THEORETICAL CROSS-SECTIONS OF SINGLE W-BOSON PRODUCTION IN PHOTON-ELECTRON COLLISIONS AND IN ELECTRON-POSITRON COLLIDING BEAM EXPERIMENTS

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(Received April 5, 1973)

Total cross-section for the production of single W-boson in high energy photon-electron collisions is computed in the limit $m_e \to 0$. Applying the Weizsäcker-Williams approximation, we obtain the asymptotic formulae for the total cross-sections of single W-boson production in high energy electron-positron colliding beam experiments.

1. Introduction

The intermediate vector boson hypothesis which assumes the existence of a charged boson ($W^{\pm}$) as a propagator in weak interactions was first formulated by Feynman and Gell-Mann [1] in 1958. Since then much attention, both experimental and theoretical, has been paid to the W-boson production processes [2]. In this paper, we consider the theoretical calculations of cross-sections for the W-boson production in the following "semi-weak" interactions:

$$\gamma + e^- \to W^- + \nu_e, \tag{1}$$

$$e^+ + e^- \to e^+ + W^- + \nu_e. \tag{2}$$

The numerical calculation for the process (2) was first performed by Choban [3] and subsequently by Berends and West [4], and by Brown and Smith [5]. A typical cross-section for the process (2) obtained by Choban is 10^{-30} cm^2, while Berends and West calculated the cross-section more carefully taking into account the gauge invariant property of the scattering amplitudes and obtained 10^{-38} cm^2 as a typical cross-section for the process (2).

The main purpose of the present paper is to discuss the analytical formulae, in particular, the asymptotic behaviour of the total cross-sections for high energy W-boson production reactions (1) and (2). We apply the standard method based on the lowest order pertur-

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bation theory in discussing the cross-section for the reaction (1) and use the Weizsäcker--Williams approximation [6] to derive the analytical formulae for the cross-section of the reaction (2).

The plan of the paper is as follows: In Section 2, we study the W-boson production in the reaction (1) and in Section 3, we discuss the W-boson production in the electron--positron colliding beam experiments (2). Finally Section 4 concludes the paper.

2. $\gamma+e^- \to W^- + \nu_e$

The corresponding lowest order contributions to process (1) are illustrated in Fig. 1. In the following, the calculation is performed by neglecting the magnetic and quadrupole moments of the W-boson. The electromagnetic interaction of the W-boson is assumed

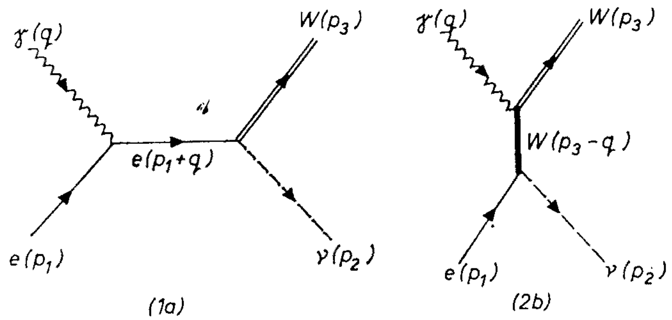

Fig. 1. Feynman diagrams contributing to the process (1)

to be pointlike. The interaction Lagrangian which describes the $W\nu_e e$ interaction is assumed to take the form [7]

$$L_{\text{int}} = G_W W^\mu \overline{\psi}_{(e)} \gamma_\mu (1+\gamma_5) \psi_{(\nu)} + \text{H.C.}, \tag{3}$$

where the fields W^μ, $\psi_{(e)}$ and $\psi_{(\nu)}$ refer to the W, e and ν_e, respectively. The quantity G_W is related to the Fermi coupling constant for the weak vector current, G_V, as follows:

$$G_W = M_W G_V^{1/2} 2^{-1/4}, \quad G_V \cong 10^{-5} M_p^{-2}, \tag{4}$$

where M_W and M_p are the W-boson and proton mass, respectively. The scattering amplitudes F_a and F_b corresponding to the diagrams (1a) and (1b) in Fig. 1, respectively, are expressed as follows:

$$F_a = ieG_W \varepsilon^\mu(q) \varphi_\alpha^\dagger(p_3) \left\{ \overline{u}(p_2)(1+\gamma_5)\gamma^\alpha \frac{1}{\not{q}+\not{p}_1 - m_e} \gamma_\mu u(p_1) \right\}, \tag{5a}$$

$$F_b = ieG_W \varepsilon^\mu(q) \varphi_\alpha^\dagger(p_3) \Big[\overline{u}(p_2) \{(2p_3-q)_\mu g^{\alpha\beta} - p_3^\beta g_\mu^\alpha - (p_3-q)^\alpha g_\mu^\beta\} \times$$

$$\times \frac{1}{(p_3-q)^2 - M_W^2} \left\{ g_{\beta\sigma} - \frac{(p_3-q)_\beta (p_3-q)_\sigma}{M_W^2} \right\} (1+\gamma_5)\gamma_\sigma u(p_1) \Big], \tag{5b}$$

where $\varphi_\alpha(p_3)$ represents the polarization vector of the W ansd ε^μ the polarization vector of the photon. The differential cross-section for the process (1) by an unpolarized photon on the unpolarized electron target can now be written in the form

$$d\sigma = \frac{1}{16m_e q_0}\frac{1}{(2\pi)^2}|M_{fi}|^2\delta^4(p_1+q-p_2-p_3)\frac{d^3p_2}{2p_{20}}\frac{d^3p_3}{2p_{30}}, \tag{6}$$

where

$$M_{fi} = F_a+F_b.$$

In Eq. (6) the invariant flux quantity is evaluated in the laboratory frame where $\boldsymbol{p}_1 = 0$. The spin-averaged squared matrix element takes the form

$$|M_{fi}|^2 = (eG)^2 T_{\mu\nu}\sum_{\text{photon spin}}\varepsilon^\mu(q)\varepsilon^{\nu\dagger}(q). \tag{7}$$

After a straightforward calculation on the traces resulting from the sum on the electron, neutrino and W-boson spins, $T_{\mu\nu}$ is expressed in the form

$$T_{\mu\nu} = \alpha(p_{3\mu}p_{1\nu}+p_{3\nu}p_{1\mu})-\beta g_{\mu\nu}+\gamma p_{3\mu}p_{3\nu}+\delta p_{1\nu}p_{1\mu}, \tag{8}$$

where

$$\alpha = \left(\frac{3S}{2M_W^2}-\frac{4M_W^2}{S}\right)\Big/T,$$

$$\beta = \left(\frac{1}{8M_W^2}+\frac{1}{S}\right)T+\frac{17}{8}-\left(\frac{3}{2}S-M_W^2\right)\Big/T-\frac{1}{2}S^2/T^2,$$

$$\gamma = \left(4M_W^2-\frac{S^2}{2M_W^2}\right)\Big/T^2,$$

$$\delta = \left(\frac{4M_W^2}{S^2}-\frac{1}{2M_W^2}\right).$$

The invariants S and T are defined as

$$S = (p_2+p_3)^2,$$
$$T = (p_3-q)^2-M_W^2. \tag{9}$$

In Eq. (8), the electron mass m_e^2 is neglected everywhere if it appears in nonsingular form. The phase volume elements in Eq. (8) are Lorentz-invariant quantities. We work in the c.m. system of the outgoing νW pair by using the formula

$$\frac{d^3p_2}{2p_{20}}\frac{d^3p_3}{2p_{30}} = \frac{1}{8}\frac{d^3P}{2P_0}\frac{S-M_W^2}{S}dSd\Omega, \tag{10}$$

where

$$P = p_2+p_3,$$

$d\Omega$ = internal angular variable of the outgoing νW pair defined at the rest frame of the pair.

