UPDATED FITS TO THE PRESENT $b \to s\ell^+\ell^-$ DATA*

F. Mahmoudi

Univ. Lyon, Univ. Lyon 1, CNRS/IN2P3, Institut de Physique Nucléaire de Lyon
UMR5822, 69622 Villeurbanne, France
and
Theoretical Physics Department, CERN, 1211 Geneva 23, Switzerland

T. Hurth

Cluster of Excellence PRISMA and Institute for Physics (THEP)
Johannes Gutenberg University, 55099 Mainz, Germany

S. Neshatpour

School of Particles and Accelerators
Institute for Research in Fundamental Sciences (IPM)
P.O. Box 19395-5531, Tehran, Iran

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We discuss the observed deviations in $b \to s\ell^+\ell^-$ processes from the Standard Model predictions and present global fits for both hadronic effects and the new physics description of these anomalies. We investigate whether the different anomalies can be described by a consistent new physics effect. We consider all the possible relevant new physics contributions to the semileptonic $b \to s$ transitions. Moreover, we study the prospects of future LHCb upgrade for establishing new physics with the theoretically clean observables.

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1. Introduction

The full angular analysis of the $B \to K^*\mu^+\mu^-$ observables was presented for the first time by the LHCb Collaboration in 2013 with 1 fb^{-1} of data [1]. While most of the results were consistent with the Standard Model (SM)

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predictions, a few deviations were observed. The largest tension was in the P_5' angular observable with 3.7σ significance in the dilepton invariant mass squared bin $q^2 \in [4.30, 8.68]$ GeV^2. Less significant tensions were observed in some of the other angular observables such as P_2. The P_5' tension was later reconfirmed by LHCb with 3 fb^{-1} of data [2], in the finer $[4.0, 6.0]$ and $[6.0, 8.0]$ GeV^2 bins, with 2.8 and 3.0σ significance, respectively. More recently, the Belle Collaboration [3] as well as the ATLAS [4] and CMS collaborations [5] have measured P_5', although with larger experimental uncertainties compared to LHCb.

As the deviation in P_5' is persisting with more experimental data and with several experimental and analysis methods, at this point, it seems unlikely that the statistical fluctuations could be the source of the tensions. Hence, either underestimated theoretical (hadronic) uncertainties or new physics (NP) effects can be responsible for the observed deviations.

The LHCb measurements with 3 fb^{-1} dataset for other $b \to s\ell^+\ell^-$ transitions indicate further deviations with the SM predictions at 2–4σ significance level in observables such as BR$(B_s \to \phi\mu^+\mu^-)$ [6], but also in the ratios $R_K \equiv \mathrm{BR}(B \to K^+\mu^+\mu^-)/\mathrm{BR}(B \to K^+e^+e^-)$ [7] and $R_{K^*} \equiv \mathrm{BR}(B \to K^*\mu^+\mu^-)\,/\,\mathrm{BR}(B \to K^*e^+e^-)$ [8]. It is important to note that the 2–3σ deviations in the theoretically clean ratios R_K and R_{K^*} cannot be explained by underestimated theoretical (hadronic) uncertainties, but the tensions in all $b \to s\ell^+\ell^-$ can be explained with a common NP effect, namely about 25% reduction in the $C_9^{(\mu)}$ Wilson coefficient [9–11] (see also Refs. [12–17]).

Besides the $R_{K^{(*)}}$ ratios which are very precisely predicted in the SM, the other observables suffer from hadronic uncertainties. The standard method for calculating the hadronic effects in the low-q^2 region for the exclusive $B \to K^*\ell^+\ell^-$ decay is the QCD factorisation framework where an expansion of Λ/m_b is employed. However, higher powers of Λ/m_b remain unknown and so far are only "guesstimated". The significance of the anomalies to a large extent depends on the precise treatment of these non-factorisable power corrections [10, 18, 19]. In the absence of concrete estimates of the power corrections, we make a statistical comparison between an NP fit and a hadronic power corrections fit to the $B \to K^*\mu^+\mu^-$ measurements [20, 21][1]. In addition, we examine whether the various observed tensions indicate a common new physics scenario and we perform NP fits in the most general case where all the relevant Wilson coefficients can receive contributions from new physics. Furthermore, the prospects of the LHCb upgrade for corroborating new physics are studied.

