DUALITY AND REGGE ANALYSIS OF INCLUSIVE REACTIONS. PART I*

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The application of duality to analysis of inclusive spectra in fragmentation and central regions is discussed.

1. Introduction

These talks will be restricted mostly to single particle inclusive processes. For this is where most of the data and consequently most of the phenomenological analyses has been concentrated so far. However, during the last talk I shall briefly discuss the two--particle inclusive processes, on which some very interesting results have started coming out of the ISR.

The first lecture will cover the following formal aspects:

1. The Kinematics for the single particle inclusive process — Fragmentation and Central Regions.

2. Mueller's Optical Theorem connecting the single particle Inclusive cross-section to 3-body amplitude.

3. Regge Expansion of the 3-body amplitude in the Fragmentation and Central Regions — Single Regge and Double Regge Expansion.

4. Dual Properties of the 3-body amplitude — the 7-Component Picture.

5. The dominant components and the exoticity criteria in the Fragmentation and Central Regions.

The second lecture will cover the duality and Regge phenomenology in the Fragmentation Region. The last one will describe the duality and Regge phenomenology in the central region and for two-particle inclusive processes.

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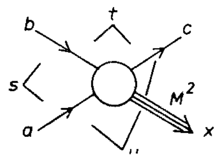
2. *Formalism*

Kinematics

Consider the process $ab \to cX$.



We have here 4 invariants — the energy square of $ab(s)$, the momentum transfer squares between $bc(t)$ and between $ac(u)$, and the missing mass (M^2). Only 3 of these 4 are independent, since

$$s+t+u = m_a^2+m_b^2+m_c^2+M^2. \tag{1}$$

Asymptotically in s, the phase space of the final particle c can be divided into 2 parts — the Fragmentation and Central Regions.

Fragmentation regions correspond to regions of finite t (b-fragmentation region) and finite u(a-fragmentation region) as $s \to \infty$. Intuitively, of course, c can be looked upon as a fragment of $b(a)$ when the momentum transfer between the two remains finite at large s. Since one can apply identical considerations to the 2 fragmentation regions, we shall save time by discussing the fragmentation of b only.

Central Region corresponds to the remainder *i.e.* where t and u are both large.

For Regge analysis, the most suitable variables for the Fragmentation region of b are s, t and M^2, and those for the central region are s, t and u. However, the experimenters mostly give their data in terms of the directly measurable quantities s and p_{L}, p_{T} — the longitudinal and transverse momenta of c. Let us discuss briefly the connection of p_{L}, p_{T} with the invariant quantities. The longitudinal momentum p_{L} occurs in 3 alternative forms — each form has some advantage over the others.

Firstly, the Feynman variable

$$x = 2p_{\mathrm{L}}^{\mathrm{CM}}/\sqrt{s}; \tag{2}$$

we have

$$t,\, u = m_{b,a}^2+m_c^2-E_{b,a}E_c+p_{b,a}p_{\mathrm{L}}, \tag{3}$$

$$\frac{ut}{s} = \kappa = m_c^2+p_{\mathrm{T}}^2, \tag{4}$$

where all the quantities on the right are in the CM system. The κ is referred to by some as the transverse mass and by others as the longitudinal mass. For small x, Eq. (3) reduces to

$$t,\, u \simeq -s^{\frac{1}{2}}(E_c \mp p_{\mathrm{L}}) \simeq -s^{\frac{1}{2}}\left[\left(m_c^2+p_{\mathrm{T}}^2+\frac{x^2 s}{4}\right)^{\frac{1}{2}} \mp \frac{xs^{\frac{1}{2}}}{2}\right]. \tag{3'}$$

We shall be interested only in the small p_{T} region, where most of the events lie anyway. Then for large s, x has a range -1 to $+1$ as shown in Fig. 1. The central region, where both t and u are large, corresponds to a small interval near $x = 0$, where t, $u \sim -s^{\frac{1}{2}}$ by Eq. (3). Strictly speaking, it goes to ∞ only in the interval

$$-\frac{4\kappa^{\frac{1}{2}}}{s} < x < \frac{4\kappa^{\frac{1}{2}}}{s}.$$

Of course, a Reggeist's definition of asymptotic is $|t|$, $|u| > 5$ GeV^2, say. Typically it corresponds to $-0.1 < x < 0.1$ for $\kappa \geqslant 0.5$ GeV^2 and large s. The region $x > 0.1$ corresponds to finite t and hence b-fragmentation, and similarly $x < -0.1$ corresponds to finite u and hence to a-fragmentation. One can check these simply by making a binomial expansion of Eq. (3′). Over the b-fragmentation region we have

$$\frac{M^2}{s} \simeq 1-x, \tag{5}$$

$$t = m_b^2+m_c^2-\frac{p_{\mathrm{T}}^2+m_c^2}{x}-m_b^2 x. \tag{6}$$

Evidently x is not very suitable for analysing the central region, as the entire region is mapped into a very small segment of x. For this purpose one uses a new variable — the CM rapidity Y.

$$Y = \frac{1}{2}\ln\frac{t}{u} \xrightarrow{\text{small } x} \frac{1}{2}\ln\frac{\left[\left(\kappa+\frac{x^2s}{4}\right)^{\frac{1}{2}}-\frac{xs^{\frac{1}{2}}}{2}\right]}{\left[\left(\kappa+\frac{x^2s}{4}\right)^{\frac{1}{2}}+\frac{xs^{\frac{1}{2}}}{2}\right]}. \tag{7}$$

For $x \sim 0.1$ we get, using the binomial expansion

$$Y \simeq -\frac{1}{2}\ln\frac{x^2s}{\kappa} \simeq -\frac{1}{2}\ln\frac{s}{\kappa}+2. \tag{7′}$$

Now, the maximum rapidity is $\sim \frac{1}{2}\ln(s/\kappa)$, which is typically 4 in the ISR energy range. Thus the region $0 < x < 0.1$ covers 2 units of rapidity at the ISR energies, which is as big as the rapidity length covered by the entire region $0.1 < x < 1$. This is illustrated in Fig. 1.

Finally for the b-fragmentation region, p_{L} is often given in the rest frame of b (*i.e.* in the lab frame for target fragmentation). One has for large s

$$t = m_b^2+m_c^2-2m_b(p_{\mathrm{L}}^{\mathrm{lab}^2}+p_{\mathrm{T}}^2+m_c^2)^{\frac{1}{2}} = m_b^2+m_c^2-2m_bE_c^{\mathrm{lab}}, \tag{8}$$

$$\frac{M^2}{s} = 1+\frac{p_{\mathrm{L}}^{\mathrm{lab}}}{m_b}-\frac{(p_{\mathrm{L}}^{\mathrm{lab}^2}+p_{\mathrm{T}}^2+m_c^2)^{\frac{1}{2}}}{m_b} = 1+\frac{p_{\mathrm{L}}^{\mathrm{lab}}-E_c^{\mathrm{lab}}}{m_b}. \tag{9}$$

Note that finite t corresponds to finite $p_{\mathrm{L}}^{\mathrm{lab}}$.

The invariant cross-section is given in the following terms, which are all equal at large s:

$$E_c\frac{d\sigma}{d^3p_c} = \frac{1}{\pi}E_c\frac{d\sigma}{dp_{\mathrm{L}}dp_{\mathrm{T}}^2} = \frac{s}{\pi}\frac{d\sigma}{dxdp_{\mathrm{T}}^2}\frac{2E_c}{\sqrt{s}} = \frac{1}{\pi}\frac{d\sigma}{dYdp_{\mathrm{T}}^2}. \tag{10}$$

One more comment about the kinematics. The fragmentation region can be further divided into two parts — large M^2 and large s/M^2 as we see from (5). Since different quantities are becoming large in the two parts, naturally the Regge expansions would be different. For instance, the single Regge Expansion applies only to the large M^2 part.

