Vol. B5 (1974) ACTA PHYSICA POLONICA No 1

DUALITY AND REGGE ANALYSIS OF INCLUSIVE REACTIONS. PART II*

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(Presented at the XIII Cracow School of Theoretical Physics, Zakopane, June 1–12, 1973)

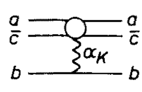
Recent developments in triple Regge analysis of inclusive spectra are presented, with particular emphasis on application of the finite mass sum rules.

Triple-Regge behaviour and application to data

When discussing the fragmentation region (or single Regge region) we parametrised the cross-section as

$$f = \frac{d^2\sigma}{dtd\left(\frac{M^2}{s}\right)} = \sum_k F_k\left(t, \frac{s}{M^2}\right) s^{\alpha_k(0)-1}, \tag{1}$$

where F_k denotes the "blob"



As far as phenomenology was concerned F_k was treated just as a normal factorisable residue function, no comment being made about its dependence on either t or s/M^2. In this case we had considered M^2 to be large; if we go to the particular region of fragmentation phase space where, in addition, s/M^2 is large then we can learn what the dependence on s/M^2 should be in this region.

* To speed up publication, proofs of this paper were read by K. Fiałkowski and A. Staruszkiewicz.

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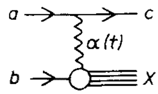
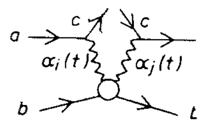
If we begin by assuming s/M^2 large and $s \gg t$ and keeping M^2 moderate then we have essentially the near forward pseudo-two-body process $a+b \to c+X$ for which the appropriate graph is



corresponding to exchange of the leading Regge trajectories in the t-channel.

On squaring and summing over all states in X we obtain



summed over i and j

$$f = \frac{d^2\sigma}{dtd\left(\frac{M^2}{s}\right)} = \frac{1}{s}\sum_{ij} \beta^i_{a\bar{c}}\xi_i(t)\beta^j_{a\bar{c}}\xi^*_j s^{\alpha_i(t)+\alpha_j(t)} \operatorname{Im}\{A_{ib\to jb}(M^2, t)\}. \tag{2}$$

The quantity $A_{ib\to jb}(M^2, t)$ is usually referred to as the forward Reggeon-particle scattering amplitude; strictly speaking, it is the analytic continuation of the maximum helicity flip amplitude in the centre-of-mass of the crossed channel $b\bar{b}\to\bar{\alpha}_i\alpha_j$. $\beta^i_{a\bar{c}}$ is the coupling of the vertex $(a\bar{c}\alpha_i)$ and $\xi_i(t)$ is the signature factor $(\tau_k + e^{-i\pi\alpha(t)}) \times \frac{1}{\sqrt{\pi}}\Gamma(1-\alpha(t))$ (this normalisation we choose now, for use later on).

When we allow M^2 to become large (but still keeping s/M^2 large) the asymptotic behaviour of $A_{ib\to jb}$ is controlled by exchange of Regge poles $\alpha_k(0)$ which can couple to $b\bar{b}$ and $\alpha_i\bar{\alpha}_j$

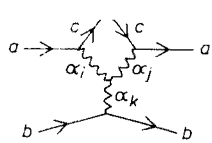
$$\operatorname{Im}\{A_{ib\to jb}(M^2, t)\} = \beta^k_{b\bar{b}} g^k_{ij}(t)\,(M^2)^{\alpha_k(0)-\alpha_i(t)-\alpha_j(t)}. \tag{3}$$

(The $-$ $[\alpha_i+\alpha_j]$ term in the exponent is connected to the fact that $A_{ib\to jb}$ corresponds to maximal helicity flip of the Reggeon legs.) Combining (2) and (3)

$$\frac{d^2\sigma}{dtd\left(\frac{M^2}{s}\right)} = \sum_{ijk} G_{ijk}(t)\left(\frac{M^2}{s}\right)^{\alpha_k(0)-\alpha_i(t)-\alpha_j(t)} s^{\alpha_k(0)-1}, \tag{4}$$

where $G_{ijk}(t) = \beta^i_{a\bar{c}}\beta^j_{a\bar{c}}\xi_i(t)\xi^*_j(t)g^k_{ij}(t)\,\beta^k_{b\bar{b}}$.

Expression (4) is called the triple-Regge formula since it describes the graph:



Comparing with (1) we see that the fragmentation residue has been expanded as

$$F_k\left(t, \frac{M^2}{s}\right) = \sum_{ij} G_{ijk}(t)\left(\frac{M^2}{s}\right)^{\alpha_k(0)-\alpha_i(t)-\alpha_j(t)} \tag{5}$$

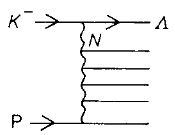
and we can now see the necessity of $-\ [\alpha_i-\alpha_j]$ term in (3) in order to ensure consistency with the fact that F_k has to be a function of the ratio M^2/s.

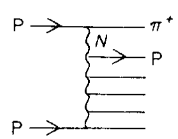
Now let us turn to direct application of equation (4) to experimental data. Immediately one problem arises, namely that it is very hard to have simultaneously both M^2 and s/M^2 large (*i. e.* both $\geqslant 5$ say). As we shall see later on the insistence on large M^2 can be relaxed provided we have some reliable procedure for "averaging" the rather fluctuating behaviour in this region.

Let us consider cases where α_i, α_j are not Pomerons. Almost all triple-Regge analyses proceed by taking M^2 above the resonance region and assuming then that the dominant exchange in the $b\bar{b}$ channel is the Pomeron *i. e.* $\alpha_k = 1$. This is simply because most analyses treat data only at one energy — or perhaps a small range in energy. By studying the M^2 dependence one can then hope to extract $\alpha_i(t)$ and compare it with the intercept of the trajectory allowed by the quantum numbers of the reaction.

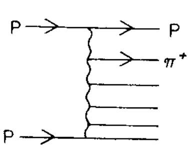
Rather than attempt any kind of comprehensive survey of triple Regge phenomenology let us briefly mention just one or two recent results.

The Rutherford-Saclay-Ecole Polytechnique Collaboration have looked at $K^-\mathrm{p} \to \overline{K^0}+X$ and $K^-\mathrm{p} \to \Lambda+X$ at 14 GeV/c [1]. In this way they study the ϱ, K^* and nucleon exchanges. Indeed the trajectories obtained from the M^2 dependence are not at all unreasonable. Their actual choice for $\alpha_k(0)$ was below 1 which is reasonable since the missing mass is not exotic but the qualitative agreement with the expected trajectory in each case was good. The result for the nucleon trajectory obtained from $K^- \xrightarrow{\mathrm{p}} \Lambda$ is especially interesting when one compares with nucleon trajectory obtained from $\mathrm{p} \xrightarrow{\mathrm{p}} \pi^+$ whose intercept had always turned to be embarrassingly low, around $-$ 1.2. The difference for $\alpha_N(t)$ in the two cases has been suggested by Chan Hong-Mo to follow from the different kinematic situations. In one case a small negative value of t is close to $t_{\min}$, in the other relatively further away. If we think of a multiperipheral mechanism, then close to $t_{\min}$, the detected particle comes out at the top rung:





Such graphs are "true" contributions to the triple-Regge limit. However, in the second case when t is not close to $t_{\min}$, the π^+ would tend to come out lower down the ladder:



which is not a genuine contributor to the triple-Regge behaviour. This latter graph, since it involves Pomeron exchange, could easily obscure the contribution from the graph with genuine nucleon exchange and hence effectively lower the observed value of $\alpha(t)$.

Another nice example is $p \xrightarrow{\pi^+} \Delta^{++}$ which has been measured at CERN at 8 and 16 GeV [2]. For t not very small we expect ϱ-A_2 exchange in the t-channel. Since the experiment is done at two widely spaced energies, $\alpha_k(0)$ can be determined rather than assumed ($\alpha_k(0) = 1$ can be justified only in the case X exotic). Again one sees an effective $\alpha(t)$ which is reasonably close to the expected ϱ-A_2 trajectory with intercept $\frac{1}{2}$.

Triple Regge analyses have been carried out for processes involving Pomeron exchange but this topic is best dealt with after introducing the Finite Mass Sum Rules.

Finite Mass Sum Rules

In two-body scattering, the development of the ideas of duality began with the writing down of the Finite Energy Sum Rules. In the last Section we have been discussing the form of the amplitude for Reggeon-particle scattering and so it is natural to ask whether similar sum rules can be stated in this case where the "dispersing variable" is the missing mass M^2 rather than the incoming-energy s; hence the name finite mass sum rule (FMSR). Of course we need to consider only one special case where the momentum transfer is zero; since we are considering forward Reggeon-particle scattering. The procedure we follow is that of Kwieciński [3] and Einhorn *et al.* [4]. Again we begin by assuming s/M^2 large and $s \gg t$ so that the inclusive cross-section is given by Eq. (2) represented by the graph immediately before it.

The next step is to write down the large M^2 behaviour of $A_{ib\to jb}(M^2, t)$ which is Eq. (3).