[1] We note that theoretical methods for estimating the hadronic power corrections have been suggested, using dispersion relations [22] and the analicity structure of the corresponding amplitudes [23].

2. Comparison of hadronic fits and new physics fits

The $b \to s\ell^+\ell^-$ transitions can be described via an effective Hamiltonian which can be separated into a hadronic and a semileptonic part [24]

$$\mathcal{H}_{\rm eff} = \mathcal{H}_{\rm eff}^{\rm had} + \mathcal{H}_{\rm eff}^{\rm sl} \,, \tag{1}$$

where

$$\begin{aligned} \mathcal{H}_{\rm eff}^{\rm had} &= -\frac{4G_{\rm F}}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1,\ldots,6,8} C_i \, O_i \,, \\ \mathcal{H}_{\rm eff}^{\rm sl} &= -\frac{4G_{\rm F}}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=7,9,10,S,P,T} \left(C_i \, O_i + C_i' \, O_i'\right) . \end{aligned} \tag{2}$$

For the exclusive decays $B \to K^*\mu^+\mu^-$ and $B_s \to \phi\mu^+\mu^-$, the semileptonic part of the Hamiltonian which accounts for the dominant contribution can be described by seven independent form factors $\tilde{S}, \tilde{V}_\lambda, \tilde{T}_\lambda$, with helicities $\lambda = \pm1, 0$. The exclusive $B \to V\ell^+\ell^-$ decay, where V is a vector meson, can be described in the SM by the following seven helicity amplitudes:

$$\begin{aligned} H_V(\lambda) &= -i\, N' \left\{ C_9^{\rm eff} \tilde{V}_\lambda\left(q^2\right) + \frac{m_B^2}{q^2}\left[\frac{2\,\hat{m}_b}{m_B} C_7^{\rm eff} \tilde{T}_\lambda\left(q^2\right) - 16\pi^2 \mathcal{N}_\lambda\left(q^2\right)\right]\right\} , \\ H_A(\lambda) &= -i\, N' C_{10} \tilde{V}_\lambda \,, \\ H_P &= i\, N' \left\{ \frac{2\, m_\ell \hat{m}_b}{q^2} C_{10} \left(1 + \frac{m_s}{m_b}\right) \tilde{S} \right\} , \end{aligned} \tag{3}$$

where the effective part of $C_9^{\rm eff}$ $\left(\equiv C_9 + Y(q^2)\right)$ as well as the non-factorisable contribution $\mathcal{N}_\lambda(q^2)$ arise from the hadronic part of the Hamiltonian through the emission of a photon which itself turns into a lepton pair. Due to the vectorial coupling of the photon to the lepton pair, the contributions of $\mathcal{H}_{\rm eff}^{\rm had}$ appear in the vectorial helicity amplitude $H_V(\lambda)$. It is due to the similar effect from the short-distance C_9 (and C_7) of $\mathcal{H}_{\rm eff}^{\rm sl}$ and the long-distance contribution from $\mathcal{H}_{\rm eff}^{\rm had}$ that there is an ambiguity in separating NP effects of the type of $C_9^{\rm NP}$ (and $C_7^{\rm NP}$) from non-factorisable hadronic contributions. The non-factorisable contribution $\mathcal{N}_\lambda(q^2)$ is known at leading order in Λ/m_b from QCDf calculations, while higher powers are only partially known [22] and can only be guesstimated. These power corrections are usually assumed to be 10%, 20%, *etc.* of the leading order non-factorisable contribution. However, instead of making an Ansatz on the size of the power corrections, they can be parametrised by a polynomial with a number of free parameters which can be fitted to the experimental data. One possible description of the power corrections is given in Ref. [25] which is described through 9 complex parameters. With such an Ansatz, the NP contributions can be embedded in the hadronic power corrections and it is possible to make a statistical comparison of a hadronic fit and an NP fit of C_9 (and C_7) to the $B \to K^* \mu^+ \mu^-$ data.

We perform such fits by means of the MINUIT minimisation tool with theoretical predictions from SuperIso v3.6 [26, 27] and considering CP-averaged $B \to K^* \mu^+ \mu^-$ observables at $q^2 < 8$ GeV^2. For the NP scenarios, we have fitted C_9 (and C_7) which assuming complex Wilson coefficient(s) involves 2 (4) free parameters and for the hadronic power corrections we have fitted 18 free parameters.

