ELECTRON-POSITRON ANNIHILATION: UNITARITY, SCALING AND ELECTRODYNAMICS AT HIGH ENERGIES

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Recent work on e^+e^- annihilation by Cabibbo, Wolfenstein and the author is reviewed. The restrictions of unitarity are analyzed and the connection between the cross sections σ_h (into hadrons) and σ_μ (into muons) is derived. The possibility of non-scaling in e^+e^- annihilation is studied and it is pointed out that it leads to no contradiction with presently available information. It is further pointed out that non-scaling could provide a cut-off mechanism for Quantum Electrodynamics.

Introduction

Recently, a great deal of attention has been focused on the large and roughly constant cross section (σ_h) of the process:

$$e^+e^- \to \text{hadrons} \tag{1}$$

compared to the smaller, and falling cross section (σ_μ) for the leptonic process

$$e^+e^- \to \mu^+\mu^-. \tag{2}$$

It is found that σ_μ agrees with the prediction based on Q.E.D. to leading order in α. (See Eq. (4).)

The large and constant cross section σ_h has caused surprise because of general expectations of "scaling behaviour" ($\sigma \propto s^{-1}$) in analogy with deep inelastic electron scattering. The explanations offered find reasons for "late" (or "senile") scaling or introduce (ad-hoc) new interactions which produce a constant cross section σ_h. Another logical possibility, which is pursued here, is that the $e\bar{e}$ annihilation is electromagnetic, but it does not scale, not even at high energies. In this case one has to explain why the photon propagator in low energy experiments looks like s^{-1}, and when and how will scaling be broken in electron scattering experiments. Further, given the assumption of non-scaling for σ_h one can ask about its general physical consequences: the main one appears to be that the hadrons

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provide a cut-off mechanism for quantum electrodynamics, thereby rendering all EM mass differences finite.

We shall first discuss the constraints imposed by unitarity between σ_h and σ_μ, given simple assumptions of minimal coupling and universality. We will then turn to study the unitarity constraints on the photon propagator given the non scaling behaviour of σ_h, and construct various logical possibilities for the photon propagator, and focus on the simplest possibility.

To start with we discuss the main assumptions made throughout. Although the assumptions are quite conservative, I shall try to spell them out in detail in order to avoid confusion with other strategies that have been proposed. We shall conclude with a discussion of the problems raised by the logical possibility described here, assuming it is true. The notes describe work which has been carried out at CERN in the first half of this year, in collaboration with Nicola Cabibbo and Lincoln Wolfenstein. Although the notes may differ in details from the exact content of the preprints CERN TH-1858 and 1881, most of the observations arose in the discussions preceding the writing of these two preprints. The surviving mistakes however are probably my own.

A. Assumptions

We assume in the first place that $e\bar{e}$ annihilation is an electromagnetic process, i.e. that it proceeds mainly through a single virtual photon intermediate state. We assume that perturbation expansions (in e) remain valid at all energies, and so, two or three photon annihilation is unimportant. We abstract then the main properties of the 1-photon annihilation graphs in Q.E.D., but we contemplate the possibility of unitarity corrections to one photon graphs. In an abstract form, we thus assume the following propeities for the amplitudes (S matrix elements) of processes (1) and (2):

1. Annihilation occurs from the $J^{PC} = 1$ state of $e\bar{e}$ (with opposite helicities of the two leptons). This is a consequence of the "minimal" coupling of the leptons to the photon $[\bar{\psi}\gamma_\mu\psi]$, which prefers, at high energies, the lepton and antilepton to *spin* in the same direction. The student who wants to check this statement is advised to work out $\bar{\psi}\gamma_\mu\psi$ for all choices of helicities of $\bar{\psi}$ and ψ!

2. Electron-muon universality, in the form that the amplitudes for process (2) and for the exclusive channels in (1) are invariant at high energies under the simultaneous replacement ($e^- \leftrightarrow \mu^-$, $e^+ \leftrightarrow \mu^+$). Again this is an abstract statement of properties of single photon annihilation graphs, in field theory.

