THE MUON $g-2$ IN PROGRESS*

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Dedicated to the memory of Maria Krawczyk

Two next generation muon $g-2$ experiments at Fermilab in the USA and at J-PARC in Japan have been designed to reach a four times better precision from 0.54 ppm to 0.14 ppm and the challenge for the theory side is to keep up in precision as far as possible. This has triggered a lot of new research activities. The main motivation is the persisting 3 to 4σ deviation between standard theory and experiment. As the Standard Model predictions almost without exception match perfectly all other experimental information, the deviation in one of the most precisely measured quantities in particle physics remains a mystery and inspires the imagination of model builders. Plenty of speculations are aiming to explain what beyond the Standard Model effects could fill what seems to be missing. Here, very high precision experiments are competing with searches for new physics at the high-energy frontier lead by the Large Hadron Collider at CERN. Actually, the tension is increasing steadily as no new states are found which could accommodate the $g_\mu-2$ discrepancy. With the new muon $g-2$ experiments, this discrepancy would go up at least to 6σ, in the case the central values do not move, up to 10σ could be reached if the present theory error could be reduced by a factor of two.

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1. Introduction

A particle with spin $\vec{s}$ like the muon exhibits a magnetic moment $\vec{\mu}$

$$\vec{\mu} = g_\mu \frac{e\hbar}{2m_\mu c}\,\vec{s}\,; \qquad g_\mu = 2\,(1+a_\mu)\,.$$

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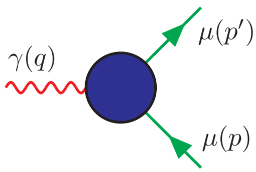
Its Dirac value $g_\mu = 2$ is modified by radiative corrections $a_\mu = (g_\mu - 2)/2 = \frac{\alpha}{2\pi} + \ldots$ known as the muon anomaly. The electromagnetic lepton vertex tested in the static limit here is the simplest object you can think of.



$$= (-ie)\,\bar{u}\left(p'\right)\left[\gamma^\mu F_1\left(q^2\right) + i\,\frac{\sigma^{\mu\nu} q_\nu}{2m_\mu} F_2\left(q^2\right)\right] u(p)\,, \quad (1)$$

where $F_1(0)$ is normalized by charge renormalization and $F_2(0)$ defines the anomaly

$$F_1(0) = 1\,; \qquad F_2(0) = a_\mu\,. \quad (2)$$

The muon anomaly a_μ is responsible for the Larmor spin precession and for its tracking, one needs polarized muons orbiting in a homogeneous magnetic field. To this end, one is shooting protons on a target producing pions which decay by the parity violating weak process $\pi^+ \to \mu^+ \nu_\mu$ into polarized muons of negative helicity which are injected into a storage ring where they decay $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$ producing positrons flying preferably in direction of the spin of the decaying muon. For π^-, helicity and electron flight direction are reversed. Indeed, the two parity violating weak decays perfectly transport the needed spin precession information.

The Larmor precession frequency $\vec{\omega}$ developing in the beam of polarized spinning muons injected into a homogeneous magnetic field $\vec{B}$ is detected by counting the positrons or electrons ejected by the decaying muons preferably along the spin vector. In storage ring type experiments as the CERN, Brookhaven and Fermilab experiments, the muon beam has to be focused by electric quadrupole fields $\vec{E}$, but the beam dynamics can be kept simple by running at the "Magic Energy" where $\vec{\omega}$ is directly proportional to $\vec{B}$. At magic energy at about 3.1 GeV, indeed we have

$$\vec{\omega}_a = \frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta} \times \vec{E}\right]^{E \sim 3.1\,\mathrm{GeV}}_{\text{at ``magic } \gamma\text{''}} \simeq \frac{e}{m}\left[a_\mu \vec{B}\right].$$

First lepton magnetic moment measurements were carried out by Stern and Gerlach in 1922 revealing the famous $g_e = 2$ factor and much later by Kusch and Foley in 1948 who first observed the anomaly $g_e = 2\,(1.00119 \pm 0.00005)$ for the electron.

A crucial point is that at 3.1 GeV, the muons life-time $\gamma\tau_\mu$ in the lab frame is by $\gamma \approx 29$ times longer than in the rest frame. This makes it possible to store and let muons circle in a storage ring.

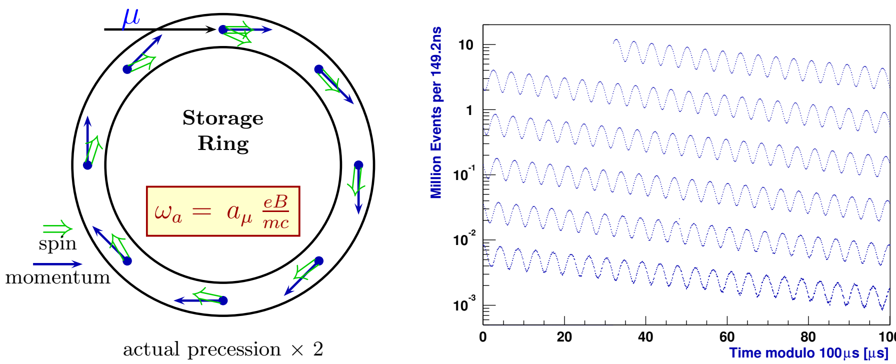


Fig. 1. Left: the Larmor precession of a muon in a storage ring. The spin is rotating $\sim 12'$ per circle. Right: the number of decay positrons with energy greater than $E_{\rm th}$ emitted at time t after muons are injected into the storage ring is $N(t) = N_0(E_{\rm th})\ \exp\left(\frac{-t}{\gamma\tau_\mu}\right)\ [1 + A(E_{\rm th})\ \sin(\omega_a t + \phi(E_{\rm th}))]$, where $N_0(E_{\rm th})$ is a normalization factor, τ_μ the muon life-time (in the muon rest frame), and $A(E_{\rm th})$ is the asymmetry factor for positrons of energy $E > E_{\rm th}$. Courtesy of the E821 Collaboration [1].

