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POSITIVE π^--π^0 CORRELATION IN AN UNCORRELATED JET MODEL

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Any uncorrelated jet model is characterized by the three distributions $P_0(n)$, $P_1(n)$ and $P_2(n)$ for the total number of pions in the final mesonic states with isospin 0, 1 and 2. It is shown that for each number of protons in the final state there exists a scaling function for the total number of charged particles.In *pp*-collisions there are three such functions (when pair production is disregarded). A measurement of these three functions would be very important since they completely determine every other distribution and therefore all correlations. It is shown that posititive $\pi^- - \pi^0$ correlations can be obtained from three Poisson-distributions $P_0(n)$, $P_1(n)$ and $P_2(n)$, provided their averages $\bar{N}_0$, $\bar{N}_1$ and $\bar{N}_2$ are not equal. This effect is similar to that of the two component model.

1. Introduction

In a previous paper [1] — to be referred to as I — we introduced exact isospin conservation in an uncorrelated jet model (UJM) for inelastic proton-proton collisions. This was done by constructing pionic states $|Im\rangle$ with definite values of the isospin (I) and its third component (m). These states were then used to write the final state as

$$|pp\rangle = A(pp)\,|00\rangle + B\left[\frac{1}{\sqrt{2}}(pp)\,|10\rangle - \frac{1}{2}(pn+np)\,|11\rangle\right] +$$
$$+C\left[\frac{1}{\sqrt{10}}(pp)\,|20\rangle - \sqrt{\frac{3}{20}}(pn+np)\,|21\rangle + \sqrt{\frac{3}{5}}(nn)\,|22\rangle\right] +$$
$$+\frac{D}{\sqrt{2}}(pn-np)\,|11\rangle, \tag{1}$$

with the normalization condition

$$|A|^2+|B|^2+|C|^2+|D|^2 = 1. \tag{2}$$

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We now extend the definition of the states $|lm\rangle$ and take it as

$$|lm\rangle = N_l^{-\frac{1}{2}} \sum_{n=0}^{\infty} \frac{C_n(l)}{n!} \int d\vec{\tau}\, Y_{lm}(\vec{\tau})\, (\vec{\tau}\cdot\vec{a}^*)^n|0\rangle. \tag{3}$$

The normalization factor is given by

$$N_l = 4\pi \sum_{k=0}^{\infty} \frac{|C_{2k+l}(l)|^2}{2^k k!(2k+2l+1)!!}, \tag{4}$$

or, what amounts to the same, by

$$N_l = 2\pi \int_{-1}^{1} P_l(y) G_l(y) dy \tag{5}$$

with

$$G_l(y) = \sum_{n=0}^{\infty} \frac{|C_n(l)|^2}{n!} y^n. \tag{6}$$

By not considering the dependence of the states $|lm\rangle$ on the particle momenta we do not have the possibility of calculating momentum distributions. It can be shown, however, that the number-distributions and -correlations are not affected by this omission, provided we have independent particle emision. With this proviso and neglecting the production of strange particles and $\bar{p}$p-pairs, Eq. (1) gives the most general final state after a proton--proton collisions. For convenience we will assume that the coefficients A, B, C and D are energy independent. The dependence of the coefficients $C_n(l)$ on the energy need not be specified.

In I we took $C_n(l) = g^n$ for all l. This led to a Poisson-like distribution for the total number of pions with an average $\bar{n} = \lambda = |g|^2$ for high energies.

Another choice is

$$C_n(l) = g^n \sqrt{(n+1)!} \text{ for all } l. \tag{7}$$

This will be called the "bootstrap-case", since it gives rise to the same multiplicity distribution as obtained in the bootstrap model [2].

The important difference with I is that it will not always be assumed that $C_n(l)$ and $C_n(l')$ are equal. There is also no good reason to assume equality of the relative phase of these two numbers, so we take it to be a random function of n. Only in the case where we assume $|C_n(l)| = |C_n(l')|$ we also take the phases equal.