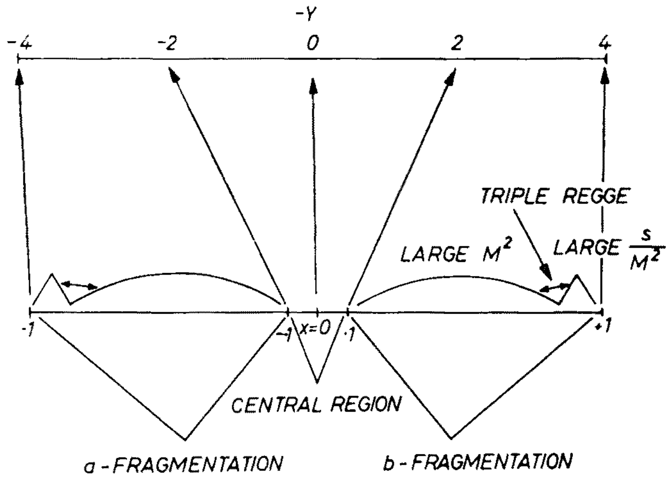


Fig. 1. The Fragmentation and Central regions on the x-plot and their mappings on to the Y-plot

I shall discuss only this part of the fragmentation region. Dick Roberts is going to discuss the large s/M^2 part and the overlap region between the 2 parts which is the so called triple Regge region.

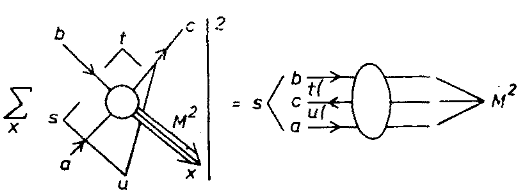
Mueller's Optical Theorem

Let us recall the standard optical theorem connecting the total cross-section to the discontinuity of the forward elastic amplitude

$$\sigma_T \propto \text{Disc. } T_{el}(s), \tag{11}$$

which follows essentially from unitarity. Mueller [1] has suggested an extension of this relation connecting the inclusive cross-section to a discontinuity of a 3-body elastic amplitude

$$E_c \frac{d^3\sigma}{dp_c^3} = \text{Disc. } T_{ab\bar{c}}(M^2) = f. \tag{12}$$



There are, of course, some additional complications for the 3-body case. On the formal side the 3-body amplitude $T_{ab\bar{c}}$ is unphysical since the sub-energies t and u are negative. Moreover, two independent variables M^2 and s have overlapping cuts, which complicates the definition of the discontinuity.

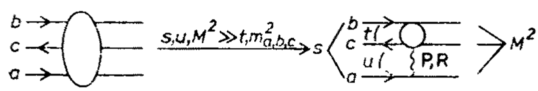
Due to these complications the Mueller theorem lacks a rigorous proof. It has been checked, however, in some field theoretic models and generally accepted as a very plausible hypothesis. The appropriate discontinuity, I am told, refers to

$$f = T_{ab\bar{c}}(s+i\varepsilon,\, M^2+i\varepsilon,\, s'-i\varepsilon) - T_{ab\bar{c}}(s+i\varepsilon,\, M^2-i\varepsilon,\, s'-i\varepsilon). \qquad (13)$$

On the practical side the 3-body amplitude is not a measurable quantity unlike the 2-body case. However, we can predict many features of the 3-body amplitude from Regge theory and Duality, and test them against the inclusive cross-section using Mueller's theorem. This is very similar to testing the exchange degeneracy and the Regge behaviour of a 2-body elastic amplitude with the total cross-section data.

Regge Expansion of the 3-Body Amplitude. *b*-Fragmentation Region (Single Regge Region)

The criterion for Regge expansion of a multiparticle amplitude is that all the invariants spanning across the Regge exchange be large compared to those lying on either end of it. Thus for s, u, $M^2 \gg t$, $m^2_{a,b,c}$ the 3-body amplitude can be approximated by the leading Regge exchanges in $a\bar{a}$:



$$f = F_P(t, s/M^2)s^{\alpha_P(0)-1} + \sum_{R=\varrho,\omega,f,A_2} F_R(t, s/M^2)s^{\alpha_R(0)-1},$$

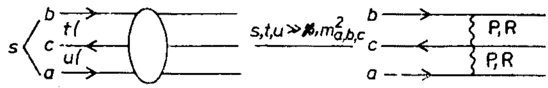
$$\alpha_P(0) = 1, \qquad \alpha_R(0) = \tfrac{1}{2}. \qquad (14)$$

Here P and R refer to the Pomeron and the leading meson trajectories (ϱ, ω, f, A_2), which we shall simply refer as the Reggeons. Asymptotically the lst term goes to a constant and the 2nd goes down as $s^{-\frac{1}{2}}$. Thus the Regge model makes specific predictions about the scaling and the non-scaling parts of the inclusive cross-section, similar to those of the total cross--section. Moreover we shall see that the duality considerations require the Reggeon contribution to vanish for some "exotic" inclusive reactions, analogous to the K^+p total cross--section case. Therefore the inclusive cross-section data here provides a rich field to test the duality and Regge hypotheses.

Central Region (Double Regge Region); Here both u and t become large along with s. Only the ratio

$$\kappa = \frac{ut}{s} = m_c^2 + p_T^2 \qquad (4)$$

is held finite. The 3-body amplitude can be approximated here by the double Regge contribution shown below.

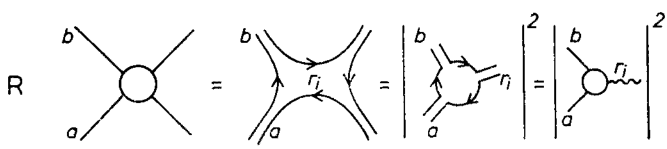


$$
\begin{aligned}
f = {} & \beta_{\mathrm{P}aa}\beta_{\mathrm{P}bb}\gamma^{c}_{\mathrm{PP}}(\kappa)u^{\alpha_{\mathrm{P}}(0)-1}t^{\alpha_{\mathrm{P}}(0)-1} && \text{Scaling} \\
& + \sum_{\mathrm{R}} \beta_{\mathrm{R}aa}\beta_{\mathrm{P}bb}\gamma^{c}_{\mathrm{RP}}(\kappa)u^{\alpha_{\mathrm{R}}(0)-1}t^{\alpha_{\mathrm{P}}(0)-1} && \text{Leading} \\
& + \sum_{\mathrm{R}} \beta_{\mathrm{P}aa}\beta_{\mathrm{R}bb}\gamma^{c}_{\mathrm{RP}}(\kappa)u^{\alpha_{\mathrm{P}}(0)-1}t^{\alpha_{\mathrm{R}}(0)-1} && \text{Non-scaling} \\
& + \sim u^{\alpha_{\mathrm{R}}(0)-1}t^{\alpha_{\mathrm{R}}(0)-1}. && \text{Tertiary}
\end{aligned}
\qquad (15)
$$

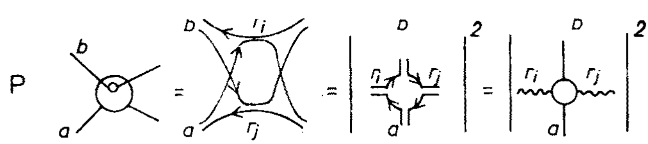
The scaling and the leading non-scaling terms correspond to Pomeron-Pomeron and Pomeron-Reggeon exchanges. The Reggeon-Reggeon exchanges constitute the tertiary terms, which could be neglected. We shall again see that duality predicts the non-scaling contributions to vanish for certain "exotic" reactions.

Dual Properties of the 3-Body Amplitude

Let us analyse the duality properties of a 3-body amplitude. The Harari-Freund 2-component duality for the 2-body amplitude, extends quite naturally in to a 7-component picture for the 3-body amplitude [2, 3]. To see this let us first recall how the two components (Regge and Pomeron) arise in a two-body amplitude



LHS is the shorthand notation. For mesonic amplitudes it amounts to denoting the external diquark lines by single lines. (a, b lie on the same quark loop, representing resonances (r_i) in s-channel. It vanishes if the system ab has exotic quantum numbers.)

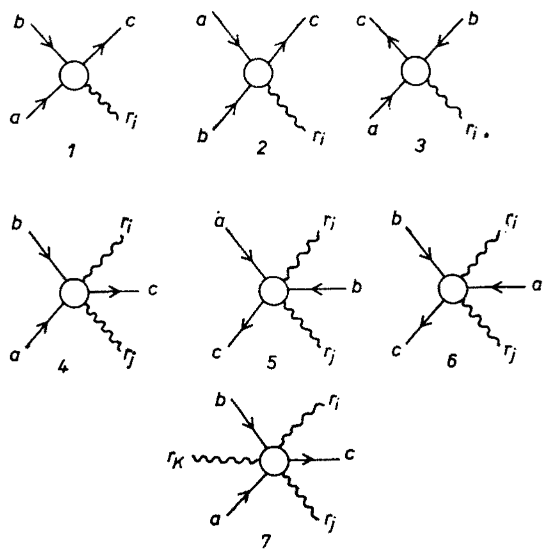


(a, b lie on separate quark loops, representing two-body intermediate states).