A precise experimental determination of a_μ has to be based on measurements of *ratios of frequencies*. From $B = \frac{\hbar\omega_p}{2\mu_p}$ and $\omega_a = \frac{e a_\mu}{m_\mu c} B$, and using $\mu_\mu = (1 + a_\mu)\ \frac{e\hbar}{2m_\mu c}$ or $\mu_\mu = (1 + a_\mu)\ \frac{\hbar}{2}\ \frac{\omega_a}{a_\mu B} = \left(\frac{1}{a_\mu} + 1\right)\ \frac{\omega_a}{\omega_p}\ \mu_p$ and eliminating the muon mass, one obtains $a_\mu = \mathcal{R}/(\lambda - \mathcal{R})$ in terms of 3 frequency measurables: $\tilde{\omega}_p = (e/m_\mu)\langle B\rangle$ the free proton NMR frequency , $\mathcal{R} = \omega_a/\tilde{\omega}_p$ the muon Larmor precession frequency , and $\lambda = \omega_L/\tilde{\omega}_p = \mu_\mu/\mu_p$ from the muonium hyperfine splitting experiment at LAMPF . The actual result from BNL (λ updated) is [1]

$$a_\mu^{\rm exp} = (11\,659\,209.1 \pm 5.4 \pm 3.3[6.3]) \times 10^{-10}\,.$$

To come are two complementary experiments: the magic γ improved muon $g-2$ experiment at Fermilab, tuning $(a_\mu - \frac{1}{\gamma^2-1}) = 0$ [2], and a novel cold muons experiment at J-PARC using a small storage ring at $\vec{E} = 0$ [3]. Both experiments attempt to improve the error by a factor 4. Most importantly, the ultra-relativistic muons (CERN, BNL, Fermilab) and the ultra-cold muons (J-PARC) experiments exhibit very different systematics and the latter will provide an important cross check of the magic gamma ones (see [4] and references therein). More on the experimental aspects and status the reader may find in the contribution by Lusiani [5], in these proceedings.

The muon anomalous magnetic moment is a number represented by an overlay of a large number of individual quantum corrections of different sign, which depend on a few fundamental parameters. In any renormalizable theory like the SM, it is an unambiguous prediction of that theory. It is an ideal monitor for physics beyond the SM. The muon $g-2$ is about a factor 19 or 46 (if theory uncertainties included) more sensitive to new physics (NP) than the electron $g-2$, as we expect $\Delta a_\ell^{\rm NP} = \alpha^{\rm NP}\, m_\ell^2/M_{\rm NP}^2$. The new muon $g-2$ search for NP will take place as usual by confronting the new experiments with the SM theory

$$\Delta a_\mu^{\rm NP} = a_\mu^{\rm exp} - a_\mu^{\rm SM}\,, \tag{3}$$

where

$$a_\mu^{\rm exp} = \frac{\omega_a/\tilde{\omega}_p}{\mu_\mu/\mu_p - \omega_a/\tilde{\omega}_p} \tag{4}$$

and

$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm weak} + a_\mu^{\rm HVP\ LO} + a_\mu^{\rm HLbL} + a_\mu^{\rm HAD\ HO}\,, \tag{5}$$

and the goal is to reach a precision $\delta a_\mu^{\rm exp} \sim 140$ ppb in experiments and $\delta a_\mu^{\rm SM} < 220$ ppb in theory. The coming round of "digging deeper" into the virtual quantum world is based on an improvement of the 5 numbers that have relevant uncertainties. These are ω_a, $\tilde{\omega}_p$ and μ_μ/μ_p, experimentally limited at 120 ppb. The expected experimental improvement will increase $\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm the}$ to 6.7σ with theoretical accuracy as today and to Δa_μ to 11.5σ if the SM prediction is improved by a reduction of the hadronic uncertainty by a factor 2, which concerns $a_\mu^{\rm HVP\ LO}$ and $a_\mu^{\rm HLbL}$. That is what we hope to achieve! A case that promises new physics to be seen with high significance.

In the following, I will focus on the parts of the SM prediction which are limiting its precision, the leading order hadronic photon vacuum polarization (LO-HVP) and the hadronic light-by-light (HLbL) contributions.

2. Evaluation of the leading order $a_\mu^{\rm had}$

The hadronic contribution to the vacuum polarization (see Fig. 2) can be evaluated, with the help of dispersion relations (DR), from the energy scan of the ratio $R(s) \equiv \sigma^{(0)}(e^+e^- \to \gamma^* \to {\rm hadrons})/\frac{4\pi\alpha^2}{3s}$ which can be measured up to some energy $E_{\rm cut}$ above which we can safely use perturbative QCD (pQCD) thanks to asymptotic freedom of QCD. Note that the DR requires the undressed (bare) cross section $\sigma^{(0)}(e^+e^- \to \gamma^* \to {\rm hadrons}) = \sigma(e^+e^- \to \gamma^* \to {\rm hadrons})\,|\alpha(0)/\alpha(s)|^2$. The lowest order HVP contribution is given by

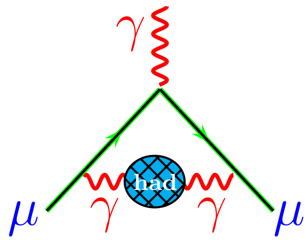


Fig. 2. Hadronic contribution to the vacuum polarization.

$$a_\mu^{\mathrm{had}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left(\int\limits_{m_{\pi^0}^2}^{E^2_{\mathrm{cut}}} \mathrm{d}s\, \frac{R^{\mathrm{data}}(s)\, \hat{K}(s)}{s^2} + \int\limits_{E^2_{\mathrm{cut}}}^{\infty} \mathrm{d}s\, \frac{R^{\mathrm{pQCD}}(s)\, \hat{K}(s)}{s^2} \right), \tag{6}$$

where $\hat{K}(s)$ is a known kernel function growing form $0.39, 0.63\ldots$ at the $m_{\pi^0}^2, 4m_\pi^2$ thresholds to 1 as $s \to \infty$. The integral is dominated by the $\pi^+\pi^- \to \rho$ resonance peak shown in Fig. 3. The $R(s)$-data are displayed in Fig. 4. I apply pQCD from 5.2 GeV to 9.46 GeV and above 11.5 GeV.