To write a dispersion relation for $A_{ib\to jb}(M^2, t)$ we need to know its analytic properties. The assumption has to be made that these are the same as for forward two-body scattering. This is supported by analyses of the six-point function in dual resonance models. As in the two-body case, we need to consider both the right- and left-hand cuts of $A_{ib\to jb}(M^2, t)$. To do this it is first convenient to define an anti-symmetric variable ν (analogous to s-u in the two-body case) given by $\nu = p_b(p_a - p_c) = \frac{1}{2}(M^2 - t - m_b^2)$. *I. e.* $\nu \to -\nu$

as $(\alpha_i + b \to \alpha_j + b) \to (\bar{\alpha}_j + b \to \bar{\alpha}_i + b)$ or equivalently

as $(a + b \to c + X) \to (c + b \to a + X)$

The absorptive part of this gives the right-hand cut	The absorptive part of this gives the left-hand cut

The large ν behaviour is of course given by the triple-Regge behaviour and we get, taking contributions from left and right hand cuts

$$A(\nu, t) = \sum_k \beta^k_{b\bar{b}} g^k_{ij}(t) \nu^{\alpha_k(0) - \alpha_i(t) - \alpha_j(t)} [\tau_i \tau_j \tau_k + e^{-i\pi(\alpha_k(0) - \alpha_i - \alpha_j)}]. \tag{6}$$

To write down the FMSR one then simply has to take a combination of amplitudes which is guaranteed to be antisymmetric in ν:

$$\int_0^N d\nu\nu^n\{\operatorname{Im} A(\nu, t)+(-1)^{n+1} \operatorname{Im} A(-\nu, t)\} =$$

$$= \sum_k \beta_{b\bar{b}}^k g_{ij}^k(t)\,[1+(-1)^{n+1}\tau_i\tau_j\tau_k]\,\frac{N^{\alpha_k(0)-\alpha_i-\alpha_j+n+1}}{\alpha_k(0)-\alpha_i-\alpha_j+n+1} \tag{7}$$

i. e.

$$\int_0^N d\nu\nu^n[f(ab \to cX)+(-1)^{n+1}f(cb \to aX)] =$$

$$= \sum_{ijk} s^{\alpha_i+\alpha_j-1} G_{ijk}(t)\,[1+(-1)^{n+1}\tau_i\tau_j\tau_k]\,\frac{N^{\alpha_k(0)-\alpha_i-\alpha_j+n+1}}{\alpha_k(0)-\alpha_i-\alpha_j+n+1}\,. \tag{8}$$

If we take a combination which is symmetric in ν *i. e.* symmetric under "$s \leftrightarrow u$" for ab scattering then the contribution from residues of the nonsense wrong signature fixed poles has to be included:

$$\int_0^N d\nu\nu^n[f(ab \to cX)+(-1)^n f(cb \to aX)] =$$

$$= \sum_{ijk} s^{\alpha_i+\alpha_j-1}\left\{H_{ij}^{(n)}(t)+G_{ijk}(t)\,[1+(-1)^n\tau_i\tau_j\tau_k]\,\frac{N^{\alpha_k(0)-\alpha_i-\alpha_j+n+1}}{\alpha_k(0)-\alpha_i-\alpha_j+n+1}\right\}. \tag{9}$$

This is just the same as for two-body scattering. The fixed poles arise from the presence of a non-zero third double-spectral function which is symmetric under "$s \leftrightarrow u$". In the language of Veneziano 4pt functions, the terms $V(s, t)$, $V(u, t)$ give the usual Regge pole behaviour while $V(s, u)$ gives rise to the fixed pole behaviour. However, if we can arrange matters such that either s or u for the ab scattering is exotic then $V(s, u)$ vanishes and hence the fixed pole contribution vanishes for the resonant part of the amplitude.

The "odd" sum rule (8) is the analogue of the "good" FESR while (9) is the analogue of the Schwarz sum rule.

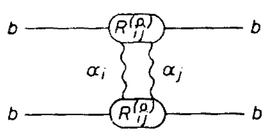
Applications of FMSR

To try to estimate the triple Regge couplings by applying the triple Regge expression (4) directly to data is difficult since its validity really requires both s/M^2 and M^2 large. The low M^2 data can now be used to feed into the l. h. s. of the FMSR and, with a suitable cut-off N, the triple-Regge couplings obtained. N has to be chosen above the resonances — say $M^2 \simeq 4$ GeV^2 but the error obtained on the estimate of the couplings will naturally be $0(M^2{}_{\text{cut off}}/s)$ which is not small for $p_{\text{lab}} < 25$ GeV/c.

Next, what about duality for Reggeon-particle scattering? The FMSR provide a method for testing whether the Harari-Freund two-component duality holds for αb scattering. According to the seven component duality scheme presented in the earlier lectures we saw that for the case of a normal Reggeon for $\alpha(t)$, the usual 2-component picture is expected to hold up; *i. e.* resonances in the missing mass dual to Regge in the $b\bar{b}$ channel, non-resonating background dual to the Pomeron. In fact we shall proceed assuming this to be the case — the assumption being justified *a posteriori*. On the other hand we also saw previously that when $\alpha(t)$ is a Pomeron, the duality situation is controversial. Einhorn *et. al.* [4] have suggested that for Pomeron-particle scattering the resonances are dual to the Pomeron. Therefore we make no duality assumption for diffraction dissociation but rather try to resolve the question by applying FMSR to the data.

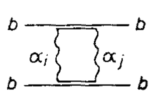
Related to this is the controversy over the smallness or otherwise of the triple-Pomeron coupling. Ther eare theoretical arguments for believing that when $t = 0, g_{PPP}(t) = 0$. It has been conjectured by many people that the triple-Pomeron vertex would therefore be small even for $t \neq 0$. This can be precisely tested by evaluating the FMSR for diffractive processes.

Next consider the even moment sum rule (9). In ordinary two-body scattering the fixed pole contributions have no physical interpretation and only tend to be a nuisance. However, the fixed pole contributions to Reggeon-particle scattering have a direct physical significance, first pointed out by Abarbanel [5]. Writing $H_{ij}^{(n)}(t) = \beta^{i}_{a\bar{c}}\xi_i(t)\,\beta^{j}_{a\bar{c}}\xi_j(t)\,R_{ij}^{(n)}(t)$ then $R_{ij}^{(0)}(t)$ is precisely the vertex for the cut generated by the exchange of α_i and α_j, according to the Gribov calculus. The cut is generated by convoluting $R_{ij}^{(0)}(t)$ with itself:



Notice that since the momentum transfers for α_i and α_j are the same, we can calculate the cut contribution only in the forward direction. At least, in principle, we can use data on inclusive reactions and *via* the FMSR calculate the two-Reggeon contribution to a total cross-section.

Essentially the only other prescription for calculating Regge-Regge cuts is the eikonal procedure. In this model we have a sum over low mass resonances (including the incoming particle itself) instead of the fixed pole contribution *i. e.*



Even if the eikonal prescription is theoretically not well justified, almost all cut estimates are, in one way or another, based on it. What is therefore interesting is to compare an estimate based on the Gribov calculus with that based on the eikonal model. This amounts to comparing, in Eq. (9) with $n = 0$, the size of $H_{ij}^{(0)}$ and the low mass integral on the l. h. s. which can be done using the $n = 1$ sum rule (8) to evaluate the Regge terms.

A special form of a FMSR is one in which the leading Regge terms, for one reason or another, are absent. This is called a superconvergent FMSR. The low mass integral which is essentially a sum of resonance production reactions, is then equal to zero, thus providing a relation between such quasi-two-body processes.

Let us consider some specific examples of applications of FMSR which we have been discussing.

Superconvergent relations

To obtain a superconvergence relation one should arrange things to ensure exotic quantum numbers in the $b\bar{b}$ channel and therefore, to leading order, the Regge terms in the FMSR vanish.

Consider $\varrho\pi$ elastic scattering [6]; by taking the combination

$$A_{\varrho^-\pi^+} + A_{\varrho^+\pi^+} - 2A_{\varrho^0\pi^+} \tag{10}$$

we guarantee pure $I = 2$ in the crossed ($\pi\pi \to \varrho\varrho$) channel. For example, these amplitudes could be extracted from inclusive data by looking at the natural parity exchange contributions of

$$\mathrm{p} \xrightarrow{\pi^+} \Delta^{++}, \quad \mathrm{p} \xrightarrow{\pi^+} \Delta^0, \quad \mathrm{p} \xrightarrow{\pi^+} \Delta^+. \tag{11}$$

Using isospin to relate the $\varrho\mathrm{p}\Delta$ vertices, and taking $n = 1$ for the odd sum rule (8) we get

$$\int_0^N \nu \frac{dM^2}{2} \left\{ \frac{1}{3} f(\mathrm{p} \xrightarrow{\pi^+} \Delta^{++}) + f(\mathrm{p} \xrightarrow{\pi^+} \Delta^0) - f(\mathrm{p} \xrightarrow{\pi^+} \Delta^+) \right\} = 0. \tag{12}$$

Next we take the resonance contributions only to the missing mass, the assumed resonances in the $\varrho\pi$ channel being π, ω, A_2. Using isospin we can relate all to the reactions $\pi^+\mathrm{p} \to (\text{resonance})^0\, \Delta^{++}$ and obtain finally:

$$-t \frac{d\sigma_\varrho}{dt}(\pi^+\mathrm{p} \to \pi^0\Delta^{++}) - (m_\omega^2 - t - m_\pi^2) \frac{d\sigma_\varrho}{dt}(\pi^+\mathrm{p} \to \omega\Delta^{++}) +$$

$$+ (m_{A_2}^2 - t - m_\pi^2) \frac{d\sigma_\varrho}{dt}(\pi^+\mathrm{p} \to A_2^0\Delta^{++}) = 0 \tag{13}$$

(the ϱ subscript is to remind that we are taking only the natural parity exchange contribution). Similarly one gets

$$-t \frac{d\sigma_\varrho}{dt}(\pi^-\mathrm{p} \to \pi^0\mathrm{n}) - (m_\omega^2 - t - m_\pi^2) \frac{d\sigma_\varrho}{dt}(\pi^-\mathrm{p} \to \omega\mathrm{n}) +$$

$$+ (m_{A_2}^2 - t - m_\pi^2) \frac{d\sigma_\varrho}{dt}(\pi^-\mathrm{p} \to A_2^0 n) = 0. \tag{14}$$

Before attempting to find whether data support this relation, it is worth pointing out the novelty of connecting reactions involving particles which belong to completely different

SU(3) classifications. The "elastic" reaction is strongly damped at small t, hence we have approximately

$$\frac{d\sigma_\varrho}{dt}(\pi^- p \to A_2^0 n) \approx \frac{m_\omega^2}{m_{A_2}^2}\frac{d\sigma_\varrho}{dt}(\pi^- p \to \omega n) \approx \frac{1}{3}\frac{d\sigma_\varrho}{dt}(\pi^- p \to \omega n). \tag{15}$$

From density-matrix analysis of ω production we know that natural parity exchange accounts for about 60 % of the reaction. Equation (15) predicts the natural parity contribution to A_2^0 production and when we compare this with the measured differential cross-section we find the prediction accounts for about only 30 %. Thus we note an

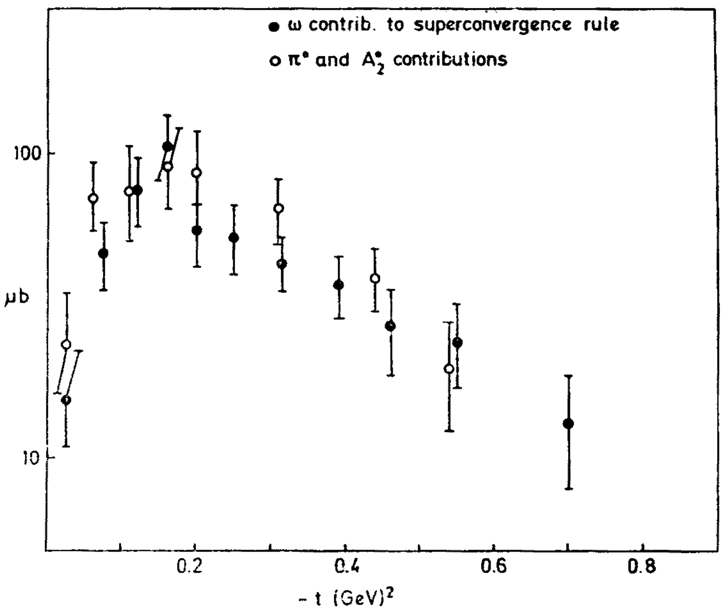


Fig. 1. Experimental values of contributions to the superconvergence relation (14) evaluated from the data

example of a systematics which we shall understand later on; high mass resonances are produced *via* an increasing proportion of unnatural parity exchange.