3. When talking about S-matrix elements for processes involving charged particles, we are commiting the sin of using very ill defined concepts. As is well known there are many dificulties, connected with the preparation of wave packets, (spurious) divergences due to soft photon emmision, etc. We sweep these under the carpet, with the statement that these are taken care of (by others!) when making radiative coreections to the raw experimental data. Most of the above can be summarized in the statement that we naively use perturbation expansions in α for the coupling of leptons to the photon and restrict attention to the leading term. The perturbation expansion can in principle fail at some higher energies, but we assume that it never does.

B. Consequences of unitarity

Why worry about unitarity? Unitarity is an expression of probability conservation and, if process (1) becomes important, that fact is expressed by saying that its probability becomes large. But because probabilities must add up to one, it necessarily follows that in such a case the probability of the leptonic process (2) taking place must be greatly modified from whatever its value is in the absence of hadronic processes (1). This is a complicated way of saying that when hadronic production becomes important, quantum electrodynamics for leptons has to suffer changes. We try below to express quantitatively these rather vague considerations.

The Unitarity bound of Cabibbo and Gatto. The maximal value of any probability is one and this is the simplest consequence of probability conservation. This implies a bound for the cross section of any single inelastic channel or the sum of cross sections for all inelastic channels, which go through a given total angular momentum intermediate state. This bound was stated for $e\bar{e}$ annihilation by Cabibbo and Gatto, who have shown that if the photon has $J^{PC} = 1^{--}$ then

$$\sigma_h \leqslant \frac{3\pi}{4} \lambda\!\!\!^{-2} = \frac{3\pi}{s}, \tag{3}$$

where σ_h is the cross section from unpolarized e^+e^- beams.

We want next to indicate that *if the bound (3) is saturated, our assumption of feeble lepton coupling to the photon no longer holds.* To see this, remember that when the inelastic cross section reaches its maximal value ("black disc", "complete absorption") the elastic cross section must equal the same value. Therefore, when $\sigma_h = 3\pi/s$ $\sigma_{\mathrm{el}}^{J=1} = 3\pi/s$. However, if perturbation theory were to hold, the $J = 1$ projection of the elastic cross section (Bhabha Scattering) would be expected to be of order (α^2/s), and therefore we get an indication that when (3) is saturated the photon couples to the leptons with coupling (much larger than e) of order unity. Note that it is inconvenient to consider Bhabha scattering in this connection since its partial wave projection diverges due to t channel photon exchange. It is much more convenient to consider for comparison purposes the cross section σ_μ of process (2) which, to leading order in α, has at high energies ($s \gg m_\mu^2$) the value:

$$\sigma_\mu^{(\mathrm{B})} = \frac{4\pi\alpha^2}{3s}. \tag{4}$$

If we insist that the estimate (4) is roughly correct, then the electrons couple to the photon with a coupling α (in the cross section) and we would naively expect that the cross section of process (1) which also starts from an electron positron pair cannot saturate the bound (3) but will be at most of order (α/s). These naive considerations are actually correct.

The unitarity of the S matrix with the additional assumption of muon electron universality allows the computation of the imaginary part of the amplitudes for the process (2) arising from the cross section σ_h of the process (1). The argument is given in detail in preprint TH-1813, and is only sketched here, as it is rather simple. Starting from the usual unitarity equations:

$$S = 1+iT,$$
$$SS^{\dagger} = S^{\dagger}S = 1, \tag{5}$$
$$T-T^{\dagger} = iT^{\dagger}T = iTT^{\dagger},$$

and sandwiching them between e^+e^- states of definite helicity (RL, say) and $\mu^+\mu^-$ states of the same helicity, in the forward direction, one can derive the equation:

$$\mathrm{Im}\, T_{\mu}(\mathrm{RL} \to \mathrm{RL}, \theta = 0) \cong \tfrac{1}{2} \sum_h |\langle h|T|e^+e^-\rangle|^2 = s\sigma(e^+e^- \to h; \mathrm{RL}), \tag{6}$$

