COMPARATIVE ANALYSIS OF THE LIGHT NUCLEI DIFFRACTIVE SCATTERING ON ^{12}C*

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The diffractive structure of experimental differential cross sections of elastic scattering of 23 light nuclei with masses $3 \leq A \leq 20$ on ^{12}C was studied for collision energies from 1 up to 200 MeV/nucleon and transferred momenta up to $q < 3$–4 fm^{-1}. A common assumption of a diffractive nature of the interaction in these conditions is confirmed by a quantitative analysis of specific features of the differential cross section such as positions of maxima and minima. Their energy evolution as well as that of the corresponding diffraction radius are parameterized with smooth functions. The presented approximation provides an adequate description of experimental data and has a certain predictive power. The observed correlations between the diffraction radius parameters and the structure features of nuclei can be used as a source of information about internal nuclear structure and also as a base for a consistency check between different experimental studies.

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1. Introduction

The small-angle elastic scattering of light nuclei is commonly believed to be of a diffractive nature. However, the behavior of cross sections in such cases was usually analyzed qualitatively (see, for instance, Ref. [1]). In addition, the range of applicability of such an approach is not well-established yet. Thus, a correct quantitative determination of the kinematical region where the diffractive mechanism of interaction dominates would be useful. Our recent analysis [2] has shown that for many pairs of interacting nuclei,

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the elastic scattering differential cross section demonstrates a systematic diffraction behavior in a very broad range of incident energies from 1–3 up to 200 MeV/nucleon, and the momentum transfer range up to 3–4 fm^{-1}. The energy evolution of the diffractive picture in this range can be parameterized with smooth functions within an approach based on the diffraction model. Although the same form of the approximation function can be used for different interacting nuclei, the obtained values of parameters demonstrate a noticeable dispersion [2]. For a reliable determination of the diffraction parameters for an arbitrary pair of nuclei, further studies are needed in order to clarify the influence of structural features on the diffractive picture and its energy evolution. To this aim, we consider here the elastic scattering of ^{12}C on various light nuclei. Since one of the two interacting nuclei is always the same, any change in the diffraction behavior is naturally expected to be caused by the features of the other partner. In particular, interesting effects might be revealed in scattering of exotic nuclei.

The data included in the analysis, listed in Table I, are taken from the EXFOR data base [3] and a review on the scattering of exotic nuclei [4].

TABLE I

Experimental data included in the analysis. Total number of incident energy values E_{i}, for which elastic scattering differential cross section has been measured, is given for each nucleus, as well as the range of energies. Entries in bold are for radioactive nuclei.

j	Nucl.	$N(E_{\mathrm{i}})$	$E_{\mathrm{i}}^{\mathrm{low}}$–$E_{\mathrm{i}}^{\mathrm{up}}$	j	Nucl.	$N(E_{\mathrm{i}})$	$E_{\mathrm{i}}^{\mathrm{low}}$–$E_{\mathrm{i}}^{\mathrm{up}}$	j	Nucl.	$N(E_{\mathrm{i}})$	$E_{\mathrm{i}}^{\mathrm{low}}$–$E_{\mathrm{i}}^{\mathrm{up}}$
1	^{3}He	20	2–150	9	**^{9}Li**	1	90	17	**^{12}Be**	1	57
2	^{4}He	22	1.3–340	10	^{9}Be	18	0.8–13	18	^{12}C	45	1.1–200
3	**^{6}He**	10	1–82	11	**^{10}Be**	3	2.3–60	19	^{13}C	7	1.3–50
4	^{6}Li	28	0.8–53	12	^{10}B	2	2–10	20	^{14}N	17	1.4–20
5	^{7}Li	19	0.6–50	13	**^{11}Li**	2	50–58	21	^{16}O	78	1.2–94
6	**^{7}Be**	7	1–40	14	**^{11}Be**	2	38–49	22	^{18}O	9	1.8–8
7	**^{8}He**	1	60	15	^{11}B	38	1–29	23	^{20}Ne	6	3–20
8	**^{8}B**	3	3–40	16	**^{11}C**	1	56				