It is now easy to calculate the distribution $P_l(n)$ of the total number of pions in the state $|l\,m\rangle$. We find

$$P_l(n) = \begin{cases} \dfrac{4\pi}{N_l} \dfrac{|C_n(l)|^2}{2^{\frac{n-l}{2}} \cdot \left(\frac{n-l}{2}\right)! \, (n+l+1)!!} & (n-l \text{ even}), \\ 0 & (n-l \text{ odd}). \end{cases} \tag{8}$$

Equation (4) shows that these distributions are properly normalized to one. Since any observable quantity can be expressed in terms of the coefficients $N_l^{-1}\,|C_n(l)|^2$ it follows from (8) that everything can be calculated, once we know the distributions $P_0(n)$, $P_1(n)$ and $P_2(n)$ and of course the weights $|A|^2$, $|B|^2$, $|C|^2$ and $|D|^2$. For reasons explained before we disregard the existence of an interference term between the A- and C-amplitudes. In particular we can calculate the probability $P_{NN'}\,(n, n_0)$ for the occurence in the final state of two nucleons (NN') and n mesons, among which n_0 neutrals. This can be done using the matrix elements which are listed in appendix A of I. For n and n_0 large enough, in order that Stirlings formula may be applied to $n!$ and $n_0!$, we obtain the following results:

$$\begin{aligned}
P_{pp}(n, n_0) &= |A|^2 \frac{P_0(n)}{\sqrt{n n_0}} + \frac{1}{8}\,|C|^2 \cdot \frac{(3n_0-n)^2}{n^2} \cdot \frac{P_2(n)}{\sqrt{n n_0}} \quad (n \text{ even, } n_0 \text{ even}), \\
P_{pp}(n, n_0) &= \frac{3}{2}\,|B|^2 \cdot \frac{1}{n} \sqrt{\frac{n_0}{n}}\, P_1(n) \quad (\text{n odd, } n_0 \text{ odd}), \\
P_{pn}(n, n_0) &= \left(\frac{3}{4}\,|B|^2 + \frac{3}{2}\,|D|^2\right) \cdot \frac{n-n_0}{n} \cdot \frac{P_1(n)}{\sqrt{n n_0}} \quad (n \text{ odd, } n_0 \text{ even}), \\
P_{pn}(n, n_0) &= \frac{9}{4}\,|C|^2 \cdot \frac{n-n_0}{n^2} \cdot \sqrt{\frac{n_0}{n}}\, P_2(n) \quad (n \text{ even, } n_0 \text{ odd}), \\
P_{nn}(n, n_0) &= \frac{9}{8}\,|C|^2 \cdot \frac{(n-n_0)^2}{n^2} \cdot \frac{P_2(n)}{\sqrt{n n_0}} \quad (n \text{ even, } n_0 \text{ even}), \\
P_{nn}(n, n_0) &= 0 \quad (n_0 \text{ odd}).
\end{aligned} \tag{9}$$

Because of the explicitly known and simple n_0-dependence we will be able to derive a number of general properties of correlation coefficients and scaling functions. This will be done in the next two sections.

2. Averages and correlations

Summing the distributions of Eq. (9) over the nucleons and the neutral pions gives the distribution of the total number of mesons $P(n)$ with the obvious result

$$P(n) = |A|^2 P_0(n) + (|B|^2 + |D|^2) P_1(n) + |C|^2 P_2(n). \tag{10}$$

For the separate distributions the averages and dispersions are defined in the usual way

$$\overline{N}_j = \sum_n nP_j(n) \quad \text{and} \quad \Delta_j^2 = \sum_n (n-\overline{N}_j)^2 P_j(n). \tag{11}$$

We will use the term "uncorrelated production" only when all three distributions $P_j(n)$ are narrow in the sense that $\Delta_j/\overline{N}_j \to 0$ for increasing energy. Otherwise we will speak of independent emission. The average total number of pions is

$$\bar{n} = |A|^2\overline{N}_0 + (|B|^2+|D|^2)\overline{N}_1 + |C|^2\overline{N}_2 \tag{12}$$

and it seems reasonable to assume that with increasing energy all three $\overline{N}_j$ depend linearly on $\bar{n}$. In this case it is clear that, even for uncorrelated production, the total dispersion