which has the familiar form of the optical theorem. Here $T(\mathrm{RL} \to \mathrm{RL}, \theta = 0)$ is the forward amplitude for process (2) with the helicity states specified, on the right hand side we have the cross section of (1) starting from an electron-positron pair in the same helicity state as on the left hand side. Only hadronic intermediate states have been included in between T and $T^{\dagger}$, an approximation which is justified when the hadronic states dominate the annihilation. Eq. (6) shows explicitly that by virtue of universality the annihilation into muons can play the role of the elastic channel. Eq. (6) also makes clear that the cross sections σ_h and σ_μ are not entirely independent for if one makes σ_h very large, one generates a large imaginary part for T, and therefore σ_μ must have a minimal value. To transform these statements into a precise form, note the following equations:

$$\sigma_h = \sigma_h(\text{unpolarized}) = \tfrac{1}{2}\, \sigma(e^+e^- \to h; \mathrm{RL}), \tag{7}$$

$$|T_{\mu}(\mathrm{RL} \to \mathrm{RL}; \theta = 0)|^2 = 48\pi s \sigma_{\mu}(\text{unpolarized}) = 48\pi s \sigma_{\mu}. \tag{8}$$

We can transform now Eq. (6) into a bound by squaring it and noting that

$$|\mathrm{Im}\, T|^2 \leqslant |T|^2. \tag{9}$$

Substituting (7) and (8) into (9) we obtain:

$$\sigma_h^2 \leqslant \frac{12\pi}{s}\, \sigma_{\mu}. \tag{10}$$

Comments:

1. Eq. (8) follows from the normalization of states, and the assumption of a $J = 1$ intermediate state. I have followed the normalization in the well known book by Pilkuhn.

2. In terms of the ratio R of the cross sections σ_h to σ_μ, the bound (10) can be reexpressed:

$$\sigma_h \leqslant \frac{12\pi}{sR}\,. \tag{11}$$

3. A better bound can be obtained by including also the leptonic channels in between

T and $T^\dagger$, it is

$$\sigma_h \leqslant \frac{12\pi}{s\left[R+4+\dfrac{4}{R}\right]} = \frac{12\pi R}{s(R+2)^2}\,. \tag{12}$$

4. A concise derivation of the bound (12) is given in preprint TH-1881. An "easy" derivation consists in neglecting t channel photon exchange graphs when computing the contribution of leptonic intermediate states in the unitarity equations (5). Then on the right hand side of (6) we have $s\sigma(e^+e^- \to h;\, \mathrm{RL}) + 2s\sigma(e^+e^- \to \mu^+\mu^-;\, \mathrm{RL})$. This leads to Eq. (12).

5. If we insist that the Born Approximation to σ_μ (4) remains correct, (10) becomes:

$$\sigma_h \leqslant \frac{4\pi\alpha}{s}, \tag{13}$$

in agreement with our naive expectations, spelled out after Eq. (4).

6. Numerically, the unitarity bound (3) would get saturated at values of $s \approx 10^5$ GeV^2, if the present cross section ($\approx 5\times 10^{-5}$ GeV^{-2}) is maintained at higher energies, whereas the bound (13) is saturated at $s \approx 2\times 10^3$ GeV^2, at which energy either σ_h would have to change its value, or σ_μ would deviate from (4), or both of these possibilities would occur. In other words, the present value of the cross section σ_h, if maintained up to $s \approx 2\times 10^3$ GeV^2, would lead to unavoidable and important modifications of Quantum Electrodynamics.

We have now expressed into a fairly precise form the general considerations mentioned at the begining of this section, and we turn now to speculate on various possibilities for the cross sections σ_h, σ_μ at higher energies.

C. The photon propagator when scaling is broken

We found above that the unitarity relation between σ_h and σ_μ can be written in the form of a bound:

$$\sigma_h \lesssim \frac{12\pi}{sR}, \tag{11}$$

where R is the ratio of σ_h to σ_μ. R has the meaning of an equivalent number of leptons which would give rise to a cross section equal to σ_h. In models in which the hadrons are made out of "quarks", or more precisely, the electromagnetic current of hadrons is due to partons, R equals the sum of the squares of charges of all possible kinds of partons which can be produced. If R is small $\sigma_\mu = \sigma_\mu^{(B)}$ and we could have defined R in terms of $\sigma_\mu^{(B)}$, however as we will contemplate large values of R, the muon cross section will deviate from its Born value, and the natural definition of R is in terms of σ_μ rather than $\sigma_\mu^{(B)}$ since the muons continue to be point like. "Scaling" corresponds to a "small" value of R, which is reached at some value $s = s_0$, after which the cross section σ_h goes down like s^{-1}. "Non-scaling" corresponds to an indefinite increase of R with s. We would like to find the high energy behaviour of the photon propagator corresponding to various choices for R. We shall give, to start with, rather general arguments about the behaviour of the photon propagator and continue afterwards with a more detailed discussion. Let us assume a power law behaviour:

$$R(s) \approx (ks)^{\nu}.$$

Then in virtue of our bound stated above

$$\sigma_h \lesssim \frac{1}{s^{1+\nu}}, \quad \text{and} \quad \sigma_\mu \lesssim \frac{1}{s^{1+2\nu}}.$$

Remembering the calculation of σ_μ in terms of the photon propagator $D(s)$, $\sigma_\mu \propto s|D(s)|^2$, we obtain the bound

$$D(s) \lesssim \frac{1}{s^{1+\nu}}.$$

These statements hold as s becomes large, along the physical cut $s > 4\mu^2 > 0$. By analyticity they imply the behaviour of the photon propagator $D(s)$ in the whole complex s-plane. It now follows from Cauchy's theorem that $D(s)$ must have pole (s) in the physical sheet with negative residue, to compensate the contribution of the pole at $s = 0$ and the physical cut. The asymptotic behaviour is crucial to allow the neglect of the contribution of the circle with infinite radius. *This argument shows that the photon propagator must have poles (or cuts) with wrong residue, on the physical sheet, if there is no scaling.* I will now anticipate the results of the next section and state the main logical possibilities. If one computes the propagator $D(s)$ to leading order in α then,

— for $0 \leqslant \nu < 1$ the pole is on negative s-axis (Landau-Pomeranchuk pole) and $D(s)$ needs to extra subtraction constant (compared to usual Q.E.D.),
— for $1 \leqslant \nu < 2$ the pole can be in the complex s-plane (Lee-Wick pole) and one extra subtraction constant is needed in $D(s)$,
— for $2 \leqslant \nu < 3$ the pole is again on negative s axis, etc.

The position of the pole is the place where the asymptotic behaviour $s^{-1-\nu}$ takes over from the behaviour s^{-1} which is in the neighbourhood of the origin. If the pole is on the negative s-axis, then, to start with, $D(s)$ must increase between $0 \geqslant s \geqslant s_{\text{pole}}$, and after $s < s_{\text{pole}}$ the $s^{-1-\nu}$ behaviour sets in. If the pole is in the complex plane with $\text{Re}\,(s_{\text{pole}}) > 0$ then $D(s)$ must increase along $0 < s < \text{Re}\,(s_{\text{pole}})$ and then decrease like $s^{-1-\nu}$ for $s > \text{Re}\,(s_{\text{pole}})$. In this case ($\text{Re}\, s_{\text{pole}} > 0$) the propagator $D(s)$ will decrease for $s < 0$, from its initial behaviour s^{-1} near the origin. The range of s where the takeover from s^{-1} to $s^{-1-\nu}$ is (in all cases) determined by the position of the pole, which is the place where the bound (11) is saturated; from the derivation of the bound we know that the amplitude T_μ and propagator $D(s)$ are purely imaginary at that value of s.

The damping of the photon propagator has useful consequences, in that it is sufficient to ensure the convergence of electromagnetic contributions to self-masses, and therefore mass differences in hadronic isospin multiplets. It is this unexpected bonus, which makes the possibility of "no scaling forever" (in σ_h) rather respectable. We spell out in the next section, in more detail, the computation of the photon propagator $D(s)$, starting from the ansatz $R(s) \propto s^{\nu}$.

D. Unitarity corrections to the photon propagator

We find the unitarity corrections to the photon propagator, to leading order in α, by the round about way of computing corrections to the Born amplitudes, generated by the optical theorem.

The Born Amplitude $T^{B}_{\mu}(\text{RL}, \theta = 0)$ is given by

$$T^{B}_{\mu}(\text{RL}, \theta = 0) = \pm 8\pi\alpha, \tag{14}$$

where note again that μ is not a Lorentz index but a reminder that we are dealing with the amplitude for process (2). The amplitude (14) can be computed by the standard rules of Q.E.D., but here we computed it from equations (8) and (4) which were assumed already (that is the reason for $\pm$).