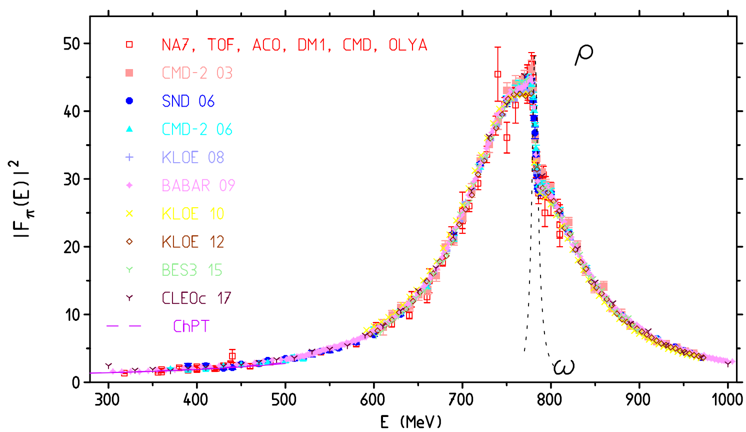


Fig. 3. A compilation of the modulus square of the pion form factor in the ρ meson region, which contributes about 75% to a_μ^{had}. The corresponding $R(s)$ is $R(s) = \frac{1}{4}\,\beta_\pi^3\,|F_\pi^{(0)}(s)|^2\,,\ \beta_\pi = \sqrt{1-4m_\pi^2/s}$ is the pion velocity.

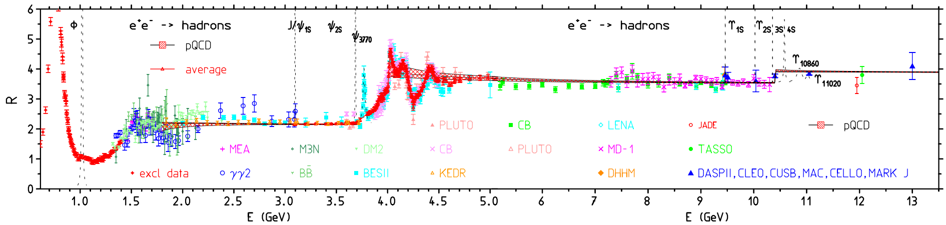


Fig. 4. The compilation of $R(s)$-data utilized.

The experimental errors imply the dominating theoretical uncertainties. As a result, I obtain [6, 7]

$$a_\mu^{\rm had} = (688.07 \pm 4.14)[688.77 \pm 3.38]\, 10^{-10}\,; \quad e^+e^-\text{-data based [incl. } \tau]\,. \quad (7)$$

Figure 5 shows the distribution of contributions and errors between different energy ranges. One of the main issues is $R(s)$ in the region from 1.2 GeV to 2.0 GeV (see Fig. 6), where more than 30 exclusive channels must be measured, and although it contributes only about 14% of the total value of $a_\mu^{\rm had}$, it contributes about 42% of its uncertainty. In the low-energy region, which is particularly important for the dispersive evaluation of the hadronic contribution to the muon $g-2$, data have improved dramatically in the past decade for the dominant $e^+e^- \to \pi^+\pi^-$ channel (CMD-2 [8], SND/Novosibirsk [9], KLOE/Frascati [10–14], BaBar/SLAC [15], BES-III/Beijing [16], CLEOc/Cornell [17]), and the statistical errors are a minor problem now. Similarly, the important region between 1.2 to 2.4 GeV has been improved a lot by the BaBar exclusive channel measurements in the ISR mode [18–21]. Recent data sets collected are: $e^+e^- \to 3(\pi^+\pi^-)$, $e^+e^- \to \bar{p}p$ and $e^+e^- \to K^0_{\rm S}K^0_{\rm L}, K^+K^-$ from CMD-3 [22, 23], and $e^+e^- \to \bar{n}n$, $e^+e^- \to \eta\pi^+\pi^-$, $e^+e^- \to \pi^0\gamma$, $e^+e^- \to \omega\eta\pi^0$, $e^+e^- \to \omega\eta$, $e^+e^- \to K^+K^-$ and $e^+e^- \to \omega\pi^0 \to \pi^0\pi^0\gamma$ from SND [24–26].

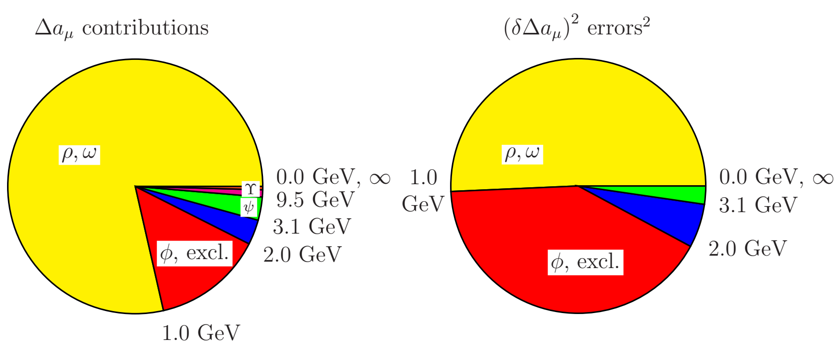


Fig. 5. Distribution of contributions and error squares from different energy regions.

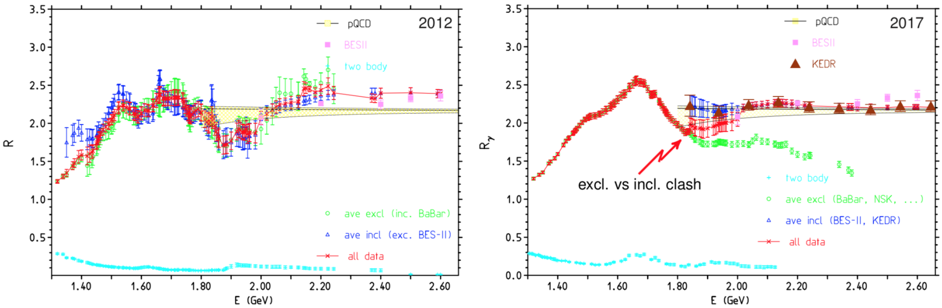


Fig. 6. Illustrating progress by BaBar and NSK exclusive channel data *vs.* new inclusive data by KEDR. Old Frascati (like $\gamma\gamma2$) and Orsay (DM2) data are superseded by much better BaBar data. The excl. data relative to the pQCD band show an over-shooting followed by an under-shooting as expected from quark–hadron duality. The KEDR point at 1.84 GeV seems to violate duality expectations.