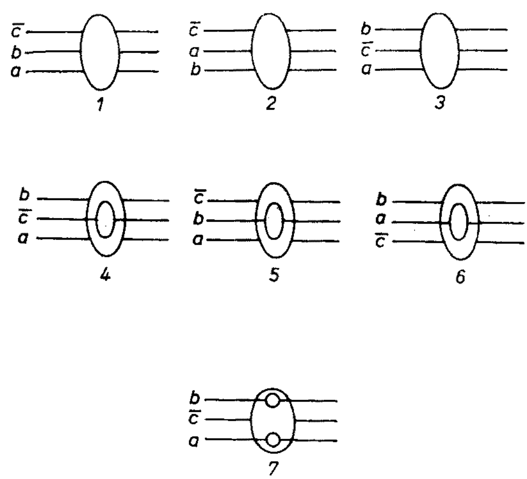
For the sake of experts, I should clarify that we interpret duality in the original spirit of small width Γ approximation. The 1st component goes like Γ^2 whereas the 2nd goes like Γ^4. Thus although in principle the 2nd component has the normal Regge poles (in addition to the Pomeron), their effect will be small compared to the 1st. This will be our interpretation of the flatness of the K^+p cross-section, for instance.

In the same spirit, we shall assume that even the Pomeron part is small compared to the 1st component, when the energies involved are extremely small.

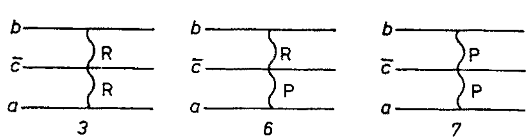
Turning to the 3-body amplitude now we see that there can be 3 independent configurations for single resonance intermediate states, 3 configurations for two-resonance states and 1 for a three resonance intermediate state.



Squaring each diagram we get



Here again particles lying on different quark loops are connected by Pomeron and those lying on the same quark loop are connected by Regge exchanges. We have, for instance



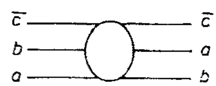
and consequently

$$\begin{aligned} 1,2,3,4 &\rightarrow t^{\alpha_R-1}u^{\alpha_R-1}, \\ 5 &\rightarrow t^{\alpha_P-1}u^{\alpha_R-1}, \\ 6 &\rightarrow t^{\alpha_R-1}u^{\alpha_P-1}, \\ 7 &\rightarrow t^{\alpha_P-1}u^{\alpha_P-1}, \end{aligned} \tag{16}$$

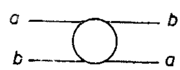
so that in a specific kinematic region only some of the 7 components will be dominant.

Note that all the 7 components above are positive as each is a square of a production amplitude. The prescription neglects 2 types of contributions.

Firstly the interference terms amongst the 7 production amplitudes. Many of them are seen to go away from the kinematics and quantum number considerations. The rest are usually assumed away. For instance the interference terms amongst 1–2 correspond to diagrams of the type



which are kinematically suppressed for high energy forward scattering. This is analogous to the suppression of the $s-u$ Veneziano term



in high energy forward scattering.

The other interference terms require a single particle or 2 particle combination to have vacuum quantum number. For instance 1–6 requires either a or $b\bar{c}$ to have vacuum quantum number. In many inclusive reactions, the single or 2 particle combinations would not in fact, have vacuum quantum numbers, thus ruling out interference terms. In general, however, the absence of interference terms have not been proved.

The second effect neglected, is the diffractive contribution to the production amplitudes. The production amplitudes 1–7 all correspond to tree diagrams and thus have no Pomeron. This is due to the perturbative approach to Unitarity employed here or in the 2-body case discussed earlier. Inclusion of Pomeron in the production amplitude is supposed to generate cuts in the elastic amplitude after unitarity summation, which were neglected in the two-component model. Nonetheless there are significant diffractive contributions in the production amplitude 1 if b and c have identical quantum numbers, which must be added as extra components. However, the diffractive components are significant only for small missing mass. Therefore, they would not be relevant for my purpose.

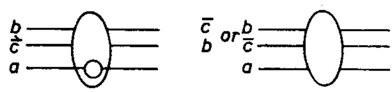
Dominant Components and Exoticity Criterion in the Fragmentation and Central Regions

We see from Eq. (16) that only a few of the 7 components are dominant in a specific kinematic region.

Central Region. Here both t and u become large. Thus 7 constitutes the scaling term whereas 5 and 6 constitute the leading non-scaling terms. Components 1–4 give the tertiary terms, and shall be neglected. The criterion for an energy independent cross-section (early scaling or Exoticity) in this region is that 5 and 6 be suppressed relative to 7 — *i.e.* the channels $a\bar{c}$ and $b\bar{c}$ be exotic.

b-Fragmentation Region (small t). We shall restrict to small values of t for which the situation simplifies very much. Kinematically the small t criterion holds over a large part of the fragmentation region, when the outgoing particle is light, *i.e.* $c = \pi$, as seen from (6).

Narrow width duality suggests that for very small t, the Pomeron exchange between $b\bar{c}$ will be suppressed relative to the Reggeon exchange (*e.g.* $4 \ll 1, 7 \ll 6$). Thus the diagrams, where bc lie on the same quark loop, will be the dominant ones. The same result also follows from the resonance production diagrams described earlier. When the momentum transfer between b and c becomes small the diagrams 2, 4, 5, 7, with an intercepting resonance, are going to vanish. This has been checked quantitatively in a model calculation [4]. Therefore the surviving components are 6 and (1+3)



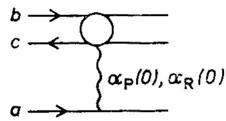
6 $t^{\alpha_R-1}u^{\alpha_P-1}$ 1 or 3 $t^{\alpha_R-1}u^{\alpha_R-1}$

which define the scaling and the non-scaling pieces in this region (as s or equivalently $u \to \infty$). Thus we have here the good old two-component model where the resonances and background in the missing mass channel ($ab\bar{c}$) are dual to Reggeon and Pomeron in $a\bar{a}$. The criteria for energy independent cross-section or "exoticity" is suppression of the Regge component relative to the Pomeron — *i.e.* $ab\bar{c}$ be exot c and $b\bar{c}$ non exotic. This criterion was suggested by Chan *et al.* [5].

I should comment upon some alternative criteria for "exoticity", which were put forward [2, 6], following the suggestion of the above criterion. They generally refer to the components 4 and 5, which would give non-scaling contributions $\sim u^{\alpha_R-1}$; and which may be comparable to 1 except at very small t. They suggest, therefore, that the combinations ab and $a\bar{c}$ should be exotic in addition to $ab\bar{c}$ so that all the non-scaling go away. These criteria have formed significant steps in the historical development of the subject. In the final analysis, however, there does not seem to be much theoretical or experimental support in their favour. For the component 6 is also expected have a nonleading term $\sim u^{\alpha_R-1}$, which would not be affected by these exoticity criteria. And this term is not suppressed relative to 4 and 5 by the duality arguments, as they occur in the same order of the coupling strength. A significant distinction is that 4 and 5 are positive whereas the non-leading term of 6 could have either sign. Experimentally, the reactions with $ab\bar{c}$ exotic and $b\bar{c}$ non-exotic show either very good scaling or sometimes a negative non--scaling piece. There is, however, no evidence so far of a positive non-scaling piece for any of these reactions.