2. Energy evolution of the diffractive picture and of the diffraction radius

In order to quantitatively prove the qualitative understanding of systematic behavior of the diffraction picture, we firstly analyzed each experimental angular distribution of the differential cross section within the well-known optical model (OM) approach. A satisfactory description of the experimental data was achieved in the considered region where the potential scattering is supposed to be the dominant interaction mechanism. An adequate physically motivated representation of data by smooth OM-curves gives us well-determined values of transferred momentum $q = 2k \sin\theta/2$ which correspond to the maxima (minima) in the differential cross-section distributions. Examples of q-positions of cross-section maxima, obtained in such a way, are shown in Fig. 1 as experimental points. The uncertainties of the obtained q-positions are related to the uncertainties of the beam energy and of the scattering angle. The q-positions of minima demonstrate the same behavior.

In the diffraction model, the q-positions of maxima (minima) in the differential cross section are equidistant

$$q_i^{\mathrm{max(min)}} = \frac{\pi\left(i + \alpha^{\mathrm{max(min)}}\right)}{R_\mathrm{d}}\,. \tag{1}$$

Here, $i = 0,1,2,\ldots$ — the sequential number of a maximum (minimum), $\alpha^{\mathrm{max(min)}}$ — the parameter of the first ($i = 0$) position and R_d — the diffraction radius. The diffraction radius is the main parameter that characterizes a system of interacting nuclei in the diffraction model. It is usually determined for the elastic scattering, however it has been also attempted to determine the diffraction radius of excited states of nuclei using inelastic scattering data [5].

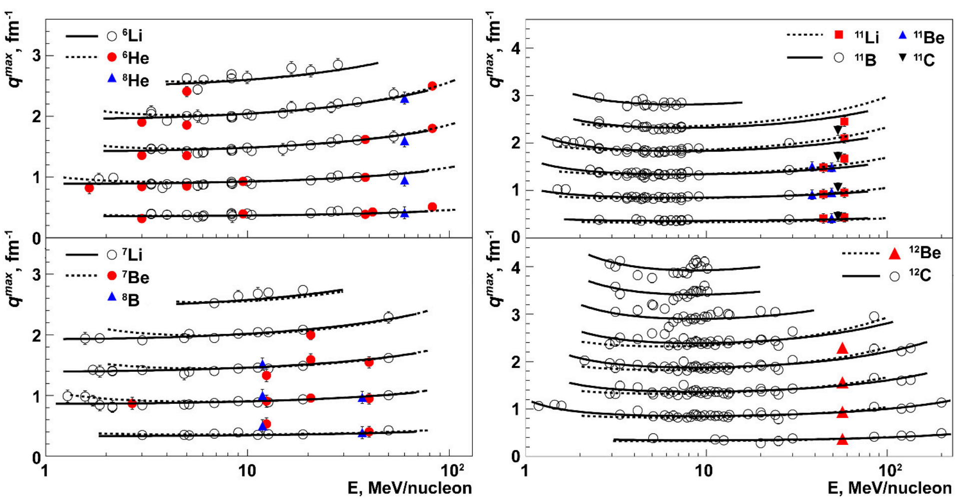

Fig. 1. q-positions of maxima of the differential cross section of the ^{12}C elastic scattering on several nuclei. The points are obtained from experimental distributions at each energy E_i separately. The curves represent the approach inspired by the diffraction model (formulae (1)–(2)). The solid curves correspond to the fits performed for a specific pair of interacting nuclei, and the dotted ones to the global fit to all the data.