The various scenarios can be compared through likelihood ratio tests via Wilks' theorem. Considering the difference in number of parameters between two scenarios and taking $\Delta\chi^2$, the p-values are obtained. The p-values imply the significance of adding parameters to go from one nested scenario to a more general case. From Table I, it can be seen that adding the hadronic parameters (16 more parameters) compared to the $C_9^{\rm NP}$ scenario does not really improve the fits as the fit is only improved by 0.76σ significance, and the NP explanation remains as a justified option for interpreting the tensions in the angular observables. This is partly due to the rather large uncertainties of the fitted parameters when using the current data which results in almost all the parameters to be consistent with zero at 1σ level. However, if in the future LHCb upgrade — with 300 fb^{-1} data — the current central values remain, then a similar statistical comparison will indicate strong preference for the hadronic explanation with a significance of 34σ compared to the NP explanation.

TABLE I

p values and significances of adding parameters to go from one scenario to another using Wilks' theorem.

	δC_9	$\delta C_7, \delta C_9$	Hadronic
Plain SM	$3.7 \times 10^{-5} (4.1\sigma)$	$6.3 \times 10^{-5} (4.0\sigma)$	$6.1 \times 10^{-3} (2.7\sigma)$
δC_9	—	$0.13 (1.5\sigma)$	$0.45 (0.76\sigma)$
$\delta C_7, \delta C_9$	—	—	$0.61 (0.52\sigma)$

3. New physics fits for different sets of observables

The tensions of the measurements with the SM predictions can be explained in a model-independent way by modified Wilson coefficients ($C_i = C_i^{\rm SM} + \delta C_i$), where δC_i can be due to some NP effects. We perform global fits by means of the calculation and minimisation of the χ^2 in which all the theoretical and experimental correlations are considered. To check whether the various anomalies point towards a consistent NP explanation, we have made the NP fits dividing the observables into two different sets, one with the very clean ratios R_K and R_{K^*} and another set with the other $b \to s\ell^+\ell^-$ observables, a full list of which can be found in [18].

First we consider the impact of NP in one Wilson coefficient at a time, where all other Wilson coefficients are kept to their SM values. In Table II we give SM pulls of the various one-operator hypotheses.

TABLE II

Best fit values in the one-operator fits considering only the observables $R_{K^*[0.045,1.1]}$, $R_{K^*[1.1,6]}$ and $R_{K[1,6]}$ (upper part), and considering all observables (under the assumption of 10% non-factorisable power corrections) except R_K and R_{K^*} (lower part). The ΔC_i in the fits are normalised to their SM values. When two numbers are mentioned for a given ΔC_i, they correspond to two possible minima.

	b.f. value	$\chi^2_{\min}$	Pull$_{\rm SM}$
ΔC_9	-0.48	18.3	0.3σ
$\Delta C_9'$	$+0.78$	18.1	0.6σ
ΔC_{10}	-1.02	18.2	0.5σ
$\Delta C_{10}'$	$+1.18$	17.9	0.7σ
ΔC_9^μ	-0.35	5.1	3.6σ
ΔC_9^e	$+0.37$	3.5	3.9σ
ΔC_{10}^μ	-1.66 -0.34	2.7	4.0σ
ΔC_{10}^e	-2.36 $+0.35$	2.2	4.0σ
ΔC_9	-0.24	70.5	4.1σ
$\Delta C_9'$	-0.02	87.4	0.3σ
ΔC_{10}	-0.02	87.3	0.4σ
$\Delta C_{10}'$	$+0.03$	87.0	0.7σ
ΔC_9^μ	-0.25	68.2	4.4σ
ΔC_9^e	$+0.18$	86.2	1.2σ
ΔC_{10}^μ	-0.05	86.8	0.8σ
ΔC_{10}^e	-2.14 $+0.14$	86.3	1.1σ

We see that NP in C_9^e, C_9^μ, C_{10}^e, or C_{10}^μ are favoured by the $R_{K^{(*)}}$ ratios with a significance of 3.6–4.0σ. NP contributions in primed operators have no significant effect in a better description of the data. In the fit to all $b \to s\ell\ell$ observables without the ratios, the C_9^μ solutions are favoured with SM pulls of 3.6 and 4.4σ in the two separate fits, respectively, but C_9^e is much less favoured. Also, the C_{10}-like solutions do not play a role in the global fit excluding the ratios.