3. Duality and Regge analyses in the fragmentation region

Standard Exchange Degeneracy Tests



$$f = F_P\left(t, \frac{s}{M^2}\right) s^{\alpha_P(0)-1} + \sum_{R=\varrho,\omega,f,A_2} F_R\left(t, \frac{s}{M^2}\right) s^{\alpha_R(0)-1} =$$

$$= F_P\left(t, \frac{s}{M^2}\right) + \sum_R F_R\left(t, \frac{s}{M^2}\right) s^{-\frac{1}{2}}, \quad R = \varrho, \omega, f, A_2. \tag{18}$$

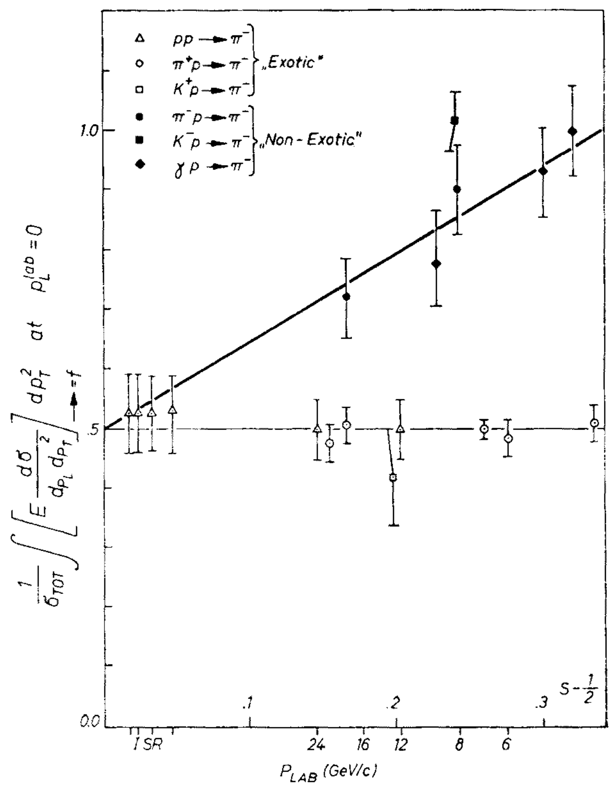


Fig. 2. Comparison of the "exotic" cross-sections $(p, \pi^+, K^+)\, p \to \pi^- X$ with the "non-exotic" ones $(\pi^-, K^-, \gamma)\, p \to \pi^- X$

Let us compare the set of "exotic" fragmentations $K^+p \to \pi^-X$, $\pi^+p \to \pi^-X$, $pp \to \pi^-X$ with the "non-exotic" ones $K^-p \to \pi^-X$, $\pi^-p \to \pi^-X$, $\gamma p \to \pi^-X$. Assuming the Pomeron to factorise, the 6 asymptotic cross-sections should have a common limit, when divided by the corresponding $\sigma_{Tot}(ap)$ say. The two-component duality predicts the "exotic" cross-sections to have reached this limiting value at small s, whereas the "non-exotic" ones are predicted to approach this from above.

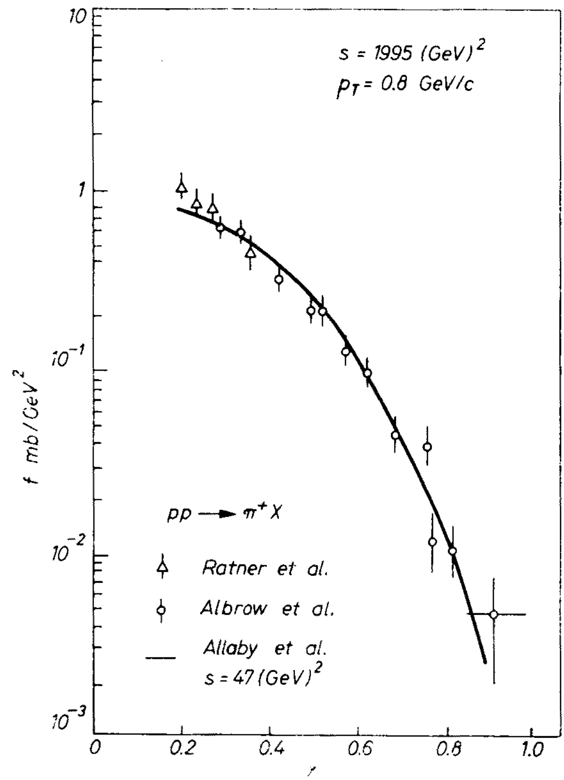


Fig. 3. Comparison of the accelerator and ISR cross-sections for the "exotic" reaction $pp \to \pi^+X$, as a function of x

The experimental cross-sections (integrated over p_T) are plotted against $s^{-\frac{1}{2}}$ in Fig 2. The data points agree with all the above features, thus supporting both duality and Pomeron factorisation. I shall come to Pomeron factorisation in a while. For the moment, however, let us concentrate on the duality part.

The Fig. 2 is the analogue of the standard EXD result with the exotic and non-exotic total cross-sections. There are however 3 significant advantages in testing EXD with the inclusive data, in comparison with the total cross-section case. Firstly, the non-scaling term in Fig. 2 is typically a 50–60% effect, in contrast with the 15–20% effect in 2-body scattering (*e.g.* the difference of $K^{\pm}p$ total cross-sections). In view of this, the observed flatness of the "exotic" cross-sections here is a remarkable achievement for two-component duality. Also in contrast with the 2-body case, we have more freedom with the quantum numbers here. Consequently a large number of "exotic" channels are experimentally accessible here, whereas we had only $\sigma_{\rm Tot}(KN)$ and $\sigma_{\rm Tot}$ (NN) in the 2-body case. The table lists the "exotic" inclusive cross-sections, where the s-independence has been checked — either by comparing data at different energies (when available) or through Pomeron factorisation. On the whole it is an impressive list. Thirdly, in contrast with the 2-body case, the s-independence for the inclusive cross-section can be checked as a function of two variables — t and s/M^2 (or equivalently p_T, x). The Fig. 3 demonstrates the equality between the 24 and 1000 GeV cross-sections for the exotic process $pp \to \pi^- X$ as a function of x.

TABLE

List of "Exotic" reactions, where s-independence has been verified — either by comparing data [7] at different energies or through factorisation*

(1)	$\pi^+ p \to \pi^- X$	(2)	$K^+ p \to \pi^- X$	(3)	$pp \to \pi^- X$
(4)	$K^+ p \to \pi^+ X$	(5)	$pp \to \pi^+ X$		
(6)	$\pi^+ p \to \Lambda X$	(7)	$K^+ p \to \Lambda X$		
(8)	$\pi^+ p \to K^0_s X$	(9)	$pp \to \Lambda X$		

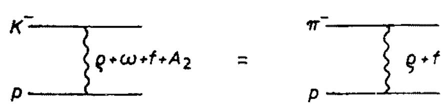
* The only "exotic" reactions, which seem to show appreciable s-dependence are $pp \to K^0 X$ and $pp \to K^+ X$. They show a negative non-scaling component (compare rising $\sigma_{\rm TOT}(K^+p)$?). There is no satisfactory explanation for this, to my knowledge. The sign suggests, however, that their origin may be different from the non-scaling components of a "non-exotic" reaction, which are positive.

Additional Exchange Degeneracy Tests

So much about the standard EXD prediction for exotic processes. There are also EXD predictions for non-exotic processes. More precisely these predictions follow from EXD, SU(3) and factorisation; factorisation for Reggeons is, of course, a relatively clean assumption.

In two-body scattering the best known examples are the equalities between non--Pomeron $K^- p$ and $\pi^- p$ cross-sections

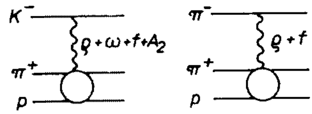
$$\sigma^{\rm N\cdot P}_{K^- p(K^- n)} = \sigma^{\rm N\cdot P}_{\pi^- p(\pi^+ p)}. \tag{19}$$



EXD gives $g_{\rho pp} = g_{A_2 pp}$, $g_{\omega pp} = g_{fpp}$, $g_{\rho KK} = g_{\omega KK} = g_{fKK} = g_{A_2 KK}$, $g_{\rho\pi\pi} = g_{f\pi\pi}$ and SU(3) gives $g_{\rho\pi\pi} = 2g_{\rho KK}$. Therefore the factor of 2 advantage for the coupling on RHS compensates having twice as many trajectories on the left. Experimentally, of course, the equalities are very well satisfied.

These predictions can again be extended to the inclusive case. One derives, for instance,

$$f_{K^-p\to\pi^-X}^{\text{non-scaling}} = f_{\pi^-p\to\pi^-X}^{\text{non-scaling}}. \tag{20}$$



The $p\pi^+$ system has the quantum numbers of a Δ^{++}. Thus its couplings obey the same EXD relations as Δ, *i.e.*

$$F_\varrho\left(\frac{s}{M^2}, t\right) = F_{A_2}\left(\frac{s}{M^2}, t\right), \quad F_\omega\left(\frac{s}{M^2}, t\right) = F_f\left(\frac{s}{M^2}, t\right),$$

analogous to the proton case. Therefore the equality follows.

