jets production.

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1. Introduction

Combining matrix elements of various multiplicities with parton showers to increase the accuracy of fully exclusive predictions have been around for over 15 years at leading order accuracy [1–10]. Since the start of the LHC, where data is gathered at an unprecedented accuracy, the need to go to next-to-leading order accuracy has been paramount. Several methods have been developed and become the new standard in direct comparisons to data [11–22]. These improved predictions are much more accurate and precise than their predecessors, allowing for a much higher scrutiny of the experimental data.

In this contribution, one of these recent developments will be discussed. In particular, we discuss the Minlo method and present results for Higgs boson production with up to two jets at NLO accuracy, without the need of a merging scale to combine the various jet multiplicities.

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2. Minlo method

Matrix elements for processes with extra jets, such as for H +jet production, diverge for observables that are completely inclusive over the extra jet, such as the H boson rapidity. One way to tame this divergence is to apply the Minlo method, which effectively damps the divergence by a suitable next-to-leading logarithmic (NLL) Sudakov form factor [23]. Using this method, the accuracy of inclusive predictions is only LO, since, after integrating out all extra radiation, effectively a term of $\mathcal{O}(\alpha_S^{3/2})$ remains [21]. By including higher-order, process-dependent terms in the Sudakov form factor, the $\mathcal{O}(\alpha_S^{3/2})$ can be removed rendering observables inclusive over radiation NLO correct (*i.e.* up to the order of $\mathcal{O}(\alpha_S^2)$). Note that even though higher-order terms are included in the Sudakov form factor, they do not improve the accuracy of the resummation, which remains, formally, only at leading logarithmic level.

In the original approach, the precise form of the Sudakov form factor to achieve NLO accuracy for inclusive observables was derived analytically. This allowed for predictions for $W/Z/H/HW$ production [21, 24], and, very recently, also WW production [25]. In all these approaches, it was possible to only cover up to one extra jet, *i.e.* starting from the NLO $B+1$ jet predictions also the B inclusive observables become NLO accurate, where B is a heavy color-singlet final state. In the following, we will show some results on an alternative method (“extended Minlo”) [26], which uses unitarity to enforce NLO $B+n$ jet predictions to be also NLO accurate for $B+(n-1)$ jet observables. Indeed, instead of explicitly computing the missing terms in the Sudakov form factor, they can be fitted by enforcing that all $B+(n-1)$ jet observables computed from the Minlo $B+n$ jet calculations are strictly identical to a dedicated NLO computation using $B+(n-1)$ jet matrix elements.

In particular, results will be shown for Higgs production in gluon fusion, in which we start from an NLO calculation for $H+2$ jets and apply the extended Minlo method to make them also accurate in $H+1$ jet predictions. The latter are already part of an NNLO+PS calculation for Higgs production [27]. In short, this allows one to have NLO accuracy for $H+2$ jets and $H+1$ jet and NNLO accuracy for inclusive H observables without the introduction of a merging scale. The results shown here are just a proof-of-concept and not yet complete enough for data comparisons: at high transverse momenta (or other large scales), the effective theory used, *i.e.* integrating out the top quark in the Higgs to gluons coupling, breaks down and improvements are needed [28, 29].

3. Higgs boson production with up to two jets at NLO accuracy

The plots presented in this section contain of a main panel (on the left) and three ratio plots (on the right). Each of the plots contain three curves, corresponding to the new extended Minlo results in red, in green the NNLOPS calculation for inclusive H production and in blue the predictions for $H+2$ jets are shown. The latter contains $H+2$ jets matrix elements together with the Sudakov resummation, but is formally not NLO accurate for lower multiplicity observables. In the main panel, the new extended Minlo results include uncertainties from scale variation, while for the other two predictions, only the central value is shown. The three ratio insets on the right contain the ratio w.r.t. the three central values, respectively. The coloured band is the uncertainty coming from scale variations in the respective results.

As a first observable, we consider the rapidity of the Higgs boson in Fig. 1. This is one of the observables that is NNLO accurate for the NNLOPS calculation for inclusive Higgs production (green curve). As can be seen from the main panel as well as the ratio plots, the extended Minlo predictions are in agreement with it as expected — both for the central value as well as for the scale variations. On the other hand, the blue curve, *i.e.* the predictions using the NLO matrix elements for $H + 2$ jets, is at most leading order accurate for this observables. In this sense, it is remarkable that its central value agrees rather well with the other two predictions (up to about 10–15%), albeit with a sizable uncertainty band.