We present in addition fits based on two-operator hypotheses in Fig. 1.

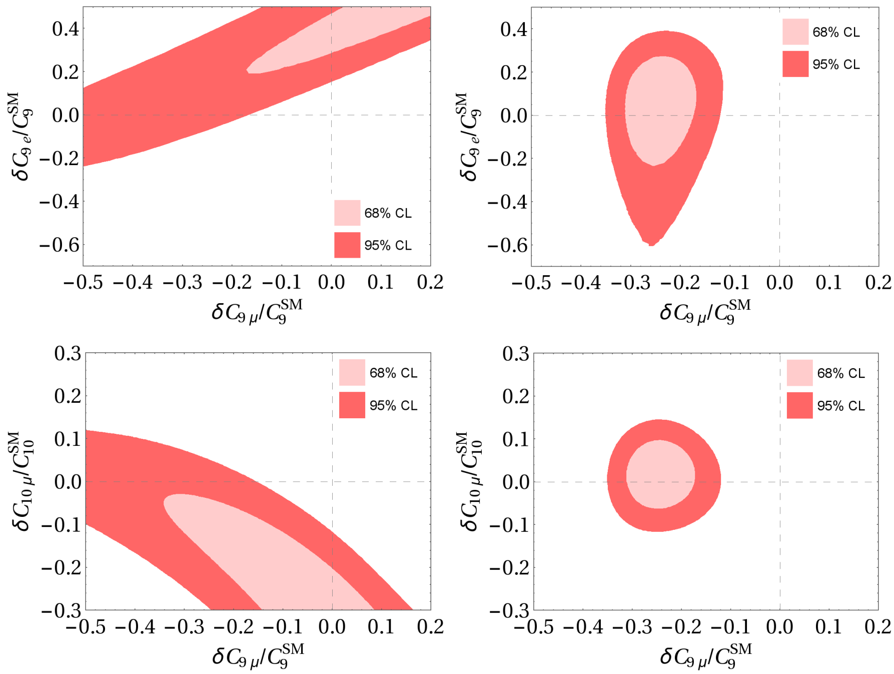

Fig. 1. Global fit results with present data, using only R_K and R_{K^*} on the left, and using all observables except R_K and R_{K^*} (under the assumption of 10% non-factorisable power corrections) on the right.

As can be seen, the two sets of fit namely considering only R_K and R_{K^*}, and considering all observables except R_K and R_{K^*} are compatible at least at the 2σ level.

4. New physics fits considering all possible operators

Finally, we expand our study by considering NP contributions in all the relevant Wilson coefficients, since there is *a priori* no reason that new physics should affect only one or two operators. The Wilson coefficients sensitive to NP are $C_7, C_8, C_9^\ell, C_{10}^\ell, C_S^\ell$ and C_P^ℓ. Therefore, there are 10 independent Wilson coefficients (considering $\ell = e, \mu$) to which we have to add the 10 chirality flipped counterparts. Since, in general, the Wilson coefficients can be complex, we will have in total 40 independent real parameters in the fit. In this case, we use all the mentioned $b \to s\ell^+\ell^-$ observables together and do not separate the very clean ratios from the rest of the observables. The results of the full fits are given in Table III. It is remarkable to notice that

TABLE III

The $\chi^2_{\rm min}$ values when varying different Wilson coefficients. In the last column, the significance of the improvement of the fit compared to the scenario of the previous line is given.

Set of WC	No. parameters	$\chi^2_{\rm min}$	Pull$_{\rm SM}$	Improv.
SM	0	105.56	—	—
$C_9^{(e,\mu)}$ real	2	79.84	4.70σ	4.70σ
$C_7, C_8, C_9^{(e,\mu)}, C_{10}^{(e,\mu)}$ real	6	79.03	3.75σ	0.08σ
All non-primed WC real	10	78.20	3.05σ	0.07σ
All WC real (incl. primed)	20	75.90	1.78σ	0.01σ
All WC complex (incl. primed)	40	67.20	0.61σ	0.01σ

TABLE IV

The best fit values for the scenario where all the Wilson coefficients are varied (corresponding to the last row in Table III with 40 free parameters).