In addition one has the equality between ϱ and f couplings to Δ, *i.e.* $F_\varrho(s/M^2, t) = F_f(s/M^2, t)$ which was not true for the proton case. This follows from having an addi-

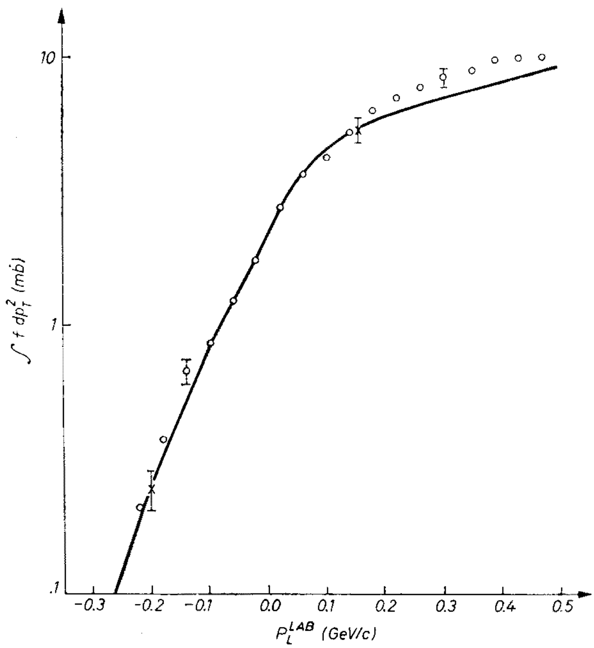


Fig. 4. The $\pi^-p \to \pi^-X$ cross-section data compared against the prediction from the $K^-p \to \pi^-X$ cross-section. The crosses denote the uncertainty in the predicted value, due to the error bars in the $K^-p \to \pi^-X$ data

tional exotic channel for Δ, namely $\pi^+\Delta^{++}$. Thus all the 4 fragmentation vertices are equal. Thus starting with the non-scaling contribution for $K^-p \to \pi^-X$ one can predict not only $\pi^-p \to \pi^-X$, but other non-exotic processes like $\gamma p \to \pi^-X$ too; that is using the couplings at the top from 2-body data. The prediction for π^-p → $\pi^- X$ has been tested by Miettinen [8]. And Chan, Lam and Miettinen [9] have checked the prediction for γp → $\pi^- X$: The results are shown in Figs 4 and 5.

What is plotted in fact is the net fragmentation cross-section — that is the predicted non-scaling part plus the scaling part obtained from the exotic process K^+p → $\pi^- X$. It

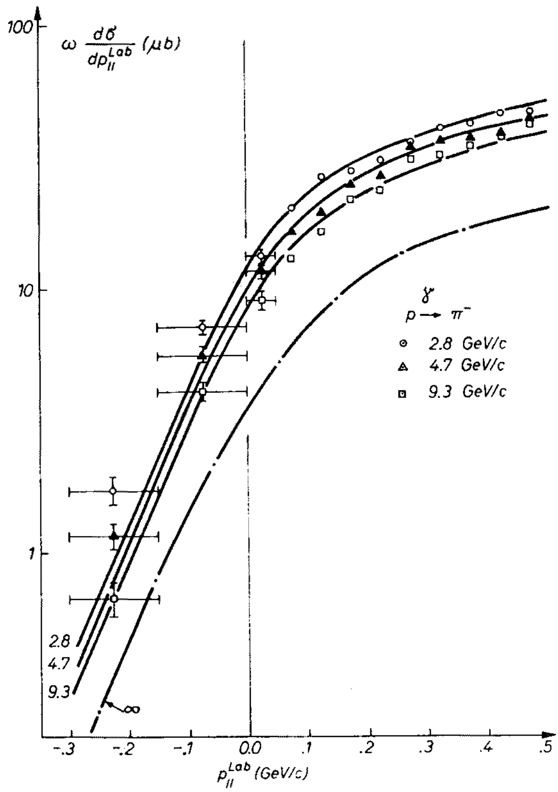


Fig. 5. The γp → $\pi^- X$ data compared against the prediction from the K^-p → $\pi^- X$ cross-section

should be remembered, of course, that the non-scaling part is roughly as big as the scaling part. Therefore the agreement for the net cross-section reflects a fairly good agreement for the non-scaling part itself. Again the relation holds over a range of p_L and p_T values. The comparison as a function of p_T has been done for the γp → $\pi^- X$ case [10].

Pomeron Factorisation

Factorisation is an important test for the nature of Pomeron singularity *i. e.* if it is a pole or a more complicated object. There are many compelling reasons against the Pomeron being exactly a factorising pole as we have heard from Prof. Le Bellac. There are in fact few reasons why it should even be dominantly a pole, except in the multiperipheral model. Many people hope, nonetheless, that it is so, as otherwise the Regge description of elastic scattering will be a pretty hopeless exercise. And besides, the multiperipheral model is perhaps the best hope we have to understand unitarity.

In 2-body scattering, the total cross-sections related by factorisation, like pp, πp, $\pi\pi$, are not all directly measurable. Therefore one can only hope to test Pomeron factorisation for processes like diffraction dissociation. But again one does not know if the Pomeron in elastic scattering and diffraction dissociation is the same object

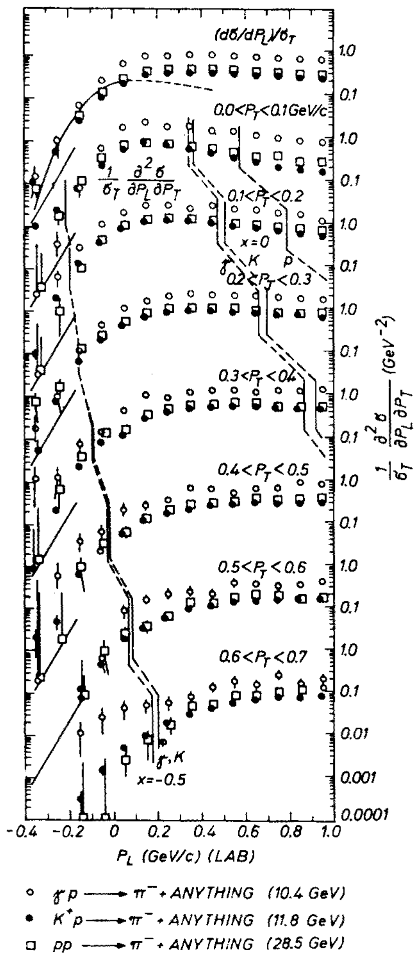


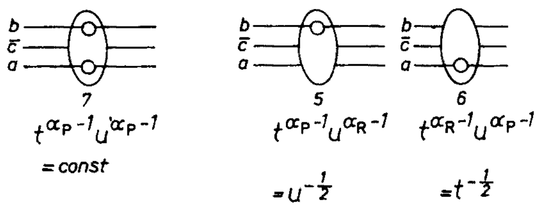
Fig. 6. Test of Pomeron factorisation between the "exotic" reactions $K^+p \to \pi^- X$ and pp $\to \pi^- X$. Note that, in contrast, the "non-exotic" cross-section $\gamma p \to \pi^- X$ is typically 50% higher. The dashed lines denote the boundary of the single Regge region

Now, the inclusive data provides many 3-body elastic amplitudes, which are related by factorisation. One can even afford the luxury of picking up only the exotic processes to ensure Pomeron dominance even at finite energy. We have seen the comparison amongst $\pi^+ p \to \pi^- X$, $K^+ p \to \pi^- X$ and pp $\to \pi^- X$. The comparison between the last two has in fact been done as a function of p_L and p_T by Swanson *et al.* [11] (Fig. 6). In all these cases there seems to be good evidence of Pomeron factorisation, that is within the experimental accuracy of 10–20%.

4. Duality and Regge analyses in the central region

For double Regge analysis we should have large s, t and u compared to $m^2_{a,b,c}$ and κ. Let us optimistically interpret large t, u as $t, u > 5\ \text{GeV}^2$. Then this criterion is satisfied at ISR; and also at some accelerator energies, particularly for an outgoing heavy particle (*e. g.* c = proton).

The "scaling" and the "leading non-scaling" terms are from components 7 and (5+6).



It is instructive to discuss the components 7 and (5+6) separately, since the data seems to agree with the prediction of the simplest dual Regge model for the latter case, but not the former.

The processes $pp \to \bar{p}X$ and $pp \to K^-X$ have all the pairs $a\bar{c}$, $b\bar{c}$ and ab exotic. Consequently a, b and $\bar{c}$ must lie on separate quark loops. Thus these cross-sections provide a direct measure of the component 7. On the other hand, the difference

$$\Delta_c \equiv f(ab \to cX) - f(ab \to \bar{c}X) \tag{21}$$

corresponds to the component (5+6), since the c and $\bar{c}$ have identical 7th component, *via* Pomeranchuk theorem.