As one can see in Fig. 1, the q-positions of diffractive maxima look equidistant at every E_{i}. At low and high energies, they shift towards larger values of q. This can be described by introducing an energy-dependent diffraction radius $R_{\mathrm{d}}(E)$ in formula (1). Based on our previous work [2], we propose the following approximation:

$$R_{\mathrm{d}}(E) = r_{\mathrm{d}} \left(1 - \beta E^{1/2}\right) \sqrt{1 - \frac{\gamma V_{\mathrm{b}}}{\mu E}} \left(A_{\mathrm{P}}^{1/3} + A_{\mathrm{T}}^{1/3}\right) , \tag{2}$$

where μ — the reduced mass of the colliding nuclei, V_{b} — the height of the Coulomb barrier between them. The parameters r_{d}, β, γ as well as $\alpha^{\max(\min)}$ have to be adjusted to the data (q-position points). We carried out a fit for each case where sufficient experimental data exist. Some examples are presented in Fig. 1 as solid curves. The proposed parametrizations describe the behavior of the data reasonably well, which proves the applicability of the diffraction-based approach in the entire kinematical region under consideration. For several nuclei (^{10}B, and all radioactive isotopes except for ^{6}He), the experimental information is scarce and for those we did not perform individual fits with formulae (1)–(2).

A global approximation was also proposed, with parameters determined in a fitting procedure which included all the q-position points at all energies E_{i} for all nuclei listed in Table I. As shown by dotted lines in Fig. 1, this approximation adequately reflects the data behavior.

Changes of the diffraction radius R_{d} in Eq. (1) with energy mean that a scattered wave interacts with an area which effectively changes its size. To disentangle common and individual characteristics of such changes, we have analyzed the energy dependence of the reduced diffraction radius $R_{\mathrm{d}}/(A_{\mathrm{P}}^{1/3} + A_{\mathrm{T}}^{1/3})$ as shown in Fig. 2. Despite some dispersion of points, one can easily see quite a similar behavior demonstrated by all studied nuclei in diffractive scattering on ^{12}C. A decrease of the diffraction radius at small energies is most probably caused by the rising influence of the Coulomb interaction. Therefore, it becomes more prominent for the nuclei with higher charges Z. The decrease at large energies has, however, no obvious explanation.

Approximation (2) with individual sets of parameters provides a reasonably good description of data. Description provided by the global parameters is slightly worse, but it also correctly reproduces the main features of the diffraction radius behavior. To improve the description by the global approximation, one would need to study in detail the behavior of the parameters in a broad range on nuclear masses.

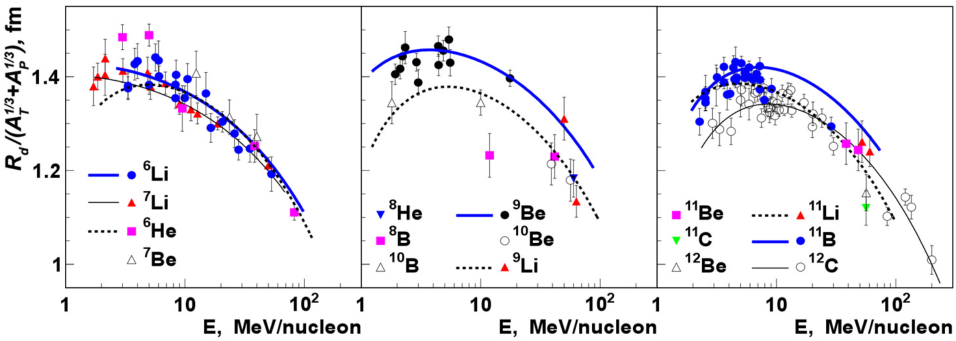

Fig. 2. Energy dependence of the reduced diffraction radius $R_\mathrm{d}/(A_P^{1/3}+A_T^{1/3})$. The points are obtained from q-positions of maxima of experimental cross sections by using formula (1). In each panel, $\alpha^{\max}$ is the same for all nuclei. The curves represent approximation (2): solid ones — with individual parameters, thick dotted ones — with global parameters.

