MULTIPOLE RADIATION IN THE BREMSSTRAHLUNG MODEL OF MULTIPLE PRODUCTION

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(*Received November 16, 1973*)

The properties of the inclusive single-particle distribution in the bremsstrahlung model are studied for the general form of the point-like current.

In the bremsstrahlung model of particle production[1] the particles are emitted by a classical current $j(x^\nu)$. The spectra of the emitted particles[2] are determined by the function

$$\varrho(k^\nu) = \int d^4z \exp(ik^\nu z_\nu) j(z^\nu), \tag{1}$$

i.e. the Fourier transform of the current. In particular, the single-particle distribution is given by

$$\frac{d\sigma}{d^3k/E} = |\varrho(k)|^2 \Omega, \tag{2}$$

where Ω describes the phase-space corrections. In the central region of the x-plot, where

$$x = \frac{k_{||}}{\sqrt{s}}, \tag{3}$$

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[1] For an extensive list of references see, *e.g.*, Ref. [3].

[2] In the following we will call the produced particles "pions" and the radiating particle a "nucleon".

Φ approaches constant and the single-particle distribution is simply proportional to $|\varrho(\boldsymbol{k})|^2$.

Thus the form of the nucleon current $j(x^\nu)$ determines the shape of particle spectra. The attractive feature of the model is that, if the nucleon current is Lorentz invariant, the single-particle distribution scales, *i.e.* in high-energy limit

$$\varrho(k^\mu, s) \xrightarrow[s\to\infty]{} \varrho(k_\perp, x), \tag{4}$$

where x is given by Eq. (3) [2].

In the existing models of particle production by the bremsstrahlung mechanism the current was assumed[3] [3–5] in the simplest form, representing the moving point--particle

$$j(z^\nu) = g\int d\tau\delta^{(4)}[z^\nu - s^\nu(\tau)], \tag{5}$$

where g is the coupling constant, $s^\nu(\tau)$ is the trajectory of the particle and τ is the proper time along the trajectory.

However, as was shown by several authors [4, 5], the bremsstrahlung model with the simple current given by Eq. (5) predicts the dip in the x-distribution of the produced particles:

$$\varrho(k_\perp, x) \underset{x\to 0}{\propto} x \xrightarrow[x\to 0]{} 0. \tag{6}$$

Such a dip has not been observed in recent experiments at ISR.

The purpose of the present note is to show that the conclusion (6) can be avoided if a more general form of the nucleon current is used[4].

The most general form of the scalar current of the point-like particle can be written in the form [6]

$$j(z^\nu) = \sum_m j_m(z^\nu), \tag{7}$$

where

$$j_m(z^\nu) = \int_{-\infty}^{\infty} d\tau D_m^{\nu_1\cdots\nu_m}(\tau)\partial_{\nu_1\cdots\nu_m}\delta^{(4)}[z^\nu - s^\nu(\tau)]. \tag{8}$$

Here

$$\partial_\nu = \frac{\partial}{\partial z^\nu} \tag{9}$$

and $D_m^{\nu_1\cdots\nu_m}$ is the completely symmetric tensor describing the m-th multipole moment of the source. The multipole tensors $D_m^{\nu_1\cdots\nu_m}$ satisfy the additional condition

$$D_m^{\nu_1\cdots\nu_m}u_{\nu_1} = D_m^{\nu_1\cdots\nu_m}u_{\nu_2} = \ldots = D_m^{\nu_1\cdots\nu_m}u_{\nu_m} = 0, \tag{10}$$

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[3] We do not discuss the bremsstrahlung models of the type proposed by Feynman [7] where the meson field is coupled to some kind of exchange current.

[4] This problem was considered also in Ref. [1].

where $u_\nu = ds_\nu/d\tau$ is the four-velocity of the nucleon. This last condition follows from the observation that, since we have

$$u^\nu \partial_\nu \delta^{(4)}[z^\nu - s^\nu(\tau)] = -\frac{d}{d\tau}\delta^{(4)}[z^\nu - s^\nu(\tau)], \tag{11}$$

any term of the form

$$\int_{-\infty}^{\infty} A u^\nu \partial_\nu \delta^{(4)}[z^\nu - s^\nu(\tau)] d\tau \tag{12}$$

can be (using integration by parts) written in the form

$$\int_{-\infty}^{\infty} \frac{dA}{d\tau}\delta^{(4)}[z^\nu - s^\nu(\tau)] \tag{13}$$

and thus reduced to the one of lower m.

Using Eqs (1), (7) and (8) we find

$$\varrho(k^\nu) = \sum_m \varrho_m(k^\nu), \tag{14}$$

where

$$\varrho(k^\nu) = i\int_{-\infty}^{\infty} d\tau \exp(ik^\nu s_\nu(\tau)) \frac{d}{d\tau}\left[\frac{F_m(\tau, k^\nu)}{k^\nu u_\nu(\tau)}\right]. \tag{15}$$

Here

$$F_m(\tau, k^\nu) = D_m^{\nu_1 \cdots \nu_m}(\tau) k_{\nu_1} \ldots k_{\nu_m}. \tag{16}$$

The standard way of calculating the integral (15) is by the sudden approximation method [5]. This can be justified if the functions $F_m(\tau, k^\nu)$ are not singular along the path of the particle. The result is

$$\varrho_m(k^\nu) = i\exp(ik^\nu s_\nu(\tau = 0)) \left.\frac{F_m(\tau, k^\nu)}{u^\nu(\tau)k_\nu}\right|_{\tau=-\infty}^{\tau=+\infty}. \tag{17}$$