Above 2 GeV, fairly accurate BES-II data [27] are available. A new inclusive determination of $R(s)$ in the range of 1.84 to 3.72 GeV has been obtained with the KEDR detector at Novosibirsk [28] (see figures 4 and 6). Recent new experimental input for HVP has been obtained by CMD-3 and SND at VEPP-2000 via energy scan and by BESIII at PEPC in the ISR setup. In Fig. 7, I show a collection of results obtained by various groups since 2009. Figure 7 illustrates the progress as well as the major uncertainties of SM predictions. Remarkable progress has been achieved by lattice QCD groups in calculating a_μ^{HVP}. Primary object for HVP in LQCD is the electromagnetic current correlator in Euclidean configuration space, which yields the vacuum polarization function $\Pi(Q^2)$ needed to calculate $a_\mu^{\mathrm{HVP}} = 4\alpha^2 \int_0^\infty \mathrm{d}Q^2 f(Q^2)[\Pi(Q^2) - \Pi(0)]$. The integrand and the need for lattice size extrapolation is illustrated in Fig. 8. Results are shown in Fig. 9. The major part of LQCD uncertainties comes from the need of extrapolations (finite volume, lattice spacing and physical parameters if not simulated at the physical point). In fact, the momentum region below $Q_{\mathrm{min}} = 2\pi/L$ (L the lattice size) which for presently accessible $Q_{\mathrm{min}} \sim 314$ MeV accounts for about 40% of the a_μ^{had} integral can only be obtained by extrapolation. The very precise RBC/UKQCD point is obtained by combining the directly accessible lattice results only (33.5%) with R-data (66.5%) where the latter are more precise.

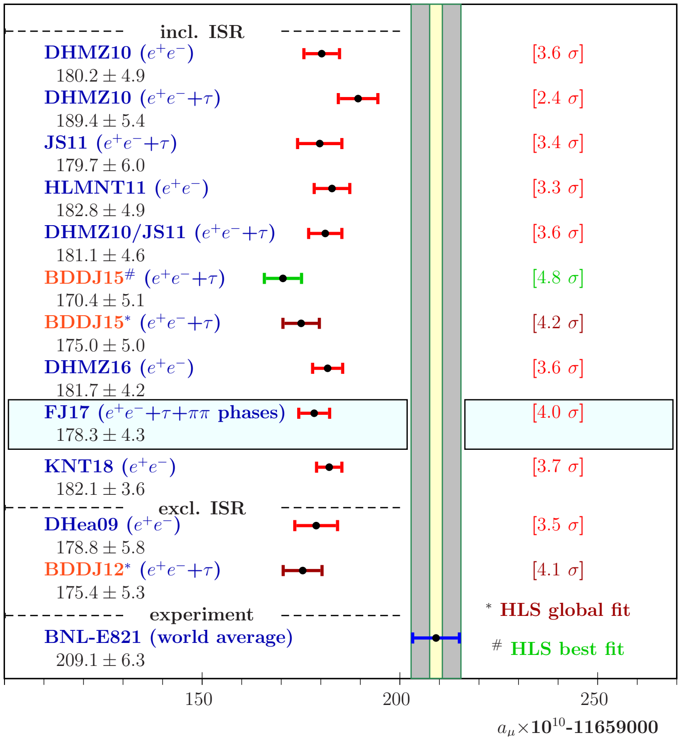


Fig. 7. Comparison with other results: DHMZ10 [29], JS11 [30], HLMNT11 [31], BDDJ15 [32], DHMZ16 [21], FJ17 [6, 7], DHea09 [33], BDDJ12 [34], KNT18 [35]. Two entries do not include IRS data. The narrow vertical band illustrates the future precision expected. Note: results depend on which value is taken for HLbL. JS11 and BDDJ13 includes $116(39)\times 10^{-11}$ [36] [JN] others use $105(26)\times 10^{-11}$ [37] [PdRV]. FJ17 includes τ spectral data [30] and $\pi\pi$ scattering phase-shift data [38].

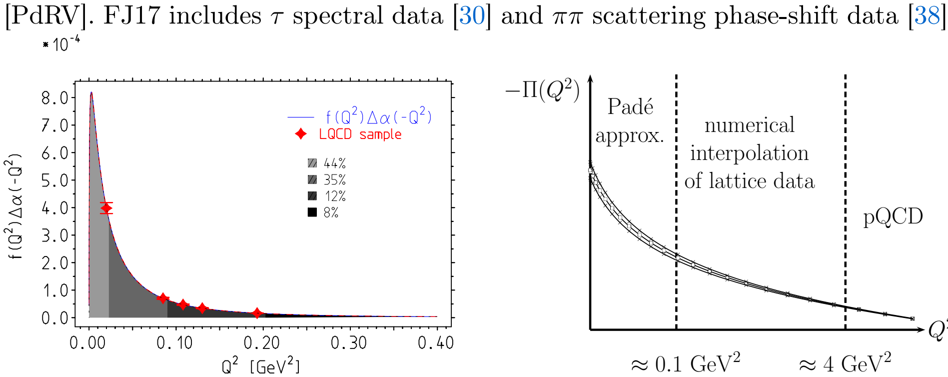


Fig. 8. Left: the $a_\mu^{\rm had}$ integrand as a function of Q^2. Ranges between $Q_i = 0.00$, 0.15, 0.30, 0.45 and 1.0 GeV and their percent contribution to $a_\mu^{\rm had}$. Right: range of direct lattice data and the need for extrapolation.