$$D = [\sum_n (n-\bar{n})^2 P(n)]^{\frac{1}{2}}$$

is large, meaning that

$$D \sim \bar{n}. \tag{13}$$

This is of great importance for the correlations between charged and neutral pions. With the definitions (n_c is the number of charged particles)

$$\begin{aligned} f_2 &= \overline{n(n-1)} - \bar{n}^2 \\ f_{2c} &= \overline{n_c(n_c-1)} - \bar{n}_c^2, \\ f_{20} &= \overline{n_0(n_0-1)} - \bar{n}_0^2, \\ f_{c0} &= \overline{n_c n_0} - \bar{n}_c \bar{n}_0, \end{aligned} \tag{14}$$

we have the identity

$$f_2 = f_{2c} + f_{20} + 2f_{c0}. \tag{15}$$

It has been pointed out [1], [3] that a small value of f_2 (like in the usual UJM) and large f_{20} and f_{2c} (as found experimentally) imply a negative correlation f_{c0} contradicting the experimental results. With the present uncorrelated production, however, this is not necessarily the case as follows from Eq. (13). This is corroborated by an explicit calculation of the correlation functions. Define v_j and μ_j by

$$\overline{N}_j = v_j\bar{n} \quad \text{and} \quad \overline{N_j^2} = \sum_n n^2 P_j(n) = \tfrac{1}{2}\mu_j n^2. \tag{16}$$

Using Eq. (9) we then obtain the following formulae

$$\bar{n}_0 = \bar{n}[\tfrac{1}{3}|A|^2 v_0 + (\tfrac{2}{5}|B|^2 + \tfrac{1}{5}|D|^2)v_1 + \tfrac{4}{15}|C|^2 v_2], \tag{17}$$

$$\bar{n}_c = \bar{n}[\tfrac{2}{3}|A|^2 v_0 + (\tfrac{3}{5}|B|^2 + \tfrac{4}{5}|D|^2)v_1 + \tfrac{11}{15}|C|^2 v_2], \tag{18}$$

$$\overline{n_c^2} = \tfrac{1}{2}\bar{n}^2[\tfrac{8}{15}|A|^2\mu_0 + (\tfrac{16}{35}|B|^2 + \tfrac{24}{35}|D|^2)\mu_1 + \tfrac{64}{105}|C|^2\mu_2], \tag{19}$$

$$\overline{n_c n_0} = \tfrac{1}{2}\bar{n}^2[\tfrac{2}{15}|A|^2\mu_0 + (\tfrac{1}{7}|B|^2 + \tfrac{4}{35}|D|^2)\mu_1 + \tfrac{13}{105}|C|^2\mu_2]. \tag{20}$$

From this it is seen that f_{c0} can be made positive by taking μ_j large enough. This is not necessary, however. Even with uncorrelated production, for which $\mu_j = 2v_j^2$, we have succeeded in choosing A, B, C, D, v_0, v_1 and v_2 such that $f_{c0} > 0$. With $|A|^2 = \frac{3}{5}$, $|B|^2 = \frac{1}{5}$, $|C|^2 = \frac{1}{5}$, $|D|^2 = 0$, $v_0 = 0$, $v_1 = \frac{5}{2}$, $v_2 = \frac{5}{2}$ we obtain

$$\bar{n}_c = 2\bar{n}_0, \tag{21}$$

$$f_{2c} = \tfrac{8}{9}\,\bar{n}^2, \tag{22}$$

$$f_{c0} = \tfrac{4}{63}\,\bar{n}^2. \tag{23}$$

The experimental values at 303 GeV/c are [4]

$$f_{2c} = 0.11\bar{n}^2 \quad \text{and} \quad f_{c0} = 0.04\bar{n}^2. \tag{24}$$

No serious effort is made to fit these data. It should be remarked, however, that they agree with the inequality

$$f_{2c} > 2f_{c0}, \tag{25}$$

which follows from Eqs (19), (20) and (21).