Since density matrix elements for A_2^0 have not been measured we have to do a little juggling before testing equation (14). We have $\varrho_{+'+'} + \varrho_{-'-'}$ for A_2^- production and using factorisation we can write

$$\frac{d\sigma_\varrho}{dt}(\pi^- p \to A_2^0 n) = \frac{d\sigma_{\varrho,f}}{dt}(\pi^- p \to A_2^- p)\,\frac{\dfrac{d\sigma_\varrho}{dt}(\pi^- p \to \pi^0 n)}{\dfrac{d\sigma_{\varrho,f}}{dt}(\pi^- p \to \pi^- p)}. \tag{16}$$

This last quantity involves using a model for πN scattering (we used Barger and Phillips 5 pole fit). At last we can test equation (14), the results being shown in Fig. 1 for $p_{lab} \simeq 7$ GeV.

Regge-Regge cuts

When the exchange channel in a reaction is exotic *e. g.* $I_t = 2$, the only contributions are those from cuts and so it is interesting to calculate the Regge-Regge cut in such a case.

If we go back to our superconvergent relation (12) but re-write it for the $n = 0$ even moment sum-rule instead then we obtain

$$\frac{s}{2}\left\{\frac{d\sigma_\varrho}{dt}(\pi^- p \to \pi^0 n) - \frac{d\sigma_\varrho}{dt}(\pi^- p \to \omega n) + \frac{d\sigma_\varrho}{dt}(\pi^- p \to A_2^0 n)\right\} =$$

$$= s^{2\alpha_\varrho(t)-1}|\xi(t)|^2(\beta_{p\pi}^\varrho)^2 R_{\varrho\varrho}^{I_t=2}(t), \tag{17}$$

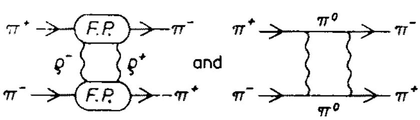
where $R_{\varrho\varrho}^{I_t=2}(t)$ is the fixed pole contribution with isospin 2 in the $\pi\pi$ or $\varrho\varrho$ channel. In fact using equation (14), the odd sum rule, we can write the R in terms of just the π^0 and ω production cross-sections

$$R_{\varrho\varrho}^{I_t=2}(t) = \frac{\pi}{2}[\beta_{\pi^+\pi^0}^\varrho]^2 \frac{(m_A^2 - m_\pi^2)\dfrac{d\sigma}{dt}(\pi^- p \to \pi^0 n) - (m_A^2 - m_\omega^2)\dfrac{d\sigma_\varrho}{dt}(\pi^- p \to \omega n)}{(m_A^2 - t^2 - m_\pi^2)\dfrac{d\sigma}{dt}(\pi^- p \to \pi^0 n)}. \tag{18}$$

The eikonal prescription in which all resonances in the intermediate state are inserted would be identical to (18). If only the elastic *i. e.* π^0 states are included, *i. e.* the Born term is just

$$B_{\varrho\varrho}^{I_t=2}(t) = \frac{\pi}{2}[\beta_{\pi^+\pi^0}^\varrho]^2 \tag{19}$$

convolution of (18), (19) gives



Insertion of the experimental resonance production cross-sections gives a value for the cut in the Gribov case to be roughly half that for π^0 eikonal prescription.

The most interesting cuts to try to estimate are those involving Pomeron-exchange. Later on we shall outline some of the problems involved in attempting to calculate the $P-P$ cut, when we discuss Pomeron-particle scattering.

Semi-local duality for Reggeon-particle scattering

We are, one by one, generalising features of duality for two-body scattering to the case of Reggeon-particle scattering. The next concept we carry over is that of semi-local duality [7]. If we write FESR for the resonance part of the two-body scattering amplitude

$$\int_0^N \mathrm{Im}\, f_{\mathrm{RES}}(\nu, t)\nu^n d\nu = \sum \beta(t)\frac{N^{\alpha+n+1}}{\alpha+n+1} \tag{20}$$

then we assume that resonance fluctuations in Im $f_{\text{RES}}(\nu, t)$ average out over relatively small energy intervals *i. e.*

$$\int_{N_1}^{N_2} \operatorname{Im} f_{\text{RES}}(\nu, t)\nu^n d\nu = \left[\sum \beta(t) \frac{N^{\alpha+n+1}}{\alpha+n+1}\right]_{N=N_1}^{N=N_2} \tag{21}$$

or

$$\langle \operatorname{Im} f_{\text{RES}}(\nu, t)\rangle = \operatorname{Im} f_{\text{REGGE}}(\nu, t). \tag{22}$$

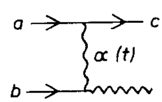
This can be seen from a plot of Im $A'^{(-)}(\nu, 0)$ [7] for πN as a function of ν where the contributions from various resonances fluctuate strongly but the amplitude is given in the mean by the smooth ϱ Regge pole term.

Perhaps the most striking demonstration of semi-local duality was when Dolen, Horn and Schmid [7] demonstrated the additive contributions of just two or three resonances to $\int \operatorname{Im} \nu B^{(-)}(\nu, t)\, dt$ could easily produce the famous W. S. zero at $\alpha(t) = 0$ at $t \simeq -0.5$ characteristic of ϱ exchange.

In the case of inclusive reactions we are looking only at the forward scattering of Reggeon-particle. If semi-local duality holds in this case then we can expect the leading Regge exchange in the $b\bar{b}$ channel to interpolate the resonances in the missing mass in an average sense; *i. e.* if no fixed poles are present

$$\left\langle \frac{d\sigma}{dt}(a+b \to c+\text{Resonances})\right\rangle \sim (\nu_{\text{RES}})^{\alpha_k(0)-2\alpha_i(t)}. \tag{23}$$

From our experience in the case of two-body scattering the averaging is to be done over a range of about 1 GeV^2 in M^2. For the quasi-two body reaction



(23) tells how the cross-section, at fixed s and t, varies with the mass of the produced resonance. We essentially have a four-point function where in addition to knowing the dependence on s and t, we know the dependence on the mass of one of the external legs.

In the situations we shall now consider we shall assume the validity of two-component duality, *i. e.* resonances in missing mass are dual to the leading Regge exchange (meson, with $\alpha_k = \frac{1}{2}$) and background dual to Pomeron ($a_k = 1$). So we can write

$$\int_0^N d\nu\nu^n \frac{d\sigma}{dM^2 dt}(a+b \to c+\text{Res}) \sim g_{\alpha\alpha}^M(t) s^{2\alpha(t)-2} N^{\frac{1}{2}-2\alpha(t)+n+1}. \tag{24}$$

In particular if there are no fixed poles present

$$\left\langle \frac{d\sigma}{dt}(a+b \to c+\text{Res})\right\rangle \sim (\nu_{\text{RES}})^{\frac{1}{2}-2\alpha(t)}. \tag{25}$$

This use of Harari-Freund duality greatly reduces the number of triple-Regge terms since we need never consider Pomeron exchange in the $b\bar{b}$ channel. Also since we are isolating one trajectory $\alpha(t)$, the number of terms is reduced. In fact we need not consider interference terms since always we look at exchange degenerate pairs like $\pi - B$ or $\omega - f$.

Looking at resonance production cross-sections offers several advantages. Very often there is no data on the general inclusive reaction *e. g.* $\pi^- p \to n +$anything but there is

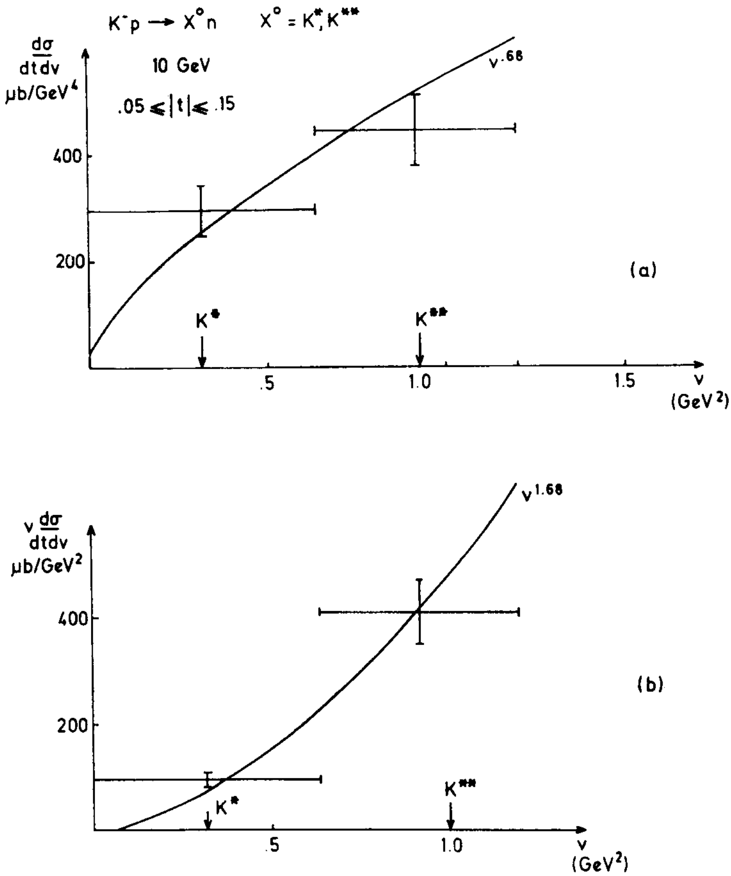


Fig. 2. Experimental $v^n\,(d\sigma_\pi/dt)\,(K^- p \to (K^*, K^{**})n)$ at 10 GeV/c for $0.05 \leqslant |t| \leqslant 0.15$ compared with the π-exchange Regge contribution. (a) $n = 0$, (b) $n = 1$

excellent data on the quasi-two-body reactions *e. g.* $\pi^- p \to \varrho n$ or fn or gn. Again for the FMSR to be valid, s must be much larger than M^2 and since M^2 is in the resonance region this is automatically guaranteed, even if s is not too large.