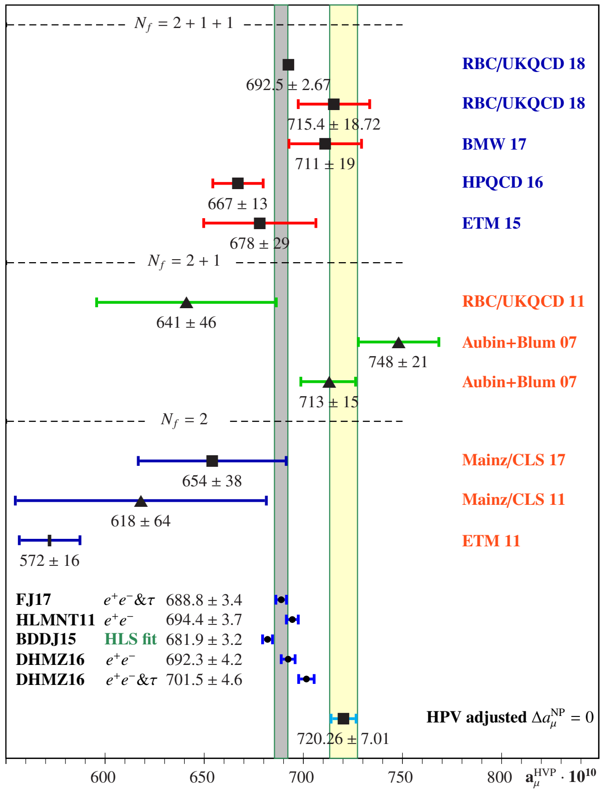
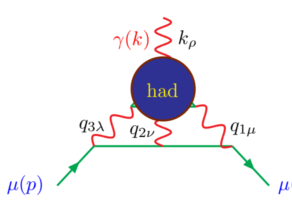


Fig. 9. Summary of recent LQCD results [39–48] for the leading order $a_\mu^{\rm HVP}$ in units of 10^{-10}. Labels: ■ marks u, d, s, c, ▲ u, d, s and ▮ u, d contributions. Individual flavor contributions from light (u, d) amount to about 90%, strange about 8% and charm about 2%. The gray vertical band represents my evaluation. The white band represents the HVP required such that the theory matches the experimental BNL result. Some recent R-data estimates are shown for comparison.

3. Hadronic light-by-light contribution: problems, results

Key object is the hadronic contribution to the full rank-four light-by-light scattering tensor ($A^\mu(x)$ denoting the photon field)



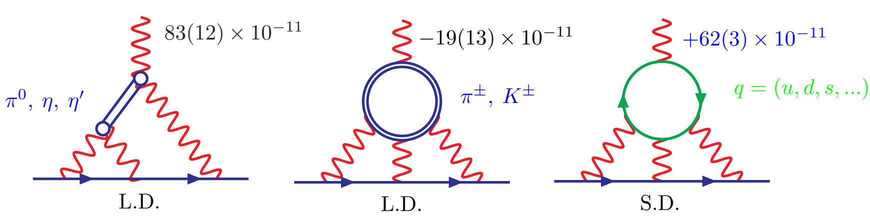
$$\langle 0|T\{A^\mu(x_1)A^\nu(x_2)A^\rho(x_3)A^\sigma(x_4)\}|0\rangle$$

which embodies the four electromagnetic current amplitude

$$\Pi_{\mu\nu\lambda\rho}(q_1,q_2,q_3) \;=\; \int \mathrm{d}^4x_1\,\mathrm{d}^4x_2\,\mathrm{d}^4x_3\,e^{i\,(q_1x_1+q_2x_2+q_3x_3)} \times\langle\,0\,|\,T\{j_\mu(x_1)j_\nu(x_2)j_\lambda(x_3)j_\rho(0)\}\,|\,0\,\rangle\,. \qquad (8)$$

The hadronic part with $j_\mu = j_\mu^{\rm had} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \ldots$ shares the following characteristic properties: (1) it is a non-perturbative object, (2) the covariant decomposition involves 138 Lorentz structures (43 gauge invariant), (3) 28 amplitudes can contribute to $g-2$, by permutation symmetry 19 thereof are independent, (4) fortunately, HLbL is dominated by the pseudoscalar exchanges π^0,η,η' described by the effective Wess–Zumino Lagrangian, (5) generally, pQCD is used to evaluate the short-distance (S.D.) tail, (6) the dominant long-distance (L.D.) part must be evaluated using some low-energy effective model which includes the pseudoscalars as well as the vector mesons ($\rho,\ldots$). The latter mediate the vector meson dominance mechanism which is providing the necessary damping of the high-energy behavior. More recently, is has been shown that a data driven dispersion relation approach is possible and very promising [49] and a number of improvements have been already obtained [50, 51].

One usually applies appropriate low-energy effective hadron theories, such as Hidden Local Symmetry (HLS), Extended Nambu–Jona-Lasinio (ENJL) models, examples of the Resonance Lagrangian Approach (RLA), or large $N_{\rm c}$ QCD inspired Ansätze and other QCD inspired modelings which amount to calculate the following type of diagrams:



The non-perturbative L.D. contributions are dominated by the π^0 exchange and require the knowledge of the off-shell $\pi^0\gamma\gamma$ form factor (see Fig. 10). A basic problem in estimating the HLbL scattering contribution we have because in contrast to the one-scale HVP, HLbL exhibits 3 different energy scales. Figure 11 illustrates the $(0,s_1,s_2)$-plane of the general (s,s_1,s_2)-domain of the π^0 form factor $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(s,s_1,s_2)$.

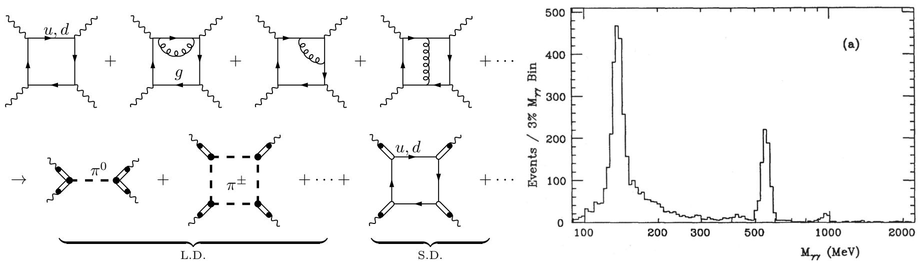


Fig. 10. Left: quark *vs.* hadron effective picture. Right: $\gamma\gamma \to$ hadrons data [Crystal Ball 1988] show almost background free spikes of the pseudoscalar mesons!