Non Scaling Component from Cross-Section Difference

We shall be concerned mainly with the pp data. Here a and b are identical. Thus (5+6) takes the simple form

$$\Delta_c = 5+6 \sim u^{-\frac{1}{2}}+t^{-\frac{1}{2}} \sim (ut)^{-\frac{1}{4}}\left\{\left(\frac{t}{u}\right)^{\frac{1}{4}}+\left(\frac{t}{u}\right)^{-\frac{1}{4}}\right\} \sim \beta(\kappa)s^{-\frac{1}{4}}\cosh\frac{1}{2}Y, \tag{22}$$

where Y is the rapidity variable. Let us compare the ISR and accelerator data on Δ_p and Δ_π with this prediction [12].

The Δ_π and Δ_p are shown on a $s^{-\frac{1}{4}}$ plot, at constant $Y = 0$ (*i. e.* $x = 0$) in Figs. 7 and 8. The data points are, indeed, suggestive of a $s^{-\frac{1}{4}}$ behaviour, although any other power between -0.1 and -0.5 (corresponding to $\alpha_R(0) = 0.8$ & 0) cannot be ruled out. Next the ISR data in Δ_p is shown on cosh $(Y/2)$ plot in Fig. 9. The energy variation within ISR is

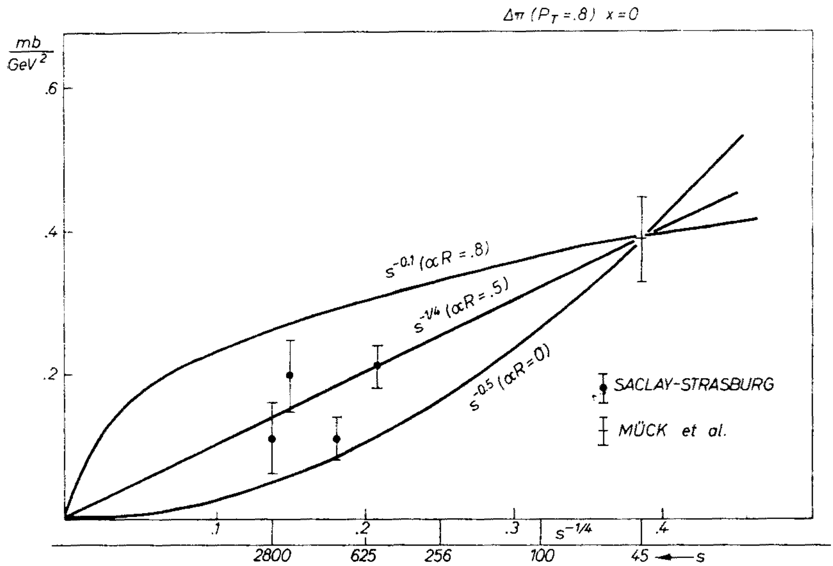


Fig. 7. The cross-section difference Δ_π on a $s^{-\frac{1}{4}}$ plot. The straight line prediction corresponds to Eg. (22). The power behaviours $s^{-0.1}$ and $s^{-0.5}$, corresponding to $\alpha_R(0) = 0.8$ and 0, are also shown for comparison

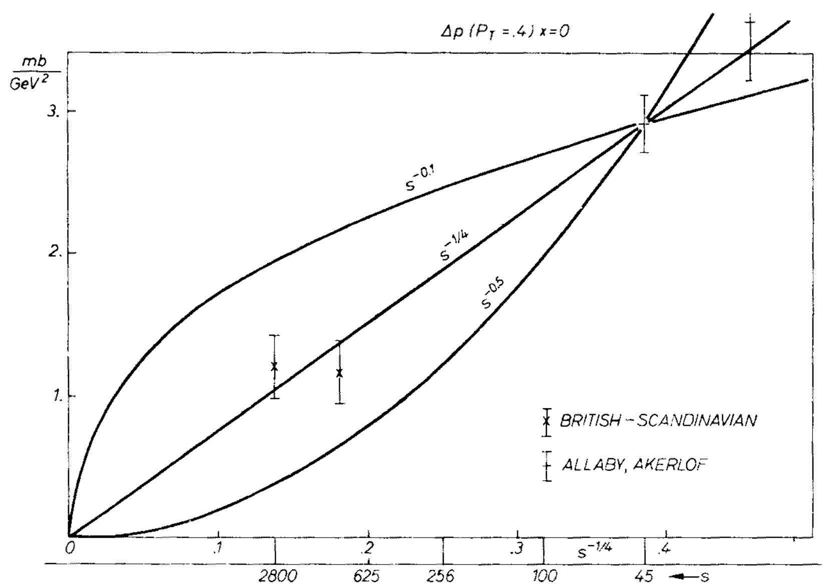
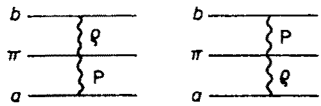


Fig. 8. Same as Fig. 7, for the cross-section difference Δ_p

taken into account by dividing Δ_p by $s^{-\frac{1}{4}}$. The data points agree well with the suggested cosh $(Y/2)$ behaviour, although again the accuracy is rather poor.

Pomeron factorisation can be tested by comparing Δ_c in pp, $\pi^{\pm}$p and $K^{\pm}$p collision. For instance, Δ_π gets contribution only from the ϱ^{-}p terms shown below.



$\beta_{\varrho P}^{ab\pi}(\kappa)s^{-\frac{1}{4}}$ $\qquad$ $\beta_{P\varrho}^{ab\pi}(\kappa)s^{-\frac{1}{4}}$

Both P–P and f–P terms drop out from the difference. Now factorisation implies

$$\beta_{\varrho P}^{ab\pi}(\kappa) = \gamma_{\varrho b}\Gamma_{\varrho P\pi}(\kappa)\gamma_{Pa}, \qquad \beta_{P\varrho}^{ab\pi}(\kappa) = \gamma_{Pb}\Gamma_{\varrho P\pi}(\kappa)\gamma_{\varrho a}. \tag{23}$$

Thus the relative amount of Δ_π can be predicted for various incident particles, taking γ's from 2-body data. One gets, for instance [12], $\Delta_\pi(pp)$: $\Delta_\pi(\pi^+p)$: $\Delta_\pi(\pi^-p) = 2:3:-2$. Data around 20 GeV ($0.4 < p_T < 0.6$) give $1.0:2.1:-1.0$ mb/GeV^2. A large number of

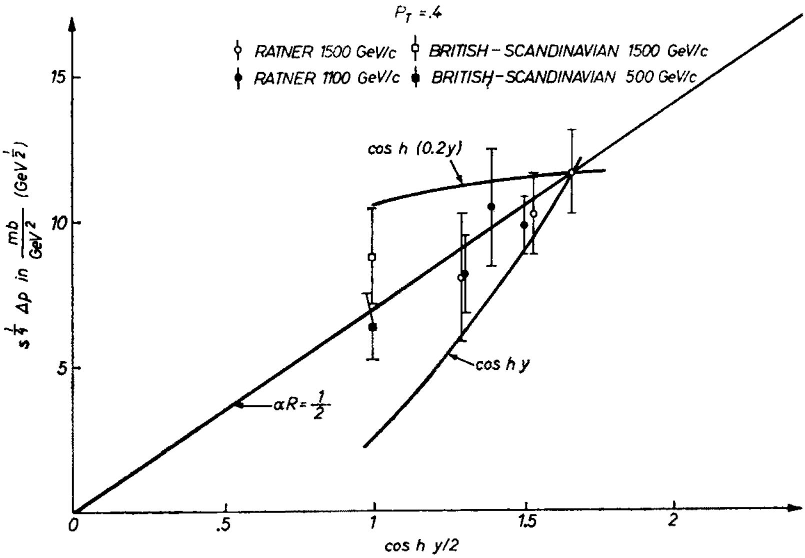


Fig. 9. The quantity $s^{-\frac{1}{4}}\Delta_p$ on a cosh $Y/2$ plot. The straight line corresponds to Eq. (22). The behaviours cosh $(0.2Y)$ and cosh (Y), corresponding to $\alpha_R(0) = 0.8$ and 0 are also shown for comparison

predictions of this type have been obtained [12], using the EXD relations for the central couplings Γ. They are expected to hold quantitatively, however, only at the NAL or Serpukhov energies.