Incidentally we are assuming everywhere that we really have genuine Regge pole-particle scattering. The FMSR are of course not valid if cuts were also important in the t-channel. To avoid such problems we try to choose examples where such cuts are thought to be rather small. For many cases there is evidence that poles are dominating the $a\bar{c}$ channel from measurements of the decay density-matrix elements.

(*i*) π-exchange

To test the above formula we should isolate a particular exchange trajectory $\alpha(t)$. For the reactions

$$\pi^- p \to (\varrho^0, f, g)n, \tag{26a}$$

$$K^- p \to (K^{*0}, K^{**0})n, \tag{26b}$$

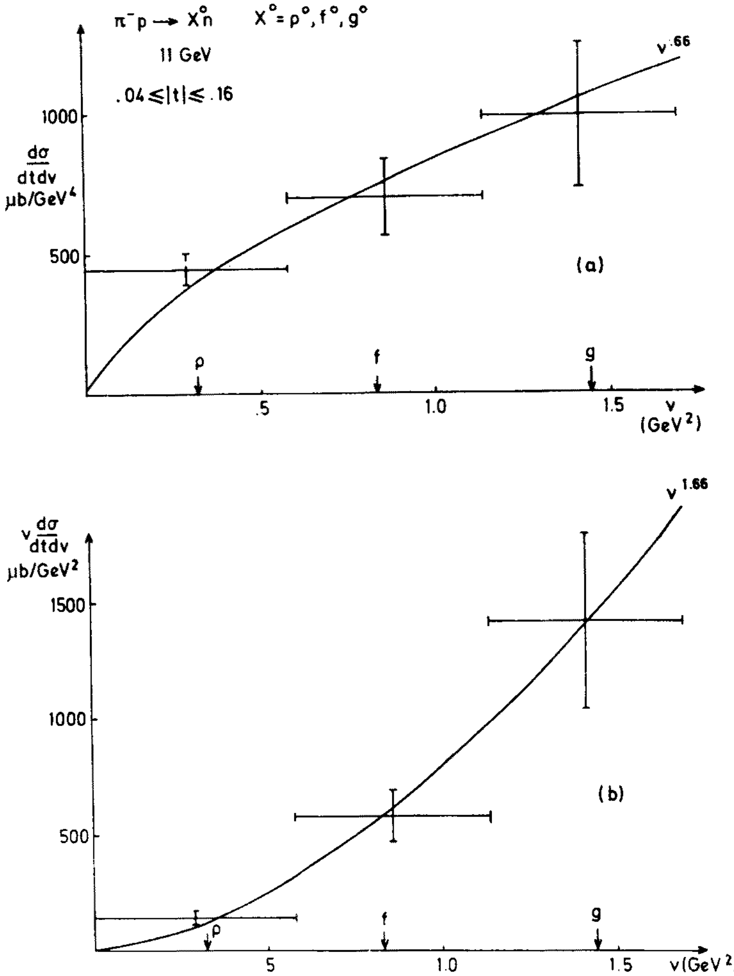


Fig. 3. Experimental $v^n(d\sigma_\pi/dt)$ $(\pi^- \to (\varrho^0, f^0, g^0)n)$ at 11 GeV/c for $0.04 \leqslant |t| \leqslant 0.16$ compared with the π-exchange Regge contribution. (a) $n = 0$, (b) $n = 1$

the small t-region is known to be dominated by π-exchange, (energy dependence, big ϱ_{00}). For (26a) we are studying $\pi^+\pi^-$ scattering and for (26b) π^+K^-, in either case the u-channel for the Reggeon-particle scattering is exotic, hence no fixed poles enter since we are looking only at s-channel resonances. So we may consider $n = 0$ as well as $n = 1$ in

$$\left\langle \nu_{\text{RES}}^{n} \frac{d\sigma}{dt} (\pi^{-}(K^{-})\text{p} \to X_{\text{RES}}^{0}\text{n}) \right\rangle = f(t) s^{2\alpha_\pi(t)-2} (\nu_{\text{RES}})^{\frac{1}{2}-2\alpha_\pi(t)+n}. \tag{27}$$

The experimental data for the l. h. s. for both reactions and for $n = 0,1$ compared with the ν behaviour given by r. h. s. of (27) are shown in Fig. 2 and Fig. 3.

(*ii*) Natural parity exchange

We may consider $f-\omega$ exchange in the t-channel for the reactions

$$\pi^{-}\text{p} \to (\pi^{-}, \varrho^{-}, A_1^{-}, A_2^{-}, g^{-})\text{p},$$

$$K^{-}\text{p} \to (K^{-}, K^{*-}, Q^{-}, K^{**-})\text{p}. \tag{28}$$

To isolate the isospin zero exchange, the correct combinations with the charge-exchange reactions were formed. Estimates for the non-diffractive Q^{-} cross-section used the observed cross-over in Q, $\bar{Q}$ production. In contrast to the previous case, neither the s nor u channel for the Reggeon-particle scattering is exotic, as only $n = 1$ can be compared.

The only essential difference between (*i*) and (*ii*) is the value of the intercept of the particular trajectory — this makes the crucial difference when we come to consider

(*iii*) Ratio of unnatural/natural parity exchange

In (*i*) we had $\pi-B$ (unnatural parity) exchange, while in (*ii*) we had $f-\omega$ (natural parity) exchange. Consequently from (24) we get the ratio

$$\frac{\left\langle \nu \dfrac{d\sigma_{\pi,B}}{dt} [\pi^{-}(K^{-}) \to X^{0}\text{n}] \right\rangle}{\left\langle \nu \dfrac{d\sigma_{f,\omega}}{dt} [\pi^{-}(K^{-}) \to X^{-}\text{p}] \right\rangle} \sim (\nu_{\text{RES}}^{2})^{2\alpha_{f,\omega}(t) - 2\alpha_{\pi,B}(t)} \sim M^{2}. \tag{29}$$

In other words, the cross-section, at fixed s and t, should show the unnatural parity contribution rapidly increasing with respect to the natural parity contribution as the mass of the produced resonance increases. This is the explanation behind the strong decreasing proportion of ϱ-exchange in going from $\pi^{-}\text{p} \to \omega\text{n}$ to $\pi^{-}\text{p} \to A_2^0\text{n}$ that we saw in the Section on superconvergent relations. In fact we can see how it drops from almost 70% to ~ 40% since semi-local duality predicts

$$\frac{\sigma_{\text{unat}}(\pi^{-}\text{p} \to A_2^0\text{n})}{\sigma_{\text{nat}}(\pi^{-}\text{p} \to A_2^0\text{n})} \sim \frac{m_{A_2}^2}{m_\omega^2} \times \frac{\sigma_{\text{unat}}(\pi^{-}\text{p} \to \omega\text{n})}{\sigma_{\text{nat}}(\pi^{-}\text{p} \to \omega\text{n})} = 3 \times \frac{\frac{1}{3}}{\frac{2}{3}} = 1.5. \tag{30}$$

Hence only about $\frac{2}{5}$ of A_2^0 production should be *via* ϱ-exchange.

We can directly test the prediction (30) by plotting the l. h. s. as a function of ν, both for π^{-} and K^{-} induced reactions. In Fig. 4 we show the situation for 8 GeV/c π^{-}p reaction, the first bin is the ratio

$$\frac{d\sigma_\pi}{dt}(\pi^- p \to \varrho^0 n) \Bigg/ \left[\frac{d\sigma_f}{dt}(\pi^- p \to \pi^- p) + \frac{d\sigma_\omega}{dt}(\pi^- p \to \varrho^- p)\right],$$

the second bin is

$$\frac{d\sigma_\pi}{dt}(\pi^- p \to f n) \Bigg/ \frac{d\sigma_f}{dt}(\pi^- p \to A_2^- p),$$

the third bin is

$$\frac{d\sigma_\pi}{dt}(\pi^- p \to g^0 n) \Bigg/ \frac{d\sigma_\omega}{dt}(\pi^- p \to g^- p).$$

Fig. 5 shows the similar plot for K^-p reaction at 10 GeV/c, where again the ratio unnatural/natural exchange is seen to increase with ν with a rate consistent with ν^2_{Res}.

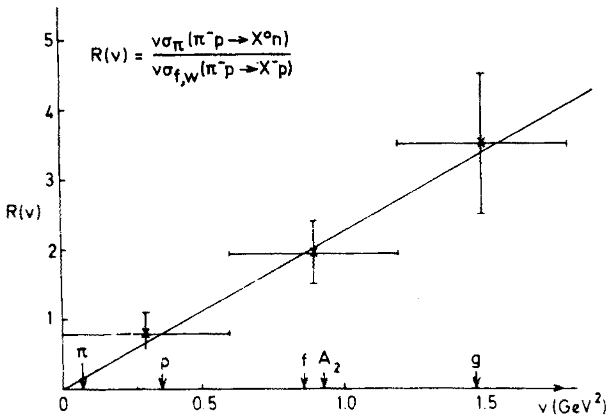


Fig. 4. Ratio of π/f–ω exchange to the reaction $\pi^-N \to XN$ at 8 GeV/c for $0.05 \le |t| \le 0.25$