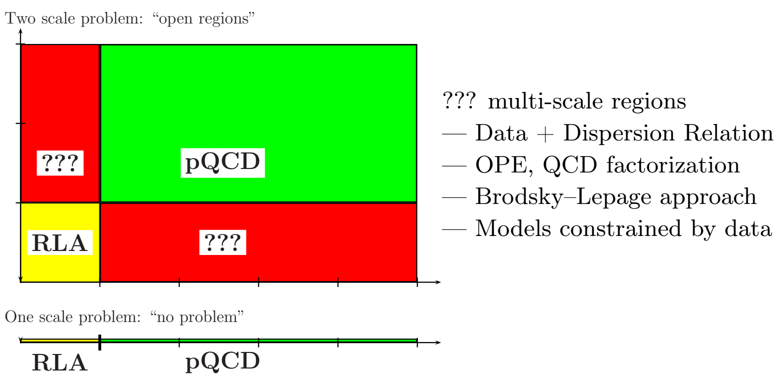


Fig. 11. The three scale HLbL exhibits not only L.D. and S.D. but also mixed regions. Possible methods are listed to the right.

Let focus on the leading π^0 exchange contribution. What do we know? Constraint I: $\pi^0 \to \gamma\gamma$ decay

— The constant $e^2\,\mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2,0,0) = \frac{e^2 N_c}{12\pi^2 f_\pi} = \frac{\alpha}{\pi f_\pi} \approx 0.025\,\mathrm{GeV}^{-1}$ is well-determined by the $\pi^0 \to \gamma\gamma$ decay rate (from Wess–Zumino (WZ) Lagrangian).

— Information on $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0)$ comes from $e^+e^- \to e^+e^-\pi^0$ experiments as shown in Fig. 12.

Constraint II: by the VMD mechanism, the related Brodsky–Lepage behavior $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0) \simeq \frac{1}{4\pi^2 f_\pi}\frac{1}{1+(Q^2/8\pi^2 f_\pi^2)} \sim \frac{2f_\pi}{Q^2}$ provides the necessary damping (cutoff) in order to obtain finite integrals (the constant WZ form factor leads to a divergent result). Variants of models satisfying the constraints I and II yield similar answers. However, ambiguities remain as only single tag data are available (one photon real) so far, as displayed in Fig. 12.

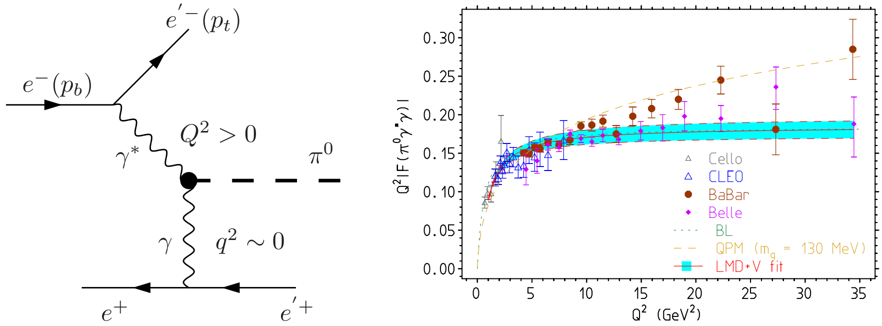


Fig. 12. CELLO, CLEO, BaBar and Belle measurements of the π^0 form factor $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, -Q^2, 0)$ at high space-like Q^2. Towards higher energies BaBar is somewhat conflicting with Belle. The latter conforms with theory expectations, which we use as an OPE constraint. More data are available for η and η' production.

Recently, the leading pseudoscalar meson exchange matrix element

$$M_{\mu\nu} = \varepsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \mathcal{F}_{\pi^{0*}\gamma\gamma}\left(m_\pi^2; q_1^2, q_2^2\right) \tag{9}$$

has been evaluated beyond the single tag case in lattice QCD [52, 53]. For the first time, $\mathcal{F}_{\pi^{0*}\gamma\gamma}(m_\pi^2; -Q^2, -Q^2)$ could be measured on the lattice and clearly discriminates all simple VMD model Ansätze! What remains is the large-$N_{\rm c}$ QCD (OPE constrained) LMD+V Ansatz [54]

$$\begin{aligned}
\mathcal{F}^{\rm LMD+V}_{\pi^{0*}\gamma^*\gamma^*}\left(p_\pi^2, q_1^2, q_2^2\right) &= \frac{F_\pi}{3} \frac{\mathcal{P}\left(q_1^2, q_2^2, p_\pi^2\right)}{\mathcal{Q}\left(q_1^2, q_2^2\right)}\,, \\
\mathcal{P}\left(q_1^2, q_2^2, p_\pi^2\right) &= h_0\, q_1^2\, q_2^2 \left(q_1^2 + q_2^2 + p_\pi^2\right) + h_1 \left(q_1^2 + q_2^2\right)^2 + h_2\, q_1^2\, q_2^2 \\
&\quad + h_3 \left(q_1^2 + q_2^2\right) p_\pi^2 + h_4\, p_\pi^4 + h_5 \left(q_1^2 + q_2^2\right) + h_6\, p_\pi^2 + h_7\,, \\
\mathcal{Q}\left(q_1^2, q_2^2\right) &= \left(M_{V_1}^2 - q_1^2\right)\left(M_{V_2}^2 - q_1^2\right)\left(M_{V_1}^2 - q_2^2\right)\left(M_{V_2}^2 - q_2^2\right)\,,
\end{aligned} \tag{10}$$

which for the pion-pole approximation $p_\pi^2 = m_\pi^2$ is well-constrained now, *i.e.* parameters h_i $(i = 0, \dots, 7)$ are rather well under control by QCD asymptotics and experimental and lattice data. QCD + constraints by data fixes $h_0 = -1$, $h_1 = 0$, h_3, h_4, h_6 are absent in chiral limit such that only h_2, h_5 and h_7 remain as essential parameters if one adopts the VMD mechanism and identifies M_{V_1}, M_{V_2} with ρ, ρ' masses.