Scaling Component (!) from Exotic Cross-Sections

Finally let us compare the data of $f(pp \to \bar{p}X)$ and $f(pp \to K^-X)$ with the prediction of component 7. The two cross-sections increase by factors of 100 and 10 respectively between accelerator and ISR, suggesting a hugė negative non-scaling piece. In dual perturbation theory there can be a Regge component in 7 in addition to the Pomeron, and it could have either sign. But the magnitude is required by the narrow width hypothesis, to be small compared to the components (5+6), *i. e.*

$$f^{\text{non-scaling}}_{(pp\to\bar{p}(K^-)X)} \ll f^{\text{non-scaling}}_{(pp\to p(K^+)X)}.$$

Experimentally, however, the non-scaling pieces — as measured by the difference between the accelerator and ISR cross-sections — show

$$f^{\text{non-scaling}}_{(\text{pp}\to\text{p}(K^-)X)} > f^{\text{non-scaling}}_{(\text{pp}\to\text{p}(K^+)X)}.$$

How to interpret this huge non-scaling piece in the component 7? There have been 2 suggestions.

(1) Tye and Veneziano [3] ascribe it to the Regge term in component 7, disregarding the narrow width duality constraint.

(2) Alternatively it has been ascribed by many authors to some kind of a threshold effect, which would not have a simple singularity structure in the Regge language.

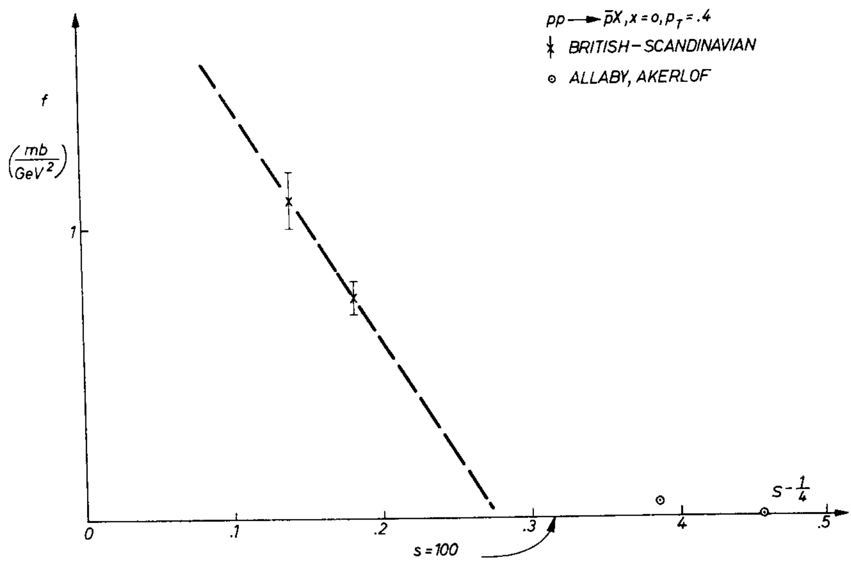


Fig. 10. The cross-section for pp → $\bar{\text{p}}X$ on a $s^{-\frac{1}{4}}$ plot

Both the alternatives are outside the scope of the simplest dual Regge approach. But one can compare the Tye-Veneziano suggestion with the energy dependence of $f(\text{pp}\to\bar{\text{p}}(K^-)X)$, as it predicts a $s^{-\frac{1}{4}}$ behaviour for the non-scaling piece. The data seems to contradict a simple $s^{-\frac{1}{4}}$ behaviour. Firstly, such a behaviour would suggest

$$\frac{f(s=2800)-f(960)}{f(960)-f(45)}(\text{pp}\to\bar{\text{p}}X) = \frac{f(2800)-f(960)}{f(960)-f(45)}(\text{pp}\to K^-X), \tag{24}$$

where each denominator is dominated by the ISR cross-section $f(960)$. The British-Scandinavian experiment suggests a 40% increase for $f(\text{pp}\to\bar{\text{p}}X)$ within ISR (LHS), but a 10% upper bound for $f(\text{pp}\to K^-X)$ (RHS). Secondly, plotting experimental $f(\text{pp}\to\bar{\text{p}}X)$ against $s^{-\frac{1}{4}}$, one sees that a linear extrapolation from the ISR point would make the cross-section negative even at $s > 100$ GeV² (see Fig. 10).

Even for pp → πX we now know that

$$f_{x=0} \neq a - bs^{-\frac{1}{2}} \tag{25}$$

contrary to some earlier claims [13]. This is seen by comparing the π^+ and π^- data on a $s^{-\frac{1}{2}}$ plot [14] Fig. 11. Linear extrapolation would make the 2-cross-sections asymptotically unequal. A much better fit is, in fact, obtained on a $s^{-\frac{1}{4}}$ plot. I feel there is no physics in

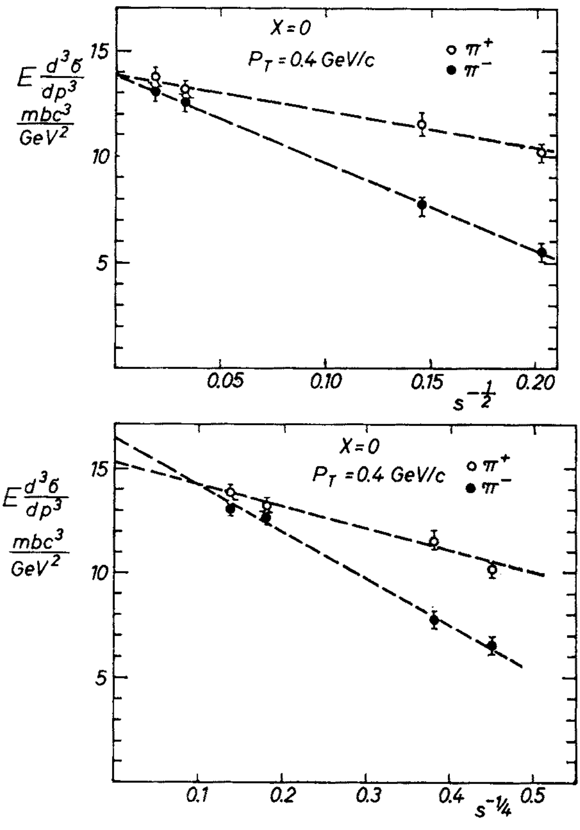
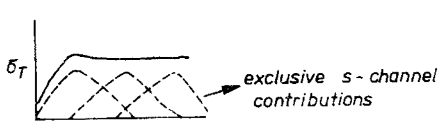


Fig. 11. The cross-sections for pp → $\pi^{\pm}X$ on the $s^{-\frac{1}{2}}$ and $s^{-\frac{1}{4}}$ plots. The two cross-sections should be equal asymptotically, which disfavours the $s^{-\frac{1}{2}}$ behaviour

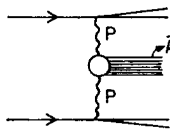
this plot. Only the rise in these cross-sections is so small compared to the error bars that a power law cannot be ruled out as long as the power is left free. The $s^{-\frac{1}{2}}$ behaviour seems to be ruled out, however.

The alternative interpretation in terms of a threshold rise, seems to be more realistic. It is analogous to the 2-body total cross-section case which shows a threshold rise before flattening



up. The flat cross-section is simple to interpret in terms of the t-channel Regge exchange, but hard in terms of the contributing s-channel states. The threshold rise, on the other hand, is very natural in the s-channel formulation, but hard to interpret in terms of Regge exchanges.

There has been several model calculations to reproduce the rise in the inclusive $\bar{p}$ cross-section, in terms of exclusive s-channels contributions of the type shown below.



The detected particle comes from the decay of a centrally produced cluster and for $\bar{p}$ or a high p_T pion the cluster has to be heavy. With a reasonable cluster mass ($\sim$ 5 GeV, say) and reasonable t-dependence for the Pomeron couplings it is possible to sustain a rise of the production cross-section up to ISR energies [15]. In this picture the rise is a kinematic effect in the sense that it depends more on the transverse mass of the detected particle than on its quantum numbers. This qualitatively agrees with the fact that ISR cross-section show a rise not only for the $\bar{p}$ but for large p_T pions as well.