Now consider again the optical theorem, Eq. (6), and let us choose $\sigma_h = \sigma^{B}_{\mu}$ (note also that (6) is written for a polarized initial state, therefore make use of (7)). Then we obtain:

$$\text{Im } T_{\mu}(\text{RL}; \theta = 0) = s\sigma^{B}_{\mu}(\text{RL}) = \frac{8\pi\alpha^2}{3}. \tag{15}$$

It is conventional to write the corrections to the photon propagator as a factor $[1+\Pi]^{-1}$, hence:

$$\text{Im } T_{\mu} = \text{Im}\left[T^{(B)}_{\mu}\frac{1}{1+\Pi}\right] = T^{(B)}_{\mu}\frac{\text{Im } \Pi^*}{|1+\Pi|^2}, \tag{16}$$

where, as in (14), we assumed the Born amplitude to be real. Comparing equations (14), (15) and (16) we see that, to leading order in α, we obtain:

$$\text{Im } \Pi = \frac{\alpha}{3}, \qquad s \gg m_{\mu}^2, \tag{17}$$

where we chose the sign in (14) to obtain a positive imaginary part for Π. As stated above the correction factor is associated with the photon propagator, which is usually written as follows

$$D(s) = -\frac{1}{s(1+\Pi(s))}. \tag{18}$$

The choice of signs is such as to ensure that the residue of the pole at $s = 0$ has the same sign[1] as the discontinuity of $\Pi(s)$. I will now digress and discuss a little bit more the photon propagator, to remind the reader of "well known" facts, which are known by the "experts" but perhaps not by the general public.

[1] I am indebted to Dr P. Grassberger (CERN) for help at this point. We want Im $\Pi > 0$ for $s > 4m_{\mu}^2$ and Im $D > 0$ as well!

We have computed Im $\Pi(s)$ corresponding to the contribution of muon pairs in e^+e^- collisions via unitarity. The real part of $\Pi(s)$ can be obtained via dispersion relations from the imaginary part. Assuming (17) to hold all the way down to threshold: $s = 4m_\mu^2$, the function $\Pi(s)$ with discontinuity given by (17) is[1]

$$\Pi(s) = -\frac{\alpha}{3\pi}\ln\left(-\frac{s}{4m_\mu^2}\right). \tag{19}$$

The assumption is not quite correct, and the precise form of $\Pi(s)$ contains threshold factors which become 1 as $s \gg 4m_\mu^2$. As we are interested here only in the high energy behaviour of $\Pi(s)$ we keep (19) which is correct asymptotically. The function $\Pi(s)$ is computed in textbooks of relativistic quantum mechanics, (e.g. Bjorken and Drell, vol. I); It corresponds to the graph in which a virtual photon dissociates into a virtual pair (bubble) which recombines again into the photon. The calculation presented here corresponds to an application of Cutkosky's rules to this diagram, which is shorter and it emphasizes the role of unitarity.

The function $\Pi(s)$ is called in Q.E.D. the photon proper self energy. One should note that one generally requires that $\Pi(0) = 0$ to guarantee that the residue of $D(s)$ given by (18) be unity at $s = 0$, which is a requirement of charge renormalization. The formula (19) does not satisfy this requirement so we should subtract the value at zero s $\Pi(s)-\Pi(0) = \ldots$, but again we shall not worry about this point for several reasons, which will become apparent later on. The main reason is that we will be interested in the high s behaviour of $D(s)$ rather than the value near $s = 0$; also our aim is to construct the propagator corresponding to σ_h, which in the most interesting case will satisfy the requirement $\Pi(0) = 0$ automatically.