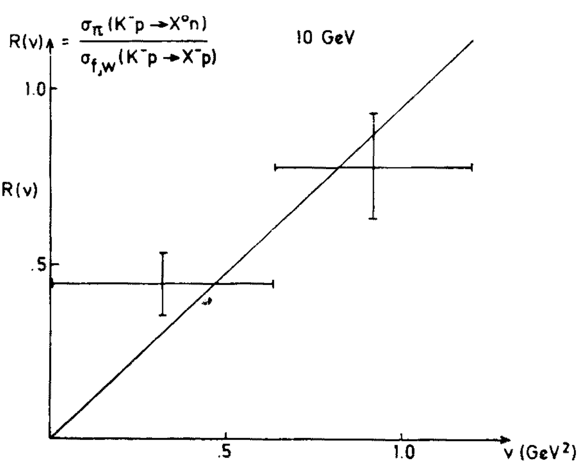


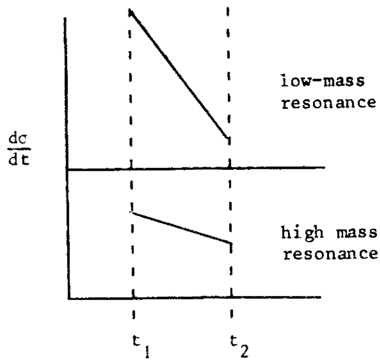
Fig. 5. Ratio of π/f–ω exchange to the reaction $K^-N \to XN$ at 10 GeV/c for $0 \le |t| \le 0.2$

(*iv*) Antishrinkage

We can express the t-dependence in Eq. (25) as

$$\left\langle \frac{d\sigma}{dt}(a+b \to c+\text{Res}) \right\rangle \sim e^{-[2\alpha' \log \nu_{\text{RES}}]t} \tag{31}$$

i. e. we obtain a logarithmic antishrinkage at the resonance mass increases. This means that higher mass resonances are expected to be less peripherally produced:



This means that if we calculate the ratio

$$\frac{\left(\frac{d\sigma}{dt}\right)_{t=t_2}(a+b \to c+\text{Res})}{\left(\frac{d\sigma}{dt}\right)_{t=t_1}(a+b \to c+\text{Res})} = R(\nu_{\text{RES}}),$$

then $R(\nu_{\text{RES}})$ should be increasing like $\nu_{\text{RES}}^{-2\alpha'(t_1-t_2)}$. There is some evidence in favour of this antishrinkage. In Fig. 6 the ratio $R(\nu_{\text{RES}})$ is plotted for $t_1 = -0.2$, $t_2 = -0.6$ for the 10 GeV K^-p data, the two bins being $K^{*-}+$ non diffractive K^- and K^{**-} + non-diffractive Q^-. Similar results are obtained for the π^-p reactions but now the f and ω exchanges must be separately treated.

Note that we are not discussing diffractive processes here. There is certainly antishrinkage seen in such reactions whose behaviour is much stronger than logarithmic and whose origin is far from obvious.

(*v*) Background

We can likewise write an expression analogous to (25) but for the background component in missing mass *i. e.*

$$\left\langle \frac{d\sigma_\alpha}{dt}\left(a+b \to c+\begin{matrix}\text{non-resonating}\\ \text{background}\end{matrix}\right)\right\rangle \sim (M^2)^{1-2\alpha(t)}. \tag{32}$$

In principle this equation could be used to separate the resonance and background components in the large M^2 regions. It seems that (32) gives a behaviour consistent with the data although any hard conclusions are difficult to make.

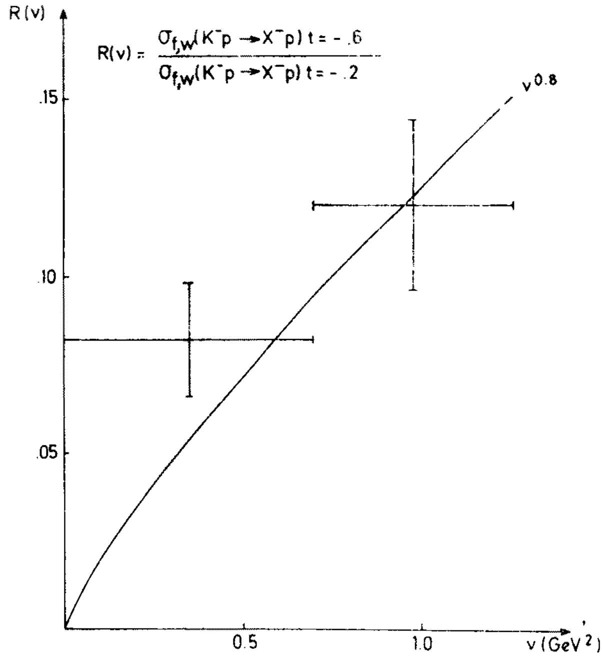
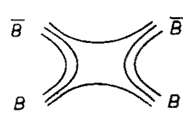


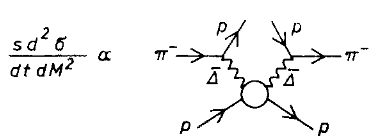
Fig. 6. Ratio of f-ω exchange contributions to the reaction $K^-p \to X^-p$ at 10 GeV/c at two values of t, -2.0 and -0.6 compared with the f-ω exchange Regge term

(*vi*) Baryon-exchange [8]

So far we have looked only at meson exchange in the t-channel; we now turn to the case where $\alpha(t)$ is an anti-baryon and we can consider the duality properties of baryon--antibaryon annihilation. There is a well-known inconsistency in the case of meson resonances formed by such annihilation for if we require the s-channel to correspond to a quark-antiquark pair then the t-channel has to be exotic:



However, all the established meson-resonances lie below $N\bar{N}$ threshold and so the prediction cannot be tested by direct annihilation experiment. The trick is to let anti-baryon be a Reggeon — whose $(\text{mass})^2$ is then negative — then mesons of any mass can be produced in the direct channel. For example consider $\pi^- p \to p_{\text{fast-forward}} + X^-$ which is assumed to proceed *via* Δ exchange in the u-channel:



i. e. α Reggeised antibaryon-baryon scattering in the forward direction. Separating the missing mass spectrum into resonance and non-resonating background, we expect from the semi-local duality considerations of the previous Section:

$$\left\langle \frac{d\sigma}{du}(\pi^- p \to p + (\text{meson resonance})) \right\rangle \sim (M^2_{\text{RES}})^{\alpha_k(0) - 2\alpha_\Delta(u)},$$

$$\left\langle \frac{d\sigma}{du}(\pi^- p \to p + \text{background}) \right\rangle \sim (M^2_{\text{RES}})^{1 - 2\alpha_\Delta(u)}, \tag{33}$$

where $\alpha_k(0)$ is the intercept of the trajectory to which the meson-resonances are dual.

The above reaction has been studied at 16 GeV/*c* where the resonances observed were π^-, ϱ^-, A_1^-, A_2^-, g^- and in the region $u \sim -0.2$ where we expect $\alpha_\Delta \simeq 0$. The experimentalists carried out a resonance-background separation so one can check both parts of

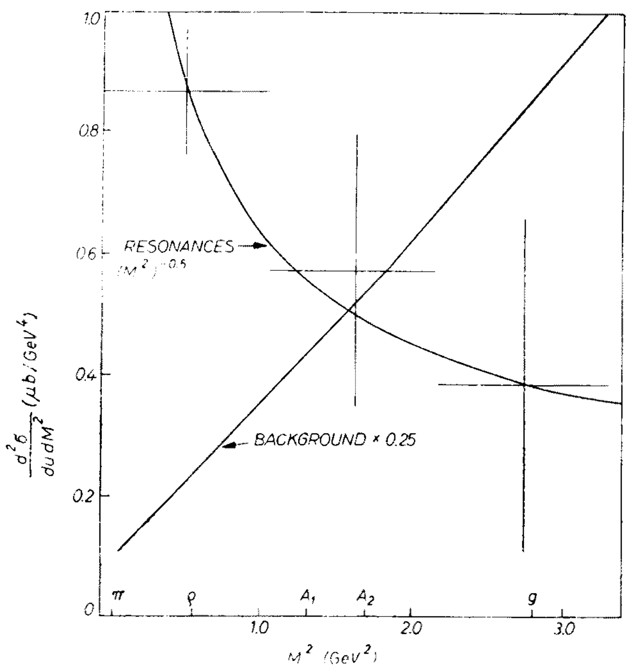


Fig. 7. Resonance and background cross-sections for $\pi^- p \to pX^-$ at 16 GeV/*c*. The resonance curve corresponds to $\alpha_R(0) = -0.5$ while the background requires $\alpha_R(0) = 1$. $u = -0.2$

Eq. (39). Fig. 7 shows the M^2 distribution for each part. The background in fact looks linear in M^2 — which is consistent with (33) when $\alpha_\Delta = 0$. Actually we would expect, on the duality argument basis, some background contribution from meson exchange corresponding to exotic resonance in the missing mass channel. This could well be present since our phenomenology will not be very sensitive to such a contribution.

The resonance cross-sections are divided into three bins corresponding to (π^-, ϱ^-), (A_1^-, A_2^-), g. The striking feature is the fall of the resonance cross-section with M^2, remembering that a normal Regge intercept of $\sim \frac{1}{2}$ would give a rise with M^2. In fact a fit with $(M^2)^{\alpha_k(0)}$ gives $\alpha_k(0) = -0.5 \pm 0.3$. So the analysis seems to give support to the duality diagram suggestion of meson-resonances in baryon-antibaryon scattering being dual to exotic exchange.

These systematics are given further support by looking at

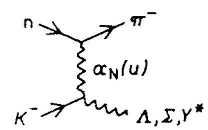
$$K^- p \to \Lambda + X^0 \quad \text{where} \quad X^0_{\mathrm{RES}} = \pi^0, \eta^0, \varrho^0, \omega, f, A^0_2 \tag{34}$$

and

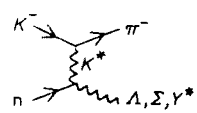
$$\pi^- p \to \Lambda + X^0 \quad \text{where} \quad X^0_{\mathrm{RES}} = K^0, K^{*0}, K^{**0} \tag{35}$$

which are proportional to $\bar{p}p$ and $\bar{Y}^*p$ forward scattering. A similar analysis to that above on data around 4.5 GeV gives $\alpha_k(0) = -1.4$ and -0.8.

We can also look at baryon-exchange but for baryon resonance production *e. g.*



In this case we have Reggeised baryon-meson scattering and so expect "normal" duality, *i. e.* $\alpha_k(0) = \frac{1}{2}$, which is consistent with the available data. On the other hand the same reaction proceeds in the forward direction *via* K^* exchange:



with again $\alpha_k(0) = \frac{1}{2}$. Therefore if we compare the reaction in the forward and backward directions we expect

$$\frac{\left\langle \frac{d\sigma}{dt} \text{ forward} \right\rangle}{\left\langle \frac{d\sigma}{dt} \text{ backward} \right\rangle} \sim (M^2)^{2\alpha_{K^*} - 2\alpha_N} \sim M^2.$$

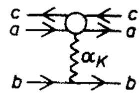
The data, even if it is rather sparse at 3 → 5 GeV, does tend to confirm this behaviour.