3. *Scaling functions*

In this section we will assume that the functions $P_j(n)$ scale, meaning that for very large energy

$$\bar{n}P_j(n) \to \psi_j(v) \quad \text{with} \quad n = v\bar{n}. \tag{26}$$

For uncorrelated production this certainly holds with

$$\psi_j(v) = 2\delta(v - v_j). \tag{27}$$

From this scaling assumption it can be shown that also the charge particle distributions satisfy KNO-scaling. Summing the distributions $P_{NN'}(n_c + n_0, n_0)$ over $n_0 = x\bar{n}$, keeping $n_c = u\bar{n}$ fixed and multiplying by $\bar{n}$, gives in the limit of high energy a function $\psi_c^{NN'}(u)$, which can be expressed in terms of the input scaling functions $\psi_j(v)$ as follows:

$$\psi_c^{pp}(u) = \frac{1}{2}|A|^2 \int_0^\infty \frac{\psi_0(u+x)}{\sqrt{x(u+x)}}\,dx + \frac{3}{4}|B|^2 \int_0^\infty \frac{\sqrt{x}\,\psi_1(u+x)}{(u+x)^{3/2}}\,dx +$$

$$+ \frac{1}{16}|C|^2 \int_0^\infty \frac{(2x-u)^2\psi_2(u+x)}{\sqrt{x}\,(u+x)^{5/2}}\,dx, \tag{28}$$

$$\psi_c^{pn}(u) = \left(\frac{3}{8}|B|^2 + \frac{3}{4}|D|^2\right) u \int_0^\infty \frac{\psi_1(u+x)}{\sqrt{x}\,(u+x)^{3/2}}\,dx +$$

$$+\frac{9}{8}|C|^2 u \int_0^\infty \frac{\sqrt{x}\,\psi_2(u+x)}{(u+x)^{5/2}}\,dx, \tag{29}$$

$$\psi_c^{nn}(u) = \frac{9}{16}|C|^2 u^2 \int_0^\infty \frac{\psi_2(u+x)}{\sqrt{x}\,(u+x)^{5/2}}\,dx. \tag{30}$$

For uncorrelated production $\psi_j(v)$ is given by Eq. (27) and the functions $\psi_c^{NN'}(u)$ show singularities in the points $u = v_j$, which are generalizations of those discussed in I. With u very close to v_j another scaling law holds, for a discussion of which we again refer to I. For the general case the moments of $\psi_c^{NN'}(u)$ can be expressed in those of $\psi_j(v)$, by making use of Eqs. (28), (29) and (30).

For $\psi_j(v)$ we have

$$\int_0^\infty \psi_j(v)dv = 2; \quad \tfrac{1}{2}\int_0^\infty v\psi_j(v)dv = v_j; \quad \int_0^\infty v^2\psi_j(v)dv = \mu_j, \tag{31}$$

where v_j and u_j are defined by (16). For the first moments of $\psi_c^{NN'}(u)$ we obtain

$$\left.\begin{aligned}\int_0^\infty \psi_c^{pp}(u)du &= 2|A|^2+|B|^2+\tfrac{1}{5}|C|^2,\\ \int_0^\infty \psi_c^{pn}(u)du &= |B|^2+\tfrac{3}{5}|C|^2+2|D|^2,\\ \int_0^\infty \psi_c^{nn}(u)du &= \tfrac{6}{5}|C|^2,\end{aligned}\right\}\ \text{Total} = 2$$

$$\int_0^\infty u\psi_c^{pp}(u)du = \tfrac{4}{3}|A|^2 v_0+\tfrac{2}{5}|B|^2 v_1+\tfrac{2}{21}|C|^2 v_2,$$

$$\int_0^\infty u\psi_c^{pn}(u)du = (\tfrac{4}{5}|B|^2+\tfrac{8}{5}|D|^2)v_1+\tfrac{12}{35}|C|^2 v_2,$$

$$\int_0^\infty u\psi_c^{nn}(u)du = \tfrac{36}{35}|C|^2 v_2. \tag{32}$$

We list these formulae at full length in order to illustrate the importance, not only of measuring the charge prong cross-sections, but also of identifying the protons in the final state. If the left hand side of Eq. (32) were known experimentally this would fix the relative weight of the different components in the state (1) and also the average multiplicities $\overline{N}_j$ for the different values of the isospin. For uncorrelated production these numbers specify everything, including the correlations (of Eq. (19) and (20)). A comparison with the experimentally determined correlations would then open the possibility of refuting the UJM.