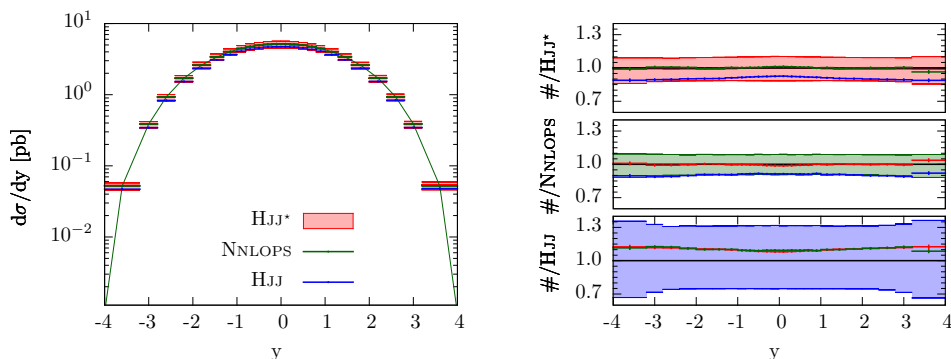


Fig. 1. (Colour on-line) Rapidity distribution of the Higgs boson.

Similar arguments also hold for the transverse momentum of the leading jet, Fig. 2. Also this observable is described at similar accuracy when considering the extended Minlo approach and the NNLOPS predictions. In both calculations, the predictions for this observable are NLO accurate. Therefore, as expected, the red and green curves are in agreement for both the central value as well as the scale uncertainty. The small discrepancy between

40 and 80 GeV can be attributed to the finite statistics used in the numerical unitarity method of the extended Minlo predictions in the merging of the NLO $H+2$ jets with the NNLOPS predictions. At a very small transverse momentum, in particular in the first 2 bins in the plot, there is also a difference visible between the red and green curves. Although the difference is large, it is well below the Sudakov peak. Therefore, subleading logarithms, non-perturbative corrections, *etc.*, can be large and are outside the scope of any of the predictions. The blue $H+2$ jets predictions are only LO accurate for this observable. Even though its curve stays within the uncertainty band of the NNLOPS and extended Minlo results, at large transverse momenta it shows a non-physically small uncertainty band from scale variations, which cannot be trusted.

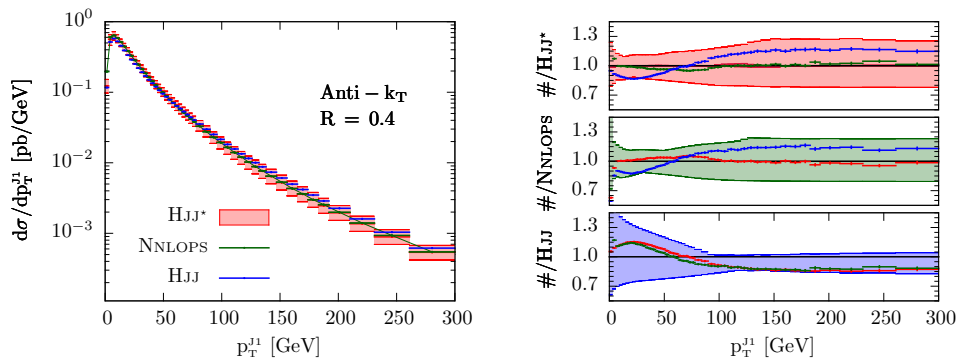


Fig. 2. (Colour on-line) Transverse momentum of the leading jet. Jets are constructed according to the anti- k_t clustering algorithm, for a radius parameter $R = 0.4$.

In contrast with the first two observables shown, for the transverse momentum of the 2nd hardest jet as shown in Fig. 3, the extended Minlo predictions agrees with the $H+2$ jet predictions, and not with the NNLOPS. Again, this is expected since the former two have the same NLO accuracy for this observable while the NNLOPS is only LO accurate here. The exception is at very small transverse momenta, $p_T^{j2} < 20$ GeV, where all three predictions differ. The reason is that NNLOPS is only LO accurate, while the two NLO accurate predictions have slightly different Sudakov factors, with terms that are different beyond the accuracy of either one of the predictions. These subleading contributions can have a sizable impact on the predictions, up to about 20% for very small transverse momenta.