One of the nice features of studying Reggeon-particle is that one is not always tied down to a nucleon "target". For example, when studying π-exchange we were considering meson-meson forward scattering. If we wish to make further tests of duality diagrams we could study reactions (34) and (35) but instead in the forward direction. In one case, (34), we would have $K^* - \bar{K}$ scattering and in the other, (35), $K^* - \pi$ scattering. The meson resonances in the first case would be dual to the $\varphi - f$ trajectory ($\alpha_k(0) \sim 0$) and in the second case to the $\varrho - f$ trajectory. Data are not too good on these reactions, the energies being rather low. However, there does seem to be some indication for an $(M^2)^{\frac{1}{2}}$ difference in the two cases although the actual values of α_k in both cases are low.

Semi-local factorisation

In the entire fragmentation region $a \xrightarrow{b} c$ we have exchange of Regge poles $\alpha_k(0)$ in the $b\bar{b}$ channel *i. e.*

$$\frac{s\, d^2\sigma}{dt\, dM^2} =$$



As we saw very early on there is very good evidence to support the idea that both exchange of Pomeron and secondary trajectories factorise. If this is true not only when M^2 is large but also down to the resonance region of M^2 then we obtain strong relations connecting various resonance production processes. This is another consequence of duality being valid in the semilocal sense.

Still assuming the Harari-Freund view of duality we need consider just meson exchange in the $b\bar{b}$ channel and then the relations become particularly simple. For example, let us consider the two charge-exchange reactions

$$\text{(a) } \pi^- p \to X^0 n, \qquad \text{(b) } \gamma p \to X^+ n. \tag{36}$$

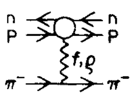
The resonances in the first case are

$$X^0 = (\pi^0, \eta), (\varrho^0, \omega), (f, A_2), g^0,$$

and in the second case

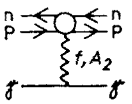
$$X^+ = \pi^+, \varrho^+, A_2^+, g^+.$$

The brackets indicate that resonances inside should be taken together in the semi-local averaging. In the first case we have



$$f(p \xrightarrow{\pi} n) = \left[F_f\left(t, \frac{M^2}{s}\right)\gamma_{f\pi\pi} + F_\varrho\left(t, \frac{M^2}{s}\right)\gamma_{\varrho\pi\pi}\right]\frac{1}{\sqrt{s}}$$

and in the second case



$$f(p \xrightarrow{\gamma} n) = \left[F_f\left(t, \frac{M^2}{s}\right)\gamma_{f\gamma\gamma} + F_{A_2}\left(t, \frac{M^2}{s}\right)\gamma_{A_2\gamma\gamma}\right]\frac{1}{\sqrt{s}}.$$

Using the usual exchange-degeneracy arguments, the ratio of the two cross-sections becomes $2\gamma_{M\pi\pi}/(\gamma_{f\gamma\gamma}+\gamma_{A_2\gamma\gamma}) = \gamma$, say, *i. e.* we get the prediction

$$\frac{\pi^- p \to (\pi^0, \eta)n}{\gamma p \to \pi^+ n} = \frac{\pi^- p \to (\varrho^0, \omega)n}{\gamma p \to \varrho^+ n} = \frac{\pi^- p \to (f, A_2^0)n}{\gamma p \to A_2^+ n} = \gamma. \tag{37}$$

We can go one step further and get even stronger relations, by once more considering semi-local duality for Reggeon-particle scattering. We write down the $n = 1$ odd sum rule for the π-exchange cross-sections. In the π induced process only the resonances ϱ^0, f, g^0

are selected and we have only the even signatured f exchange in the $\pi\pi$ or $\gamma\gamma$ channel. We then obtain:

$$\frac{\nu \dfrac{d\sigma}{dt}(\pi^- p \to \varrho^0 n)}{\nu \dfrac{d\sigma}{dt}(\gamma p \to \varrho^+ n)} = \frac{\nu \dfrac{d\sigma}{dt}(\pi^- p \to f n)}{\nu \dfrac{d\sigma}{dt}(\gamma p \to A_2^+ n)} = \frac{2\gamma_{f\pi\pi}}{\gamma_{f\gamma\gamma}}. \tag{38}$$

Data for the photoproduction reactions are very poor and only limited tests can be made of (38). Within the rather large errors the data does agree with the claims made. Of course, we can also connect π and K induced resonance production reactions in an exactly similar way, but the novelty is missing in this case since such reactions are connected anyway by SU(3). In the above relations only a higher symmetry model *e. g.* quark model could connect the photon and pion induced processes.

In fact one can in principle relate the production of meson resonances with that of baryon resonances — we shall meet such an example in the next section.

Pomeron-particle scattering

This is perhaps the area where there is most interest in triple-Regge couplings. Consequently several attempts have been made to extract the values of the couplings and make rather strong statements about duality for Pomeron-particle scattering. In particular, attention has been centred around the relative importance of the PPP (triple Pomeron) and PPM terms.

The full expression for the triple-Regge formula for the case $a = c$ is

$$\frac{d^2\sigma}{dt\,d\left(\dfrac{M^2}{s}\right)} = f_{\mathscr{P}}\left(t, \frac{M^2}{s}\right) + f_M\left(t, \frac{M^2}{s}\right)\frac{1}{\sqrt{s}}, \tag{39}$$

where

$$f_{\mathscr{P}}\left(t, \frac{M^2}{s}\right) = G_{\mathrm{PPP}}(t)\left(\frac{M^2}{s}\right)^{1-2\alpha_{\mathrm{P}}(t)} + G_{MM\mathrm{P}}(t)\left(\frac{M^2}{s}\right)^{1-2\alpha_M(t)} \tag{40}$$

and

$$f_M\left(t, \frac{M^2}{s}\right) = G_{\mathrm{PP}M}(t)\left(\frac{M^2}{s}\right)^{\frac{1}{2}-2\alpha_{\mathrm{P}}(t)} + G_{MMM}(t)\left(\frac{M^2}{s}\right)^{\frac{1}{2}-2\alpha_M(t)}. \tag{41}$$

If we take t to be, say, -0.25 then the shapes are those shown in Fig. 8.

So the small M^2/s region (< 0.1) is described by PPP and PPM (*i. e.* diffraction) and the larger M^2/s region by MMP and MMM. This means that at relatively low energies ($\lesssim 30$ GeV), it is the low M^2 region which is described by the PPP and PPM, and it is this same region which shows the lumpy behaviour due to resonances. A good way of "smooth-

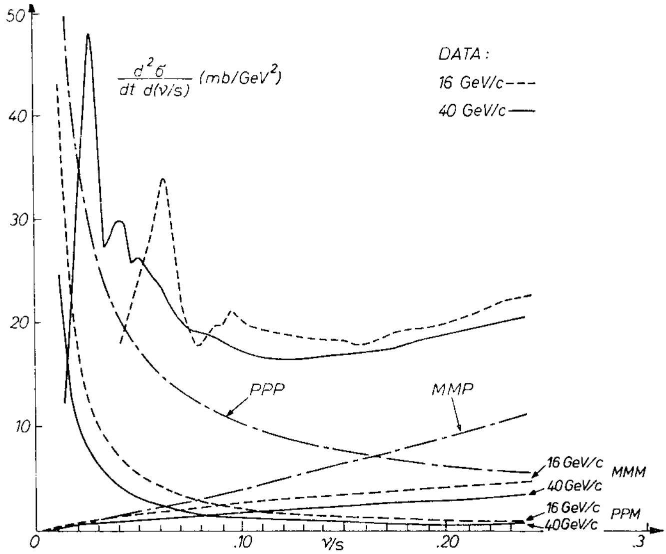


Fig. 8. Cross-sections for $\pi^- p \to pX$ at 16 and 40 GeV/c. The contributions from the four terms in the Regge fit are shown. The PPP and MMP terms are energy independent; the PPM and MMM terms behave as $s^{-\frac{1}{2}}$

ing" out these rapid fluctuations is to work with the finite mass sum rules. Take $n = 1$ and $a = c$ in (8); we get

$$I_1(t, N, s) = \int_0^N d\left(\frac{\nu}{s}\right)\left(\frac{\nu}{s}\right)\frac{d^2\sigma}{dtd\left(\frac{\nu}{s}\right)} = I_{1\mathscr{P}}(t, N) + I_{1M}(t, N)\frac{1}{\sqrt{s}} \tag{42}$$

with

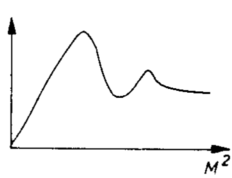
$$I_{1\mathscr{P}}(t, N) = G_{\mathrm{PPP}}(t)\frac{N^{3-2\alpha_{\mathrm{P}}(t)}}{3-2\alpha_{\mathrm{P}}(t)} + G_{MM\mathrm{P}}(t)\frac{N^{3-2\alpha_M(t)}}{3-2\alpha_M(t)} \tag{43}$$

and

$$I_{1M}(t, N) = G_{\mathrm{PP}M}(t)\frac{N^{2.5-2\alpha_{\mathrm{P}}(t)}}{2.5-2\alpha_{\mathrm{P}}(t)} + G_{MMM}(t)\frac{N^{2.5-2\alpha_M(t)}}{2.5-2\alpha_M(t)}, \tag{44}$$

$$\left(N \geqslant \frac{4}{s}; \quad N \leqslant \frac{1}{4}\right).$$

The shape of the mass spectrum say for pp → p at energies below 30 GeV looks typically like



i. e. "resonances" ($N^*(1400)$, (1520), (1690)) sitting on top of background. Early analyses of such data attempted to separate the two contributions ("resonance" and background) and then invoked the Harari-Freund duality for Pomeron-particle scattering to associate each contribution with the PPM and PPP terms respectively. At small t and for $p_{lab} \simeq 30$ the "resonance" contribution was apparently dominating — this was then taken as evidence for the dominance of PPM over PPP at small t — which might be expected if the latter actually vanished at $t = 0$.