One important issue concerns the need of analytic continuation, as illustrated in Fig. 13. In principle, this should be answered within the dispersive approach to HLbL or in lattice QCD (see *e.g.* [53, 55, 56]). So far, most estimates adopt the pion-pole approximation (except [36, 57]) and apply VMD

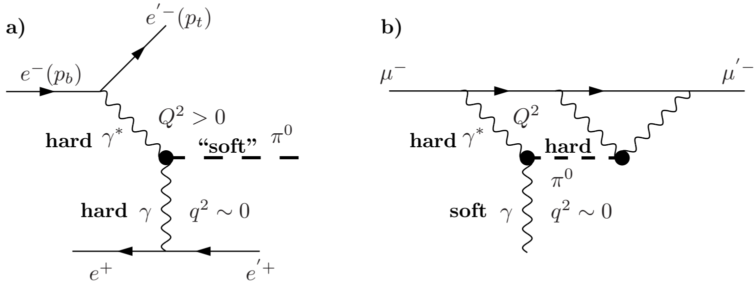


Fig. 13. Measured is $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, -Q^2, 0)$ at high space-like Q^2, needed at external vertex of the $g-2$ diagram is $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(-Q^2, -Q^2, 0)$ or $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(q^2, q^2, 0)$ if integral is evaluated in Minkowski space.

dressing at the external vertex (except [58]). Adopting a LMD+V fit, my estimation for the leading LbL contribution from PS mesons is

$$a_\mu\left[\pi^0, \eta, \eta'\right] \sim (95.45 = [64.68 + 14.87 + 15.90] \pm 12.40) \times 10^{-11}\,.$$

Table I lists a number of results for the π^0-exchange contribution using very different approaches.

TABLE I

Some results for the leading π^0-exchange contribution to the HLbL.

Model	$a_\mu^{\pi^0} \times 10^{10}$	Ref.
EJLN/BPP	5.9(0.9)	[59, 60]
Non-local quark model	6.72	[61]
Dyson–Schwinger eq. approach	5.75	[62]
LMD+V/KN	5.8–6.3	[54]
MV: LMD+V+OPE[WZ]	6.3(1.0)	[58]
Form factor inspired by AdS/QCD	6.54	[63]
Chiral quark model	6.8	[64]
Magnetic susceptibility constraint	7.2	[36, 57]

Besides the pseudoscalar contributions π^0, η, η', one similarly can estimate the axial-mesons a_1, f_1, f_1', the scalars a_0, f_0, f_0', $\pi^\pm, K^\pm$-loops and residual quark-loop contributions. Tensor mesons [50] and a NLO [65] contribution are also to be included. I then estimate

$$\begin{aligned} a_\mu^{\rm HLbL} &= [95.5(12.4) + 7.6(2.7) - 6.0(1.2) - 20(5) + 22.3(4) \\ &\quad +1.1(0.1) + 3(2)] \times 10^{-11} = 103.4(28.8) \times 10^{-11}\,. \qquad (11) \end{aligned}$$

I have scaled up the quadratically combined error on the l.h.s. by a factor 2 on the r.h.s. to account for uncertainties which are difficult to be quantified more precisely. For details, I refer to Sect. 5.2.10 of my book [7].

4. Theory vs. experiment: do we see new physics?

Table II compares SM theory with the BNL experimental result.

TABLE II

Standard Model theory and experiment comparison in units 10^{-10} (see also [66–69]).

Contribution	Value	Error	Ref.
QED incl. 4-loops + 5-loops	11 658 471.886	0.003	[70–72]
Hadronic LO vacuum polarization	689.46	3.25	[6]
Hadronic light-by-light	10.34	2.88	[7]
Hadronic HO vacuum polarization	−8.70	0.06	[6]
Weak to 2-loops	15.36	0.11	[73]
Theory	11 659 178.3	4.3	—
Experiment	11 659 209.1	6.3	[1]
Theor.-exp. 4.0 standard deviations	−30.6	7.6	—

What may the 4σ deviation be: new physics? a statistical fluctuation? underestimating uncertainties (experimental, theoretical)? *Do experiments measure what theoreticians calculate?* Could it be unaccounted real photon radiation effects?

At the present/future level of precision, a_μ depends on all physics incorporated in the SM: electromagnetic, weak, and strong interaction effects and beyond that *all possible new physics* we are hunting for. Figure 14 illustrates past and expected progress in "the closer we look the more there is to see". Here, we are and hope to go on. It contrast with the same status for the electron: Fig. 15 shows that a_e still is and remains mainly a QED test.

Note added: a new more precise value of α from atomic interferometry with Cs^{133} has been obtained at the University of California Berkeley [74]: $\alpha^{-1}(\mathrm{Cs18}) = 137.035999046(27)$ giving an a_e prediction $a_e = 0.00115965218157(23)$ such that $a_e^{\mathrm{exp}} - a_e^{\mathrm{the}} = (-84 \pm 36) \times 10^{-14}$ a 2.3σ deviation. Previously with $\alpha^{-1}(\mathrm{Rb11}) = 137.035999037(92)$, we had $a_e = 0.00115965218165(77)$ and with $a_e^{\mathrm{exp}} - a_e^{\mathrm{the}} = (-92 \pm 82) \times 10^{-14}$ a 1.1σ deviation. Although the central value moved closer to experimental value, the deviation has increased owing to the more precise value of α. Note that the a_e "discrepancy" is of opposite sign to the one of a_μ!

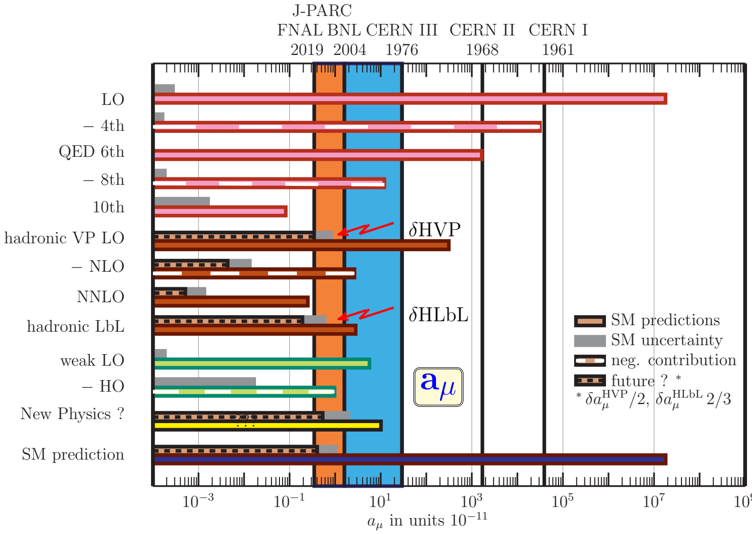


Fig. 14. Past and future $g-2$ experiments testing various contributions. New physics $\stackrel{?}{=}$ deviation $(a_\mu^{\rm exp} - a_\mu^{\rm the})/a_\mu^{\rm exp}$. Limiting theory precision: hadronic vacuum polarization (HVP) and hadronic light-by-light (HLbL) (see also [75]).