The most interesting result from this exercise is the plot of Fig. 4. In this plot, the transverse momentum of the Higgs boson is plotted, requiring two additional jets in the event, vetoing events with more or fewer jets. As expected, when the transverse momentum of the Higgs boson is small

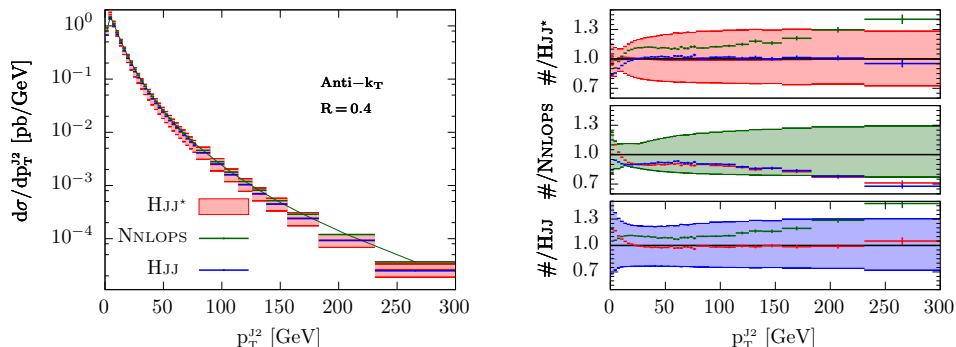


Fig. 3. (Colour on-line) Transverse momentum of the second leading jet. Jets are constructed according to the anti- k_t clustering algorithm, for a radius parameter $R = 0.4$.

(compared to the typical transverse momenta of the jets), the extended Minlo results agree with the calculation for $H+2$ jets. However, when the transverse momentum of the Higgs gets larger, the events are dominated by the Higgs recoiling against a hard jet, that either splits into two, or radiates a softer secondary jet. The latter approach is better described by the NLO predictions for $H+1$ jet: indeed, the extended Minlo results agree with the NNLOPS results for inclusive H production.

A much larger set of results and comparisons can be found in Ref. [26].

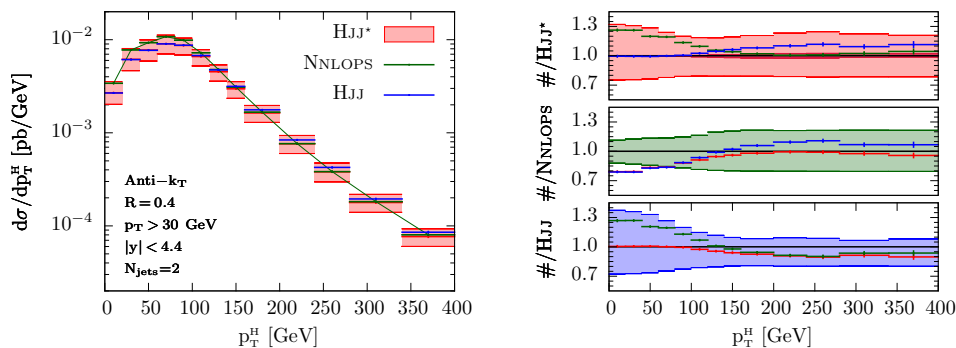


Fig. 4. Transverse momentum distribution of the Higgs boson in 2-jet events. Jets are constructed according to the anti- k_t clustering algorithm, for a radius parameter $R = 0.4$. Jets are required to have transverse momentum $p_T \geq 30$ GeV and rapidity $|y| \leq 4.4$.

4. Conclusions

During the last decade(s), fully exclusive predictions for event rates and differential distributions have both improved enormously in precision as well as accuracy.

In this paper, I discussed what is currently the most accurate predictions for Higgs boson production in gluon fusion, merging NLO $H + 2$ jets with NNLO H results, without the introduction of a merging scale. Using unitarity in a numerical way, no new analytic calculations are needed to extend this approach to other processes.

It took about ten years in going from the first NLO results matched to a parton shower [30–32] to fully fledged automation [33–36]. With the first results for next-to-next-to-leading order predictions matched to parton showers coming available today [24, 27, 37–40], it will be interesting to see how far we can push the field in the coming decade.

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