In fact it turned out that data at fixed energies on pp → p and π^- p → p could be well described by just two terms: PPM to describe the low M^2 region and MMP to describe

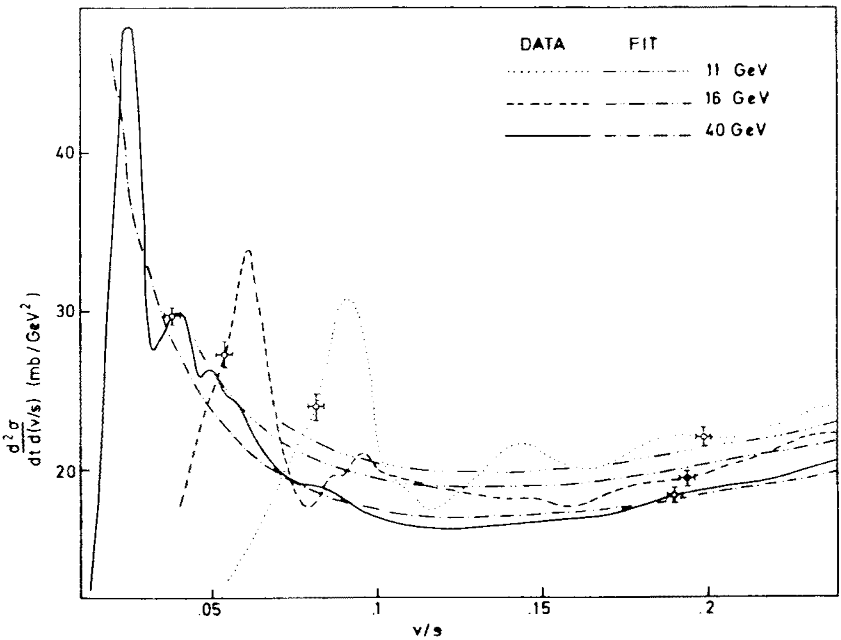


Fig. 9. Cross-sections for π^-p → pX at 11, 16, 40 GeV/c

the larger M^2 region. (The fits only go up to $M^2/s \sim 0.25$ to 0.30 since expansion is valid only up to $O(M^2/s)$. On the other hand the data pp → p at $p_{lab} \simeq 30$ GeV and π^-p → p at Serpukhov could be equally well described by pure PPP and MMP.

Clearly, as can be seen from (39)–(41) or (42)–(44), the only reliable way of separating the contributions from PPP and PPM is to look at the energy dependence at fixed values of M^2/s and t. Furthermore the separation can be clean only if the energy interval is the widest possible. This, in turn, requires data at low energies ($\lesssim$ 10 GeV) where low M^2/s means low M^2 *i. e.* resonance region. This means that the triple-Regge expression itself should not be used for the analysis since it requires $M^2 > \sim 4$ GeV^2 and so the FMSR provide a way of using this low energy, low M^2 data together with the high energy data to get a reliable estimate of the triple Regge-couplings.

This was the method used by Chan *et al.* [9] when analysing $\pi^- p \to pX$ from $p_{lab} \simeq 8$ to 40 GeV for $t \simeq -0.25$. Using equation (42), $I_1(t, N, s)$ at each value of N was separated into a scaling piece, I_{1p} and non-scaling piece I_{1m}. The ν/s dependence of each was then fitted according to equations (43) and (44). Thus one ended up with a reliable estimate of the 4 triple-Regge couplings PPP, MMP, PPM, MMM. (No interference terms were included.) These values can then be inserted into the triple-Regge expression (39), (40), (41) and we can thus calculate $f(t, \nu/s, s)$ and its various moments. These are then expected to represent the semi-local average over ν of the actual measured values, at any energy.

From Figs 9, 10 one can, in fact, see that the Regge fit remarkably well describes, in a semi-local average sense, the resonance region of the inclusive spectrum. This is another example of the success of semi-local duality for Reggeon-particle scattering.

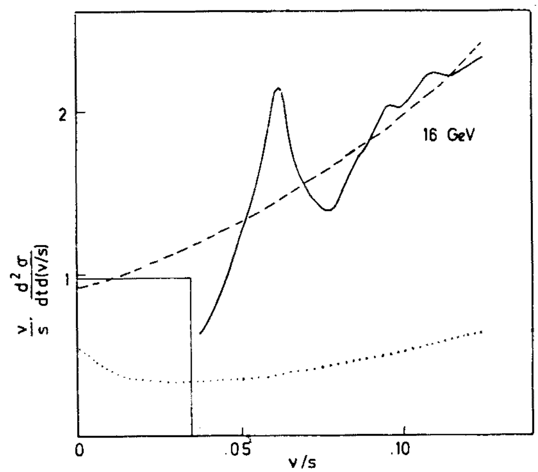


Fig. 10. First moments of cross-sections at 11, 16 and 40 GeV/c; histogram denotes π-contribution. Dashed curves represent results of Regge fit; dotted curves represent contribution from meson-exchange term only

Since we have not assumed Harari-Freund duality, the next thing to ask is whether the Regge behaviour which, on average, describes these resonances is mainly PPP or PPM. Looking at Fig. 9 we see very little energy dependence throughout the whole ν/s region (up to 0.25). This tells us that the important terms are PPP and MMP. In fact if we plot the (PPM and MMM) contributions and compare with the resonance (π, ϱ, A_1, A_3) cross--section alone, as shown in Fig. 11 we see that up to $M^2 \simeq 2.7$ GeV^2, the estimated resonance contribution is equal to or more than twice the meson-exchange contribution. One may therefore conclude that resonances which are diffractively produced are mostly dual to Pomeron exchange. This abnormal component of duality is suggested by the scheme of Einhorn *et al.* [4].

Some cautious remarks are in order here:

1) The resonance-background separation is open to some question. These diffractive resonances do not sit on a simple background and there may be large errors in the estimate of resonance cross-section.

2) The lumps in the missing mass A_1, A_3 are perhaps not really resonances. Our conclusion may be turned around to say that this is further evidence against an interpretation of genuine resonances.

3) The above analysis used $\pi^- p$ data up to $s > 76$. There now exists pp → p data at $s = 930$ (ISR) which can be used with data at 24 GeV. A preliminary analysis suggests

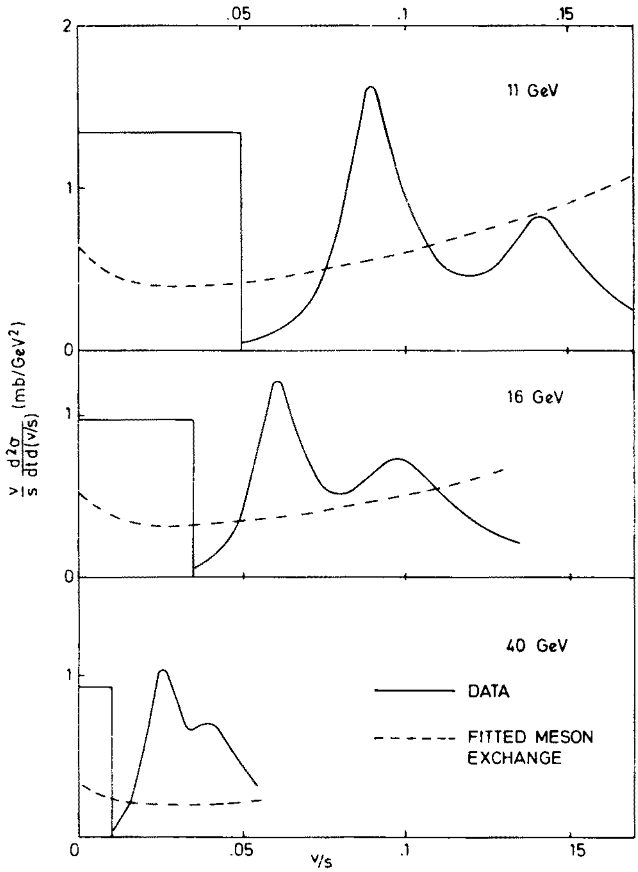


Fig. 11. Resonance contributions of the cross-sections at 11, 16 and 40 GeV/c obtained by the resonance--background separation described in text. Histogram is the pion contribution. Dashed curve is the meson-exchange contribution of the Regge fit

perhaps a decrease in the ratio PPP/PPM than that obtained above. Nevertheless the fact that the ISR data is $\sim \frac{1}{2}$ the cross-section at 24 GeV in the region $M^2/s < 0.7$ rules out dominance by the PPM term.

4) The above analysis was carried out for $0.17 \leqslant |t| \leqslant 0.34$. The A_1 has an anomalously high slope in $d\sigma/dt$ which meant that our conclusions about duality properties of resonances were using only a fraction of the resonant cross-section. In fact our triple--Regge expressions cannot hope to describe, in average, such violent t-behaviour.

Nevertheless the exercise nicely demonstrates the dual properties of the missing mass spectra. We can furthermore ask what semi-local factorisation has to say in this case of diffractive scattering. If we consider only small M^2/s with assumed Pomeron exchange only, we can relate the missing mass spectra of $\pi^-p \to pX$ and $pp \to pX$:

$$f(p \xrightarrow{p} p) = \frac{\gamma_{\mathscr{P}p}}{\gamma_{\mathscr{P}\pi}} f_{\mathscr{P}}(p \xrightarrow{\pi^-} p) + \frac{\gamma_{fp}}{\gamma_{f\pi}} f_M(p \xrightarrow{\pi^-} p) \frac{1}{\sqrt{s}} . \tag{45}$$

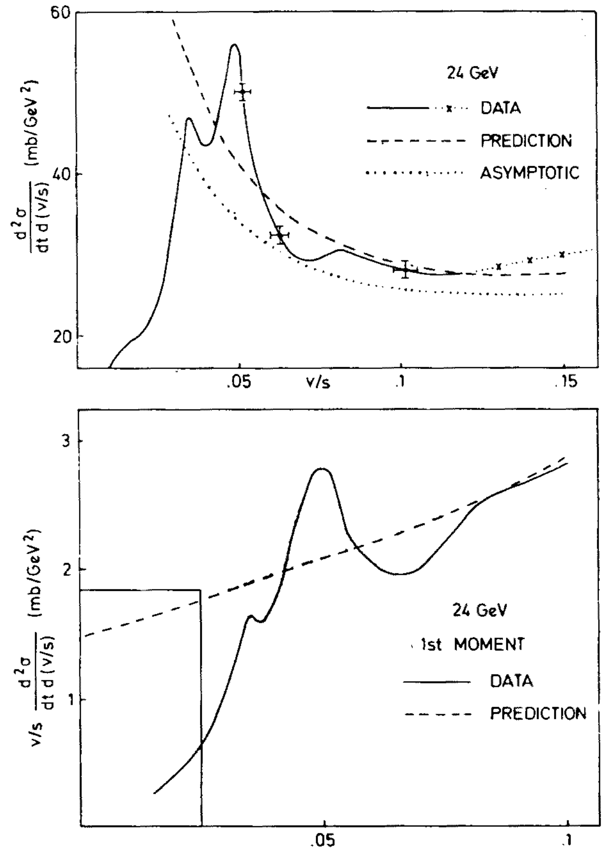


Fig. 12. Cross-section for $pp \to pX$ at 24 GeV. Data is compared with the prediction obtained from the fit to $\pi^-p \to pX$ using factorisation. Typical experimental errors are indicated

This is shown in Fig. 12.