The equations (28), (29) and (30) are of the type of Abel's integral equation. They can be inverted to give $\psi_j(v)$ in terms of $\psi_c^{NN'}(u)$. A full measurement of the three $\psi_c^{NN'}(u)$ therefore suffices to fix all other distributions and correlations, in particular the average number of neutral pions for a fixed number of charged particles, *i. e.*, $\bar{n}_0(n_c)$. For the function used by Buras and Koba [5] to parametrize the scaling function for the charged particles, including the protons, this program can be executed completely. They take

$$\psi_c(u) = \alpha u e^{-\beta u^2}, \tag{33}$$

where $\alpha = 4\beta = 25\pi/16$ is determined by (31). Because $\psi_c(u = 0) = 0$ a comparison with (28), (29) and (30) shows that $|A|^2 = |B|^2 = |C|^2 = 0$ and $|D|^2 = 1$ is the only possibility. So

$$\alpha u e^{-\beta u^2} = \frac{3}{4} u \int_0^\infty \frac{\psi_1(u+x)}{\sqrt{x}\,(u+x)^{3/2}}\, dx, \tag{34}$$

with the inversion

$$\psi_1(v) = \frac{32\alpha}{3\pi} v^{3/2} \int_v^\infty \frac{u e^{-\beta u^2}}{\sqrt{u-v}}\, du. \tag{35}$$

From this we can calculate $\bar{n}_0(n_c)$ with the result that for high energies

$$\bar{n}_0(n_c) = \tfrac{3}{8} \bar{n} \int_u^\infty e^{-\beta(x^2-u^2)} dx. \tag{36}$$

This is a decreasing function of $n_c = u\bar{n}$ with a maximum $\bar{n}_0(0) = \frac{3}{10}\bar{n}$. The experimental high energy data show a quite different behaviour [6]. Also the relative abundance and the correlations between charged and neutral pions are completely different from the experimental values

$$\bar{n}_c = 2\bar{n}_0 \quad \text{and} \quad f_{c0} = 0.04\bar{n}^2. \tag{37}$$

Formula (33) of Buras and Koba leads to the following values of these quantities

$$\bar{n}_c = 4\bar{n}_0 \quad \text{and} \quad f_{c0} = -0.024\bar{n}^2. \tag{38}$$

From this we conclude that either the KNO-scaling limit has not yet been reached at 300 GeV/c, or that the total scaling function $\psi_c(u)$ cannot be zero in $u = 0$. This latter condition indeed puts very strong constraints on the coefficients $|A|^2$, $|B|^2$, $|C|^2$ and $|D|^2$, as can be seen from Eqs (28), (29) and (30) and remembering that $\psi_j(v) > 0$.

It is possible to choose these coefficients and the functions $P_0(n)$, $P_1(n)$ and $P_2(n)$ in such a way that an increasing function $\bar{n}_0(n_c)$ is obtained. An example is the bootstrap model of Eq. (7). This, however, has the disadvantage that the total scaling function for the charged pions, although it exists, takes the rather unrealistic form

$$\psi_c(u) = e^{-\frac{1}{2}u} K_0\left(\frac{u}{2}\right), \tag{39}$$

which is singular in $u = 0$.

An attempt to fit all existing data, including branching ratios, will be postponed till later.

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REFERENCES

[1] Th. W. Ruijgrok, D. W. Schlitt, *Acta Phys. Pol.*, **B4**, 953 (1973).
[2] See *e. g.* H. Satz, *Acta Phys. Pol.*, **B5**, 3 (1974).
[3] D. W. Schlitt, Th. W. Ruijgrok, unpuolished; J. Karczmarczuk, J. Wosiek, *Acta Phys. Pol.*, **B5**, 105 (1974).
[4] For a compilation see A. Krzywicki, *Nucl. Phys.*, **B58**, 633 (1973).
[5] A. J. Buras, Z. Koba, *Nuovo Cimento Lett.*, **6**, 629 (1973).
[6] E. L. Berger, D. Horn, G. H. Thomas, *Phys. Rev.*, **D7**, 1412 (1973).