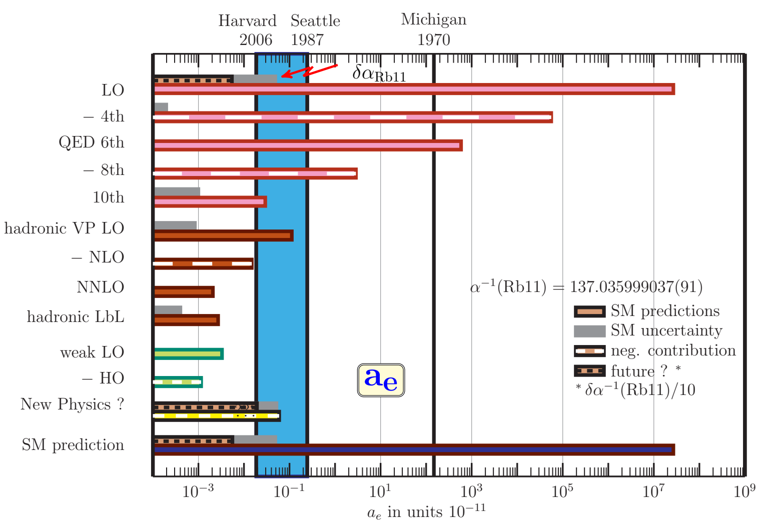


Fig. 15. Status and sensitivity of the a_e experiments testing various contributions. The error is dominated by the uncertainty of α(Rb11) from atomic interferometry. No "new physics" $\stackrel{?}{=}$ deviation $(a_e^{\rm exp} - a_e^{\rm the})/a_\mu^{\rm exp}$. The gray/blue band illustrates the improvement by the Harvard experiment. Note the very different sensitivities to non-QED contributions in comparison with a_μ (for entries see *e.g.* [6, 7, 75]).

5. Prospects

A "new physics" interpretation of the persisting 3 to 4σ gap requires relatively strongly coupled states in the range below about 250 GeV. Search bounds from LEP, Tevatron and specifically from the LHC already have ruled out a variety of Beyond the Standard Model (BSM) scenarios, so much hat standard motivations of SUSY/GUT extensions seem to fall in disgrace. There is no doubt that performing doable improvements on both the theory and the experimental side allows one to substantially sharpen (or diminish) the apparent gap between theory and experiment.

In any case, a_μ constrains BSM scenarios distinctively and at the same time challenges a better understanding of the SM prediction. The two complementary experiments on the way, operating with ultra-hot muons [2] and with ultra-cold muons [3], respectively, especially could differ by possible unaccounted real photon radiation effects. Provided the deviation is real, and theory and needed hadronic cross section data can be improved as expected, the muon $g-2$ experiments could establish $\Delta a_\mu^{\rm NP}$ at about 10 standard deviations.

A remark concerning HVP issues in the standard data based time-like approach is in order here:

(i) How to combine a pretty large number of data-sets to a truly reliable R-function? What is the true uncertainty? What part is reliably taken from pQCD? Including or excluding outdated (= older less precise) data-sets? Bare *versus* physical cross sections, how reliable is VP subtraction?

(ii) Radiative corrections specifically for the ISR method, sQED issues *etc.* The ISR method requires one order in α more precise RC calculation relative to the SCAN method, at least full 2-loop Bhabha and/or $e^+e^- \to \mu^+\mu^-$ as well as ISR–FSR interference in the $\pi^+\pi^-$ channel. What about RC to other more complicated channels (see *e.g.* [76, 77])? What about disentangling 30 channels and recombining them in the 1 to 2 GeV region (quantum interference, missing parts, double counting issues)?

(iii) What precisely do we need in the DR? The 1PI "blob", which is not a measurable quantity. Need undressing from QED effects, photon VP subtraction, FSR modeling, ρ^0–γ mixing? Do we do this at the sufficient precision?

(iv) Non-convergence of Dyson series for OZI suppressed narrow resonances (see *e.g.* [68]).

(v) Missing data compatibility among different experiments. Here, global fit strategies (see *e.g.* [32, 34]) can help to learn more about possible problems.

Of course, I think we are doing the best to our knowledge. However, there is no unambiguous method to combine systematic errors. Uncertainties are definitely squeezed beyond what can be justified beyond doubts, I think.

Therefore, the very different Euclidean approaches, lattice QCD and the proposed alternative direct measurements of the hadronic shift $\Delta\alpha(-Q^2)$ [78], in the long term will be indispensable as complementary cross-checks.

For future improvements of the HLbL part, one desperately needs more information from $\gamma\gamma \to$ hadrons (see *e.g.* [79]) in order to have better constraints on modeling of the many relevant hadronic amplitudes. The dispersive approach to HLbL [49, 51] is able to allow for real progress since contributions which were treated so far as separate contributions will be treated "rolled into one" (as entirety). Note that HLbL depends on 19 independent amplitudes which contribute to $g-2$, while HVP depends on a single one. Last but not least: do theoreticians calculate what experiments measure (form factor *vs.* cross section)?

$$a_\mu^{\rm the} = a_\mu^{\rm SM\ virtual}\,[= F_2(0)] + \underbrace{\Delta a_\mu^{\rm SM\ real\ soft\ \gamma}}_{???}\ \underbrace{[\text{dep. on exp. setup}]}_{\rm Fermilab\ vs.\ J-PARC} + \Delta a_\mu^{\rm NP}\,.$$

A lot remains to be done while a new $a_\mu^{\rm exp}$ is in sight.

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