All observables ($\chi^2_{\rm SM} = 105.6$, $\chi^2_{\rm min} = 67.2$)				
	δC_7		δC_8	
$\mathrm{Re}(\delta C_i)$	0.02 ± 0.01		0.03 ± 0.35	
$\mathrm{Im}(\delta C_i)$	0.01 ± 0.17		-1.10 ± 0.68	
	$\delta C_7'$		$\delta C_8'$	
$\mathrm{Re}(\delta C_i)$	0.02 ± 0.03		-0.13 ± 1.18	
$\mathrm{Im}(\delta C_i)$	-0.07 ± 0.02		-0.45 ± 1.50	
	δC_9^μ	δC_9^e	δC_{10}^μ	δC_{10}^e
$\mathrm{Re}(\delta C_i)$	-1.25 ± 0.17	-0.45 ± 0.54	-0.20 ± 0.20	4.39 ± 3.27
$\mathrm{Im}(\delta C_i)$	0.40 ± 4.27	-2.54 ± 0.47	0.02 ± 2.55	-0.29 ± 3.00
	$\delta C_9'^\mu$	$\delta C_9'^e$	$\delta C_{10}'^\mu$	$\delta C_{10}'^e$
$\mathrm{Re}(\delta C_i)$	0.10 ± 0.31	0.00 ± 1.41	-0.10 ± 0.17	0.00 ± 1.41
$\mathrm{Im}(\delta C_i)$	0.43 ± 0.59	0.32 ± 4.63	-0.14 ± 0.24	0.00 ± 5.01
	$\delta C_{Q_1}^\mu$	$\delta C_{Q_1}^e$	$\delta C_{Q_2}^\mu$	$\delta C_{Q_2}^e$
$\mathrm{Re}(\delta C_i)$	-0.07 ± 0.02	-3.57 ± 0.96	0.10 ± 0.14	-0.01 ± 10.58
$\mathrm{Im}(\delta C_i)$	0.00 ± 0.19	-3.53 ± 0.48	-0.01 ± 0.11	-0.02 ± 7.77
	$\delta C_{Q_1}'^\mu$	$\delta C_{Q_1}'^e$	$\delta C_{Q_2}'^\mu$	$\delta C_{Q_2}'^e$
$\mathrm{Re}(\delta C_i)$	0.07 ± 0.02	0.00 ± 1.41	-0.06 ± 0.14	0.00 ± 1.41
$\mathrm{Im}(\delta C_i)$	0.00 ± 0.19	-3.61 ± 0.94	0.02 ± 0.11	-0.07 ± 9.58

no real improvement in the fits are obtained when going beyond the $C_9^{(e,\mu)}$ case. Moreover, as can be seen from the table, the tension with the SM falls below 1σ when all the Wilson coefficients are varied. In Table IV, we show the best fit values for the Wilson coefficients in the most general fit with 40 free parameters. Due to the large number of free parameters, many of the parameters are not well-constrained. The loose constraints for the imaginary part of many of the Wilson coefficients can be strengthened by taking into account further CP violating observables which have been neglected in this study. An interesting result is that a significant contribution also from the electron scalar coefficient is favoured.

5. Prospects of future LHCb upgrade

In the case of R_K and R_{K^*}, reduced experimental errors will be very effective in establishing the new physics explanation for the anomalies since in these observables, the theory uncertainties are very small and the overall errors are dominated by the experimental one. When fully upgraded, the LHCb detector is expected to collect a total integrated luminosity of 300 fb^{-1}. The expected future luminosities of 12, 50 and 300 fb^{-1} will give access to the discussed observables where the statistical error will be decreased by a factor of 2, 4, and 10, respectively (assuming either no correlation or 50% correlation between each of the bin/observable measurements). We have considered the prospect of the NP fit to C_9^μ with the future data on R_K and R_{K^*} assuming the current central values remain and the systematic errors are either unchanged or reduced by a factor of 2 or 3. As shown in Table V where the results for NP in ΔC_9^μ are given, only a small part of the 300 fb^{-1} is needed to establish NP in the $R_{K^{(*)}}$ ratios even in the pessimistic case that the systematic errors are not reduced by then at all.