3. Influence of nuclear structure

The parameter r_d in Eq. (2) is worth a particular consideration. It can be related to the effective nuclear matter distribution in a nucleus tested by the ^{12}C probe: the bigger r_d is, the more space in the interaction area corresponds to one nucleon. In Fig. 3, we compare the r_d values obtained for each nucleus with the remaining parameters fixed at the global values: $\alpha^{\max}=0.649(5)$, $\beta=0.0286(3)$ $(\mathrm{MeV/nucleon})^{-1/2}$, $\gamma=0.425(8)$.

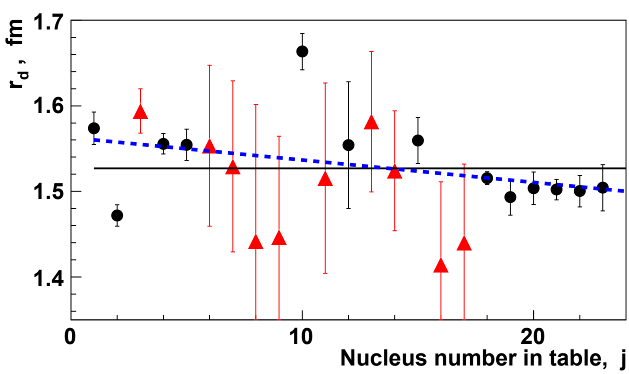

Fig. 3. (Color online) Diffraction radius parameter r_d (see formula (2)) for different nuclei scattered on ^{12}C. The x-axis coordinate of each point corresponds to the sequential number of the nucleus in Table I. Radioactive nuclei are marked by gray/red triangles. The horizontal line represents the global value $r_\mathrm{d}=1.527(4)$ fm. The dashed/blue line is to guide the eye.

Most of the individual r_d values appear to be very close to the global value $r_\mathrm{d} = 1.527(4)$ fm. A tendency of r_d to decrease with the nuclear mass, as indicated by the dashed/blue line in Fig. 3, can also be noticed. Two nuclei noticeably deviate from the common pattern: the r_d of ^{4}He is significantly smaller, while that of ^{9}Be is significantly larger than the average. The reasons for such deviations must be related to specific features of these nuclei. It is worth mentioning *e.g.* that the alpha particle is known to be a very tightly bound nucleus, while ^{9}Be differs from an unbound ^{8}Be by only one weekly-bound neutron and may be substantially clusterized. From this point of view, the rather small values of r_d obtained for weekly-bound radioactive nuclei ^{8}B, ^{9}Li, ^{11}C and ^{12}Be do not look logical, even taking into account their large uncertainties. This may indicate some systematic effect which was unaccounted for in the experimental studies.

In general, radioactive nuclei which are mostly weekly bound do not present larger reduced diffraction radii, except perhaps for ^{6}He. Thus, the nucleon separation energy $S_{n(p)}$ cannot be the only important feature. However, for all radioactive nuclei except ^{6}He, the experimental uncertainties should be significantly decreased to make any firm conclusion possible.

4. Conclusions

A systematic analysis of large amount of experimental data confirms the diffractive nature of the ^{12}C elastic scattering on nuclei of masses $3 \leq A \leq 20$ in the range of momentum transfer $q \leq$ 3–4 fm^{-1} for incident energies from 1–3 up to 200 MeV per nucleon. The observed equidistant positions of maxima and minima of the differential cross section in the scale of transferred momentum are consistent with the diffraction model with an energy-dependent diffraction radius. The proposed approximation of the diffraction radius energy dependence provides a satisfactory description of the data. In the considered range of nuclear masses, the data suggest a rather stable pattern of the diffraction radius energy dependence. The dispersion of the obtained parameters is weak and an approximation with a global set of parameters adequately describes all data. Significant deviations are observed only for ^{4}He and ^{9}Be, which must be related to their internal structure.

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