An integration over d^3P and dS is performed in the following way:

$$\frac{d^3P}{2P_0}dS\delta^4(p_1+q-S) = \int_{S_{\min}}^{S_{\max}}\int_{-\infty}^{\infty} d^4P\delta(P^2-S)\theta(P_0)\delta^4(p_1+q-S)dS =$$

$$= \int_{S_{\min}}^{S_{\max}} \delta(P^2-S)\theta(P_0)dS = 1. \tag{11}$$

An integration over $d\Omega$ is straightforward. The total cross-section is now expressed in the form

$$\sigma_\gamma(S) = \frac{\alpha G_W^2}{32M_W^2}f(x), \tag{12}$$

where

$$f(x) = \left(3+\frac{15}{x}-\frac{16}{x^2}-\frac{8}{x^3}\right)\ln x+\left(-\frac{13}{2}+\frac{19}{x}-\frac{33}{2x^2}+\frac{4}{x^3}\right),$$

and

$$x = S/M_W^2.$$

Therefore, at high energies when $S \gg M_W^2$, the total cross-section is, of the order of magnitude

$$\sigma_\gamma(S) \sim \frac{3\alpha G_W^2}{32M_W^2}\left(\ln\frac{S}{M_W^2}-\frac{13}{6}\right). \tag{13}$$

3. $e^+ + e^- \to e^+ + W^- + \nu_e$

In the lowest order in e and G_W, there are four Feynman diagrams which contribute to this process. They are illustrated in Fig. 2. The diagrams (2c) and (2d) in Fig. 2 have the s-channel photon pole. Therefore, the corresponding scattering amplitudes contain a factor of $1/s$, with s the total energy. Since the total energy s, in this case, should be larger than the W-boson mass, M_W, which is probably larger than 2–3 GeV/c^2 [2], the contributions of diagrams (2c) and (2d) to the cross-section are expected to be much smaller than those of the diagrams (2a) and (2b) in Fig. 2. Therefore we ignore their contributions to the cross-section.

It is not difficult to obtain the exact formulae for the differential cross-section for the reaction (2) [8]. We apply, however, the Weizsäcker-Williams approximation for the positron current, since it allows the entire analytical calculations to be more manageable. Applying the Weizsäcker-Williams approximation, we replace the positron current by

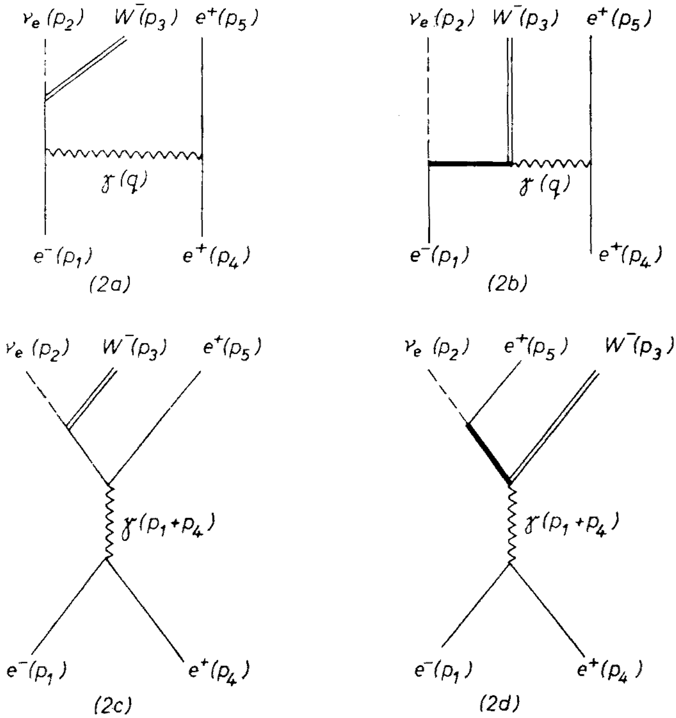

Fig. 2. Feynman diagrams contributing to the process (2)

a photon with an 《equivalent spectrum》:

$$\frac{\alpha}{\pi}\frac{E^2+(E-\nu)^2}{E^2}\ln\frac{E}{m_e}, \tag{14}$$

where E = energy of the incident positron, ν = energy loss of the positron or energy of the 《photon》.