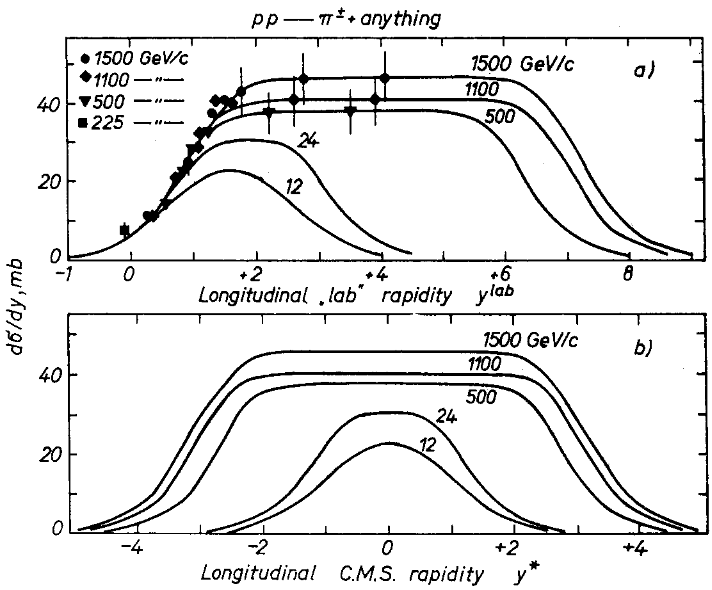


Fig. 12. The ISR cross-section for pp $\to \pi X$ showing the central plateau on a rapidity plot

Of course, this late onset of the leading Regge behaviour is a big surprise, in view of our experience with 2-body phenomenology. The large variable for double Regge expansion is u (or t)/κ. As $u = \sqrt{\kappa s}$ at $x = 0$, it is, of course, expected that the larger the κ, the higher will be the s for Regge behaviour to set in, what is surprising however is the magnitude of this scale. It has not yet set in for the $\bar{p}$ cross-section at ISR, for which $u/\kappa \sim 20$.

Let me indulge in a little bit of platitude. It is always possible to formally express a threshold rise in terms of t-channel Regge singularities. For instance, a cross-section of the type

$$f = \frac{q^2}{s} C \tag{26}$$

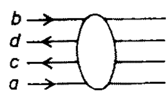
which shows a threshold rise can be expanded in power series of s. As a result we get a Pomeron accompanied by an infinite sequence of daughters. Caneschi [16] was perhaps the first to suggest an exclusive s-channel model for the rise of the $\text{pp} \to \bar{\text{p}}X$ cross-section, of the type discussed above, and to express the threshold rise in terms of the Pomeron daughters. Such a daughter structure, of course, has no predictive value. The reason I brought this up is the following. In the double Regge analysis of the central region by Chan *et al.* (Ref. [12]), we had expressed the rise in $\bar{\text{p}}$ in terms of an undefined Regge singularity Q. This has been unfortunately interpreted by some as suggesting some new dynamical singularity in the complex l-plane. We believe, however, that Q is nothing more than this non interesting daughter structure.

Let me finish with a happy note for the Mueller-Regge approach. The threshold effects are expected to be small for the ISR cross-sections for small p_T pion. They do show the energy and rapidity independence predicted by the P–P term. This is the famous central plateau in the rapidity plot (Fig. 12).

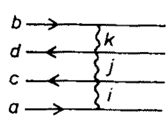
5. 2-Particle inclusive reactions

For the 2-particle Inclusive Process $ab \to cdX$ the invariant cross-section is related by Mueller's optical theorem to a 4-body amplitude

$$E_c E_d \frac{d\sigma}{dp_c} dp_d = f_{cd}\,. \tag{27}$$



When all the sub-energies are large, one can do the Regge Expansion



$$f_{cd} = \sum_{i,j,\kappa} \beta_{ij\kappa}(\kappa_c, \kappa_d) s_{a\bar{c}}^{\alpha_i - 1} s_{cd}^{\alpha_j - 1} s_{b\bar{d}}^{\alpha_k - 1}, \tag{28}$$

where one has specifically taken the limit $s_{a\bar{c}}$, $s_{b\bar{c}}$ and s_{cd} large and the transverse masses $\kappa_c = (s_{a\bar{c}} s_{cd})/s_{a\bar{d}}$, $\kappa_d = (s_{dc} s_{bd})/s_{b\bar{c}}$ fixed.

Both c and d in the Central Region

A particularly interesting situation is when both c and d are in the central region, so that Pomeron dominates the links i and j. Then

$$f_{cd} = \beta_{\mathrm{PPP}}(\kappa_c, \kappa_d) + \beta_{\mathrm{PRP}}(\kappa_c, \kappa_d) s_{cd}^{\frac{1}{2}}, \tag{29}$$

$$Y_c = \tfrac{1}{2} \ln (s_{b\bar{c}}/s_{a\bar{c}}), \qquad Y_d = \tfrac{1}{2} \ln (s_{b\bar{d}}/s_{a\bar{d}}),$$

$$s_{cd} = \left[\kappa_c \kappa_d \frac{s_{a\bar{d}} s_{bc}}{s_{a\bar{c}} s_{b\bar{d}}} \right]^{\frac{1}{2}} = (\kappa_c \kappa_d)^{\frac{1}{2}} e^{(Y_c - Y_d)}. \tag{30}$$

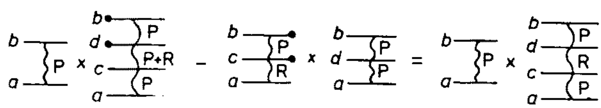
Thus

$$f_{cd} = \beta_{\mathrm{PPP}}(\kappa_c, \kappa_d) + \beta_{\mathrm{PRP}}(\kappa_c, \kappa_d) e^{-\frac{1}{2}(Y_c - Y_d)}. \tag{31}$$

The 2-particle inclusive cross-section will be translational invariant — *i. e.* independent of their individual rapidities, but only a function of the difference. This is analogous to the prediction of the central plateau in the single particle distribution.

If we assume Pomeron factorisation then

$$\sigma_{\mathrm{T}} \times f_{cd} - f_c \times f_d = \gamma_{\mathrm{P}a}^2 \gamma_{\mathrm{P}b}^2 \Gamma_{\mathrm{PR}c}(\kappa_c) \Gamma_{\mathrm{PR}d}(\kappa_d) e^{-\frac{1}{2}(Y_c - Y_d)}. \tag{32}$$



This is the correlation function G_{cd}. Alternatively it is defined as

$$R_{cd} = \frac{G_{cd}}{f_c f_d} = \frac{\sigma_{\mathrm{T}} f_{cd}}{f_c f_d} - 1 = \frac{\Gamma_{\mathrm{PR}c}(\kappa_c) \Gamma_{\mathrm{PR}d}(\kappa_d)}{\Gamma_{\mathrm{PP}c}(\kappa_c) \Gamma_{\mathrm{PP}d}(\kappa_d)} \, e^{-\frac{1}{2}(Y_c - Y_d)}. \tag{33}$$

Thus, in this model the 2-body correlation function in the central region has the following properties: (1) Energy Independence, (2) Translational Invariance, (3) Exponential fall off with rapidity difference (SRC) with range 2.

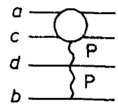
All the above features seem to agree roughly with the Pisa-Stony Brook and the CERN–Hamburg–Vienna ISR data. We have seen these data already during the talks of Prof. Le Bellac and Prof. Schmitz. One can in fact predict the magnitude of the correlation functions for 2-pions, taking the $\Gamma_{\mathrm{PR}\pi}(\kappa)$ terms from the data of Fig. 7, for instance. But I do not know if both single pion and 2-pion data are available for identical p_{T} cuts.

One should bear in mind that in this model the correlations are normalised with respect to the total cross-sections, whereas in the data they are mostly normalised with respect to the inelastic cross-sections.

It will be interesting to have data on $\pi^-\pi^-$ or $\pi^+\pi^+$ correlation, which is predicted to be zero or small due to exoticity.

c in the Fragmentation and d in the Central Region

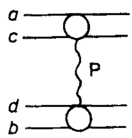
Here f_{cd} is given by the following graph:



For a factorising Pomeron there should be zero correlation. The Pisa-Stony Brook and CERN–Holland–Lancaster–Manchester data are consistent with this. In both cases the central particle is a pion. Similar measurements for a central $\bar{p}$ may be useful in determining the nature of the secondary contributions responsible for the rise in $pp \to \bar{p}X$.

c and d in the Fragmentation Regions of a and b

Now f_{cd} is given by:



Again one expects zero correlation for a factorising Pomeron. Unfortunately there are no ISR data yet. But the 20 GeV data on $pp \to \pi^-\pi^-X$ seem to be consistent with this [17], although at this energy there is not very much space available for double fragmentation. Note that there is no secondary trajectory contribution here, as the 4-body system is exotic.

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