TABLE V

Pull$_{\rm SM}$ for the fit to ΔC_9^μ based on the ratios R_K and R_{K^*} for the LHCb future scenarios with $12, 50$ and 300 fb^{-1} data, assuming current central values remain. In each scenario, the three R_K and R_{K^*} bins/observables are assumed to have no correlation (50% correlation).

ΔC_9^μ	Syst. Pull$_{\rm SM}$	Syst./2 Pull$_{\rm SM}$	Syst./3 Pull$_{\rm SM}$
12 fb^{-1}	6.1σ (4.3σ)	7.2σ (5.2σ)	7.4σ (5.5σ)
50 fb^{-1}	8.2σ (5.7σ)	11.6σ (8.7σ)	12.9σ (9.9σ)
300 fb^{-1}	9.4σ (6.5σ)	15.6σ (12.3σ)	19.5σ (16.1σ)

We also checked the prospects of NP in C_{10} and the results are very similar to C_9 and therefore — while it will be possible to establish NP with R_K and R_{K^*} — the preferred scenarios would not be possible to clarify. In addition to the clean ratios, we have also included the rather clean observable $\mathrm{BR}(B_s \to \mu^+\mu^-)$ and redone the NP fit to check whether the two scenarios (NP in C_9 or C_{10}) can be differentiated. However, as can be seen in Table VI, including or excluding $\mathrm{BR}(B_s \to \mu^+\mu^-)$ cannot clarify the preferred NP scenario even with the prospected 300 fb^{-1} which motivates the search for other clean ratios (see Ref. [11] for a detailed study).

TABLE VI

Predictions of $\mathrm{Pull}_{\mathrm{SM}}$ for the fit to ΔC_9^μ and ΔC_{10}^μ with the ratios R_K and R_{K^*} [and also $\mathrm{BR}(B_s \to \mu^+\mu^-)$] for the LHCb upgrade scenarios with $12, 50$ and 300 fb^{-1} luminosity collected, assuming current central values remain.

	$\mathrm{Pull}_{\mathrm{SM}}$ with R_K and R_K^* [and $\mathrm{BR}(B_s \to \mu^+\mu^-)$] prospects		
LHCb lum.	12 fb^{-1}	50 fb^{-1}	300 fb^{-1}
C_9^μ	7.4σ [7.4σ]	12.9σ [12.9σ]	19.5σ [19.5σ]
C_{10}^μ	8.1σ [7.6σ]	13.9σ [13.5σ]	20.8σ [20.6σ]

6. Conclusions

In view of the persisting deviations with the SM predictions in the rare $B^0 \to K^{*0}\ell^+\ell^-$ data accumulated by the LHCb experiment during the first run, we address the question of whether these deviations originate from new physics or from unknown large hadronic power corrections by performing global fits to NP in the Wilson coefficients and to unknown power corrections, and doing a statistical comparison. Our analysis shows that adding the hadronic parameters does not improve the fit compared to the NP fit. Hence, our result is a strong indication that the NP interpretation is still a valid option, even if the situation remains inconclusive.

Assuming new physics to be responsible for the observed anomalies, we have performed model-independent NP fits to different sets of Wilson coefficients separating the very clean observables from the rest. We showed that while the two operator NP fits are consistent at 2σ level for the two different sets of observables, for the one operator fit they give a less coherent picture than often stated where the very clean ratios (in addition to the C_9^ℓ explanation) indicate preference for a scenario with modified C_{10}^ℓ which is not observed for the fit to the rest of the $b \to s\ell^+\ell^-$ observables.

Moreover, we also did a global fit using all the $b \to s\ell^+\ell^-$ observables allowing for NP effect from all the relevant operators $(O^{\ell(\prime)}_{7-10,Q_1,Q_2})$ and showed that NP in C_9^μ remains as the most preferred scenario, and significant contribution to the electron scalar coefficient is also favoured.

Finally, we showed that in the future LHCb upgrade if the central values remain, even with the partial 12 fb^{-1} data, new physics can be established. Although, in order to identify the preferred new physics scenario, ratios of further observables which so far have not been measured are needed.

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