These quantities are defined at the rest frame of the electron which emits the outgoing νW pair. The cross-section for the process (2) is now expressed in the form [9]

$$d\sigma(s) = \frac{\alpha}{\pi}\frac{s^2+(s-v)^2}{s^2}\frac{dv}{v}\ln\frac{E}{m_e}\,d\sigma_\gamma(v), \tag{15}$$

where $s = 2m_eE+2m_e^2$ = total energy, $v = 2m_e\nu+m_e^2$ = s-channel invariant in the 《photon》-electron collision.

In Eq. (15), the following relation is used for $d\sigma_\gamma(v)$ which holds in the reference frame where $\boldsymbol{p}_1 = 0$:

$$S = (q+p_1)^2 = q^2+m_e^2+2m_e\nu \simeq v, \tag{16}$$

with $q^2 \to 0$ in the spirit of the Weizsäcker-Williams approximation. An integration over v in Eq. (15) is performed over the region

$$M_W^2 \lesssim v \lesssim s. \tag{17}$$

Therefore,

$$\sigma(s) = \frac{(\alpha G)^2}{32\pi M_W^2} \ln \frac{E}{m_e} \int_1^c \left(\frac{2}{x} - \frac{2}{c} + \frac{x^2}{c^2}\right) f(x) dx, \tag{18}$$

where $x = S/M_W^2, \quad c = s/M_W^2$.

At high energies when $s \gg M_W^2$, the total cross-section is given by the asymptotic formula

$$\sigma(s) \simeq \frac{3(\alpha G)^2}{32\pi M_W^2} \ln \frac{E}{m_e} \left\{\left(\ln \frac{s}{M_W^2}\right)^2 - \frac{35}{6} \ln \frac{s}{M_W^2}\right\}. \tag{19}$$

This equation can also be written as

$$\sigma(E) \simeq \frac{3(\alpha G)^2}{8\pi M_W^2} \ln \frac{E}{m_e} \left\{\left(\ln \frac{2E}{M_W}\right)^2 - \frac{35}{12} \ln \frac{2E}{M_W}\right\}, \tag{20}$$

where E is the laboratory energy of the colliding electron. From Eqs (4) and (20), we obtain, to the order of magnitude

$$\sigma(E) \simeq 10^{-38} \ln \frac{E}{m_e} \left\{\left(\ln \frac{2E}{M_W}\right)^2 - \frac{35}{12} \ln \frac{2E}{M_W}\right\} \text{cm}^2. \tag{21}$$

Note that the asymptotic formula obtained by Choban [3] on the basis of the unitarity argument reads

$$\sigma(E) \sim (E/M_W)^2 \ln^2 \frac{E}{M_W}. \tag{22}$$

Although these leading terms appear likewise in our case in Eq. (18), they cancel out with each other. Our leading term in Eq. (20) agrees in its form with the asymptotic formula quoted by Lee and Yang [10] for the neutrino-induced production of W-boson. The order of magnitude in Eq. (21) is in reasonable agreement with the value obtained by Berends and West [4].

4. Conclusion

We have calculated the total cross-section for single W-boson production in photon--electron collisions in the limit $m_e \to 0$ (Eq. (12)) and then discussed its asymptotic behaviour (Eq. (13)). Using the formula for the total cross-section for single W-boson production in photon-electron collisions, we obtained the asymptotic formulae for the total cross-

-section for single W-boson production in electron-positron colliding beam experiments (Eqs (20), (21)).

The author would like to thank Professor S. Homma for introducing him to this problem and for many helpful discussions.

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