If we just simply compare $f(p \xrightarrow{p} p)/\sigma_{TOT}(pp)$ and $f(p \xrightarrow{\pi^-} p)/\sigma_{TOT}(\pi^-p)$ over the M^2 spectrum then we are directly comparing the two pieces of experimental data. Fig. 13 shows this comparison and it is remarkable how local the agreement is between a meson resonance and baryon resonance distribution.

Having discussed the odd-moment sum rule for Pomeron-particle scattering and its consequences we now turn to the even moment sum rule and the possibility of calculating Pomeron-Pomeron cuts.

Take the Pomeron contribution of the inclusive cross-section at a fixed value of t, $A_P(t, \nu)$ which is got by extrapolating at fixed ν or M^2 the s behaviour:

$$s \frac{d^2\sigma}{dt dM^2} = A_{\mathscr{P}}(t, \nu) s^{2\alpha_{\mathscr{P}}(t)-1} + A_M(t, \nu) s^{2\alpha_M(t)-1}. \quad (46)$$

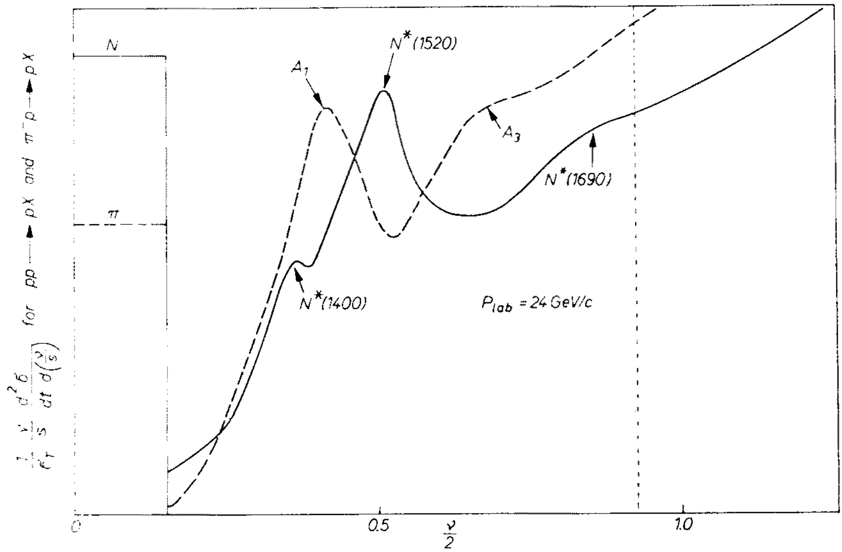


Fig. 13. Cross-section for pp → pX at 24 GeV compared with that for π^-p → pX at 25 GeV

Note that this involves knowing the value of Pomeron trajectory at that value of t. Then the fixed pole contribution to Pomeron-proton scattering is given by

$$\int_0^N d\nu A_{\mathscr{P}}(t, \nu) = G_{PPP}(t) \frac{N^{2-2\alpha_P(t)}}{2-2\alpha_P(t)} + G_{PPM}(t) \frac{N^{1\frac{1}{2}-2\alpha_P(t)}}{1\frac{1}{2}-2\alpha_P(t)} + H_{PP}(t). \quad (47)$$

Notice that the first term on the r. h. s. is $\propto N^{-2\alpha_P' t}/(-2\alpha_P' t)$ and so if the Pomeron were very flat ($\alpha' \simeq 0$), this term would be huge and would require a huge negative fixed pole contribution to cancel it. The situation at $t = -0.25$ as a function of α_P' is shown in Fig. 14.

So if the slope α_P' was taken to be 0.4 then the strength of the Pomeron-Pomeron cut would be essentially zero. At a value of $\alpha' \simeq \frac{1}{4}$, the magnitude of H is approximately the same as the low mass integral (which is dominated by the elastic contribution) but of opposite sign. However, the sign does not enter into the cut calculation since the convolution involves $|H_{PP}(t)|^2$. In such a case the P−P cut strength would be the same in either the Gribov or eikonal prescriptions. Anyhow it is somewhat distressing to see how very sensitive the value of the fixed pole residue is to the value of the Pomeron slope. However, if the triple-Regge analysis itself can estimate $\alpha_P(t)$ as well as the couplings then we have some hope. The proton spectra at ISR show a diffraction peak at small M^2/s — even

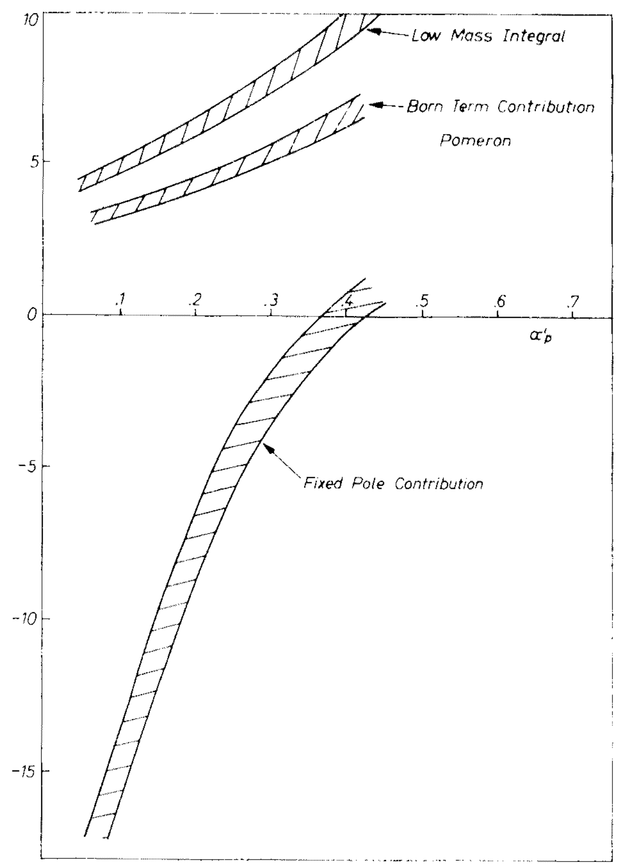


Fig. 14

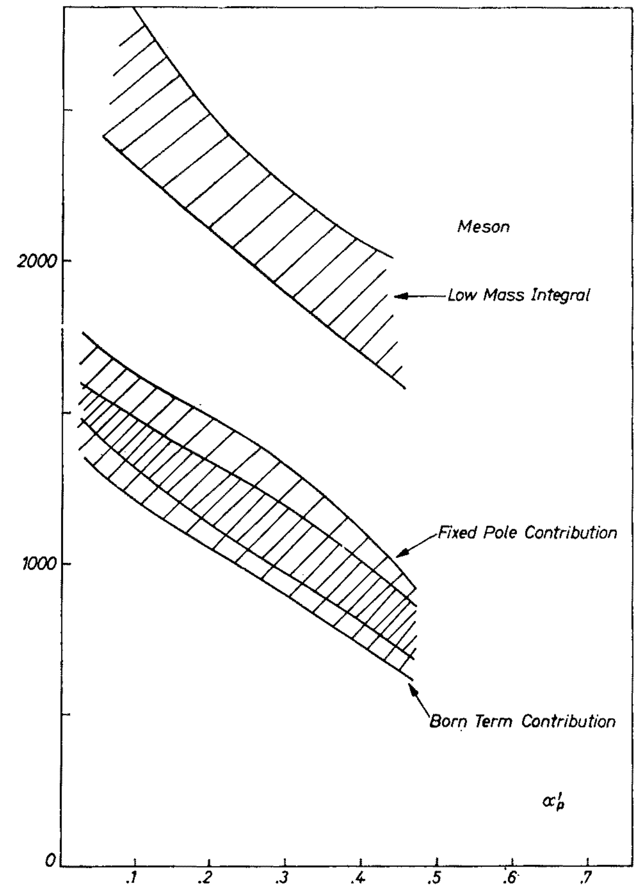


Fig. 15

Fig. 14. Various contributions to the FMSR (47) as a function of the Pomeron slope
Fig. 15. Various contributions to the FMSR (48) as function of the Pomeron slope

at large t. This allows a fairly accurate determination of $\alpha(t)$ at each to value and hence of α_P. The resulting value seems to be close to $\frac{1}{4}$, *i. e.* giving rough equality between the two cut prescriptions.

It is not enough to know $H_{pp}(t)$ at one or two values of t. To do the convolution we must integrate over t so if the triple-Pomeron coupling does vanish at $t = 0$ (we still do not know this to be true) then this crucially affects the value of the PP cut.

We can use the results of the same analysis to extract the two-meson cut — in this case $(f, \omega)-(f, \omega)$ to pp scattering. Using

$$\int_0^N d\nu A_M(t, \nu) = G_{MMP}\frac{N^{2-2\alpha_M}}{2-2\alpha_M} + G_{MMM}\frac{N^{1\frac{1}{2}-2\alpha_M}}{1\frac{1}{2}-2\alpha_M} + H_{MM}(t) \tag{48}$$

we find that $H_{MM}(t)$ and the Born term contribution (elastic scattering) to the low mass integral are almost equal with the same sign and roughly half of the low-mass integral itself see Fig. 15. Hence this implies the magnitudes of the Regge-Regge cut according to the two prescriptions (Gribov *vs.* eikonal) are not very different, although analyses at other, smaller values of t should be carried out. Finally we should remark that we have assumed only poles to be exchanged in the $b\bar{b}$ channel. Our procedure may be regarded as the first step in an iterative calculation — the next step being the inclusion of the cut calculated in the 1st step.

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