To investigate the behaviour of $\varrho(k^\nu)$ at high energies, it is therefore enough to calculate the function $F_m(\tau, k^\nu)/u^\nu k_\nu$ before and after the collision. At high energies we have

$$u^\nu k_\nu = \begin{cases} \dfrac{k_\perp^2+\mu^2}{2M}\dfrac{X}{x} + \dfrac{M}{2}\dfrac{x}{X} & \text{for } k_{||}/u_{||} > 0, \\ -\dfrac{k_0\sqrt{s}\,X}{2M} & \text{for } k_{||} \approx 0, \\ -\dfrac{sX}{4M}x & \text{for } k_{||}/u_{||} < 0, \end{cases} \tag{18}$$

where $k_\perp$ is the transverse momentum of the pion with respect to the direction of the nucleon in the c.m. system of the collision. M is the mass of the nucleon and μ that of the pion. x is the scaled longitudinal momentum $k_{||}$ of the pion (again) with respect to the nucleon direction in the c.m. system. X is the ratio of the nucleon momentum to the initial momentum. s is the total c.m.s. energy square.

To evaluate the numerator in formula (17) we denote by

$$D_m^{a_1 \cdots a_m} \tag{19}$$

the values of the multipole tensor in the rest system of the nucleon. The z-axis is chosen along the direction of the momentum of the nucleon in the c.m. system. Since the scalar product

$$D_m^{\nu_1 \cdots \nu_m} k_{\nu_1} \dots k_{\nu_m} \tag{20}$$

is an invariant, it can be calculated in any frame of reference. We choose the rest frame of the nucleon. In this frame, only the three-momentum of the pion is needed in order to evaluate the product (20). Its transverse component is, of course, the same as in the c.m. system of the collision. The longitudinal component is given by

$$\bar{k}_{||} = \begin{cases} \dfrac{k_\perp^2 + \mu^2}{2M} \dfrac{X}{x} - \dfrac{M}{2} \dfrac{x}{X} & \text{for } k_{||}/u_{||} > 0, \\[2ex] -\dfrac{k_0 \sqrt{s}\, X}{2M} & \text{for } k_{||} \approx 0, \\[2ex] -\dfrac{sX}{4M} x & \text{for } k_{||}/u_{||} < 0. \end{cases} \tag{21}$$

Consequently, for $x \to 0$ and $k_{||}/u_{||} < 0$ we have

$$\bar{k}_{||} \simeq k_\nu u^\nu. \tag{22}$$

Thus for particles produced with small x or backwards to the direction or motion of the source (in c.m.s.) the formula (17) gives

$$\varrho(k^\nu) = i \exp(ik^\nu s_\nu(\tau = 0)) \left[d_m(\tau = +\infty) (k_\mu u_f^\mu)^{m-1} - \right.$$
$$\left. - d_m(\tau = -\infty)(k_\mu u_i^\mu)^{m-1} \right], \tag{23}$$

where $d_m = D_m^{\mu_1 = 3, \dots, \mu_m = 3}$. For $m = 0$ we recover the old formula, and indeed we see that it scales (non-scaling terms vanish like $1/s$) and vanishes for $x = 0$.

However, for $m \neq 0$ the situation is different. The case $m = 1$ provides the most interesting situation: $\varrho_1(k^\nu)$ has a finite value at $x = 0$ unless the two terms in (23) cancel exactly.

The radiation of higher multipoles gives the single particle density which is singular for $x = 0$ and consists of non-scaling terms. Since such behaviour is in disagreement with the experiments at ISR, we conclude that either the multipole terms with $m \geqslant 2$ are not present or that there are indeed some very characteristic cancellations. The possible physical meaning of the absence of such singular terms remains, however, an entirely open and interesting question.

Up to now we have discussed only the radiation of isolated poles. The formalism presented can be used for investigation of the radiation from the source with any spatial distribution, by means of the usual multipole expansion. In this way we find the density of particles radiated from the spatially distributed m-poles in the following form

$$\varrho_m(k^\nu) = i \exp(ik^\nu s_\nu(\tau = 0)) \left[\sigma(\bar{k}_f) d_m(\tau = +\infty)(k_\mu u_f^\mu)^{m-1} - \sigma(\bar{k}_i) d_m(\tau = -\infty) \times \times (k_\mu u_i^\mu)^{m-1}\right], \tag{24}$$

where $\bar{k}$ is the momentum of the particle in the rest system of the source and $\sigma(\bar{k})$ is the Fourier transform of the spatial distribution of the source. Provided that the dimensions of the source are finite, $\sigma(\bar{k})$ vanishes in the limit of infinite $|k|$ faster than any inverse power of $|\bar{k}|$. Thus any source with finite spatial dimensions and density which can be represented as a pole of finite order, gives one-particle density which scales and vanishes at $x = 0$. It is also obvious that one can get any one-particle density $\varrho(k^\nu)$ by combining appropriately radiations from an infinite set of continuously distributed poles.

To conclude, we have shown that the single particle distributions in high-energy collisions can be described by a bremsstrahlung model in which the source has a pole-dipole structure. The experimental data seem to exclude the multipole radiation of higher order.

The presented formalism can also be used in more realistic situations when the source has some spatial distribution.

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