Before proceeding we wish to generalize formula (17) to the case in which σ_h is arbitrary. Let us again use $R(s)$

$$R(s) = \frac{\sigma_h}{\sigma_\mu},$$

where, σ_μ is the actual cross section for reaction in process (2), which in terms of the Born cross section is given by

$$\sigma_\mu = \sigma_\mu^{(B)} \frac{1}{|1+\Pi|^2}, \tag{20}$$

since the photon propagator is modified as in Eq. (18), and is squared in the cross section. Repeating the steps (15), (16), (17) with $\sigma_h = R(s)\sigma_\mu$, we obtain

$$\text{Im}\,\Pi(s) = \frac{\alpha}{3} R(s), \qquad s \gg m_\mu^2, \tag{21}$$

and we can compute $\Pi(s)$ for various choices of $R(s)$ by a subtracted dispersion relation, or even by mere "inspection". For example if

$$R(s) = (ks)^\nu,$$

[1] See footnote on the preceding page.

then

$$\mathrm{Im}\,\Pi(s) = \frac{\alpha}{3}(ks)^{\nu}, \tag{22}$$

and

$$\Pi(s) = \frac{\alpha}{3\sin\pi\nu}(-ks)^{\nu} + \text{polynomial in } s, \tag{23}$$

where we remember that $e^{i\pi\nu} = \cos\pi\nu + i\sin\pi\nu$, and $(-x)^{\nu} = e^{i\pi\nu}|x|^{\nu}$. Note that for integer ν the expression (23) diverges, and therefore we have to compute $\Pi(s)$ separately in the case

$$\mathrm{Im}\,\Pi(s) = \frac{\alpha}{3}(ks)^{n}$$

with integer n. Then we have:

$$\Pi(s) = -\frac{\alpha}{3\pi}(ks)^{n}\ln(-ks) + \text{polynomial in } s. \tag{24}$$

The polynomials in (23) and (24) are determined by the subtraction constants; they have no imaginary part. The degree of the polynomials is at most equal to n. We can now write down the photon propagators corresponding to the $\Pi(s)$ given by (23) for $\nu < 1$ and (24) for $n = 0, 1$.

$$D(s) = -\frac{1}{s\left(1 - \dfrac{\alpha R}{3\pi}\ln(-ks)\right)} \qquad (\nu = 0), \tag{25}$$

$$D(s) = -\frac{1}{s\left(1 - \dfrac{\alpha}{3\sin\pi\nu}(-ks)^{\nu}\right)} \qquad (0 < \nu < 1), \tag{26}$$

$$D(s) = -\frac{1}{s\left(1 - \dfrac{\alpha}{3\pi}(ks)\left[\ln(-ks) + a\right]\right)} \qquad (\nu = 1), \tag{27}$$

where in (25) we assumed a constant R ($n = 0$ in (24)); $R = 1$ corresponds to Eq. (19), and in (27) $a\,\dfrac{\alpha}{3\pi}(ks)$ is the polynomial in s.

One can note immediately that (25) and (26) have a zero in the denominator for $s < 0$: this is (called) the Landau-Pomeranchuk pole. From (25) we set:

$$1 - \frac{\alpha R}{3\pi}\ln(-ks) = 0,$$

$$-s_{\text{pole}} = (k)^{-1}\exp\left(\frac{3\pi}{\alpha R}\right) \tag{28}$$

in the case $R = 1\,k^{-1} = 4m_\mu^2$ and the pole is very far indeed (10^{400} GeV^2). Note that one can argue that near the pole the approximation of keeping only linear terms in α in $\mathrm{Im}\,\Pi$ (see (17)) is not good, and perhaps the pole is not on the axis. The existence of a pole (or some other singularity) with wrong residue hinges *only* on the asymptotic decrease of the photon propagator a little bit faster than s^{-1}, even by a logarithm, and that fact seems to me to be guaranteed by the mere positivity of cross sections. So we apparently cannot escape Landau and Pomeranchuk. Not ealso that the pole moves rapidly nearer the origin if R gets large. E.g. if $k^{-1} = 1$ GeV^2 and $R = 10(-s_{\text{pole}}) \approx 10^{40}$ GeV^2 and for $R \approx 100$ the pole is at 10^4 GeV^2; larger R would move it right into the origin. If we can be sure of one thing, is that there is no such pole very near the origin; the photon propagator is represented very well by $-s^{-1}$.

For the propagator (26), the pole is at

$$(-s)_{\text{pole}} = k^{-1}\left(\frac{3\sin\pi\nu}{\alpha}\right)^{1/\nu}. \tag{29}$$

With $k \approx 1$ GeV^2 and $\nu = 1/2$, $(-s_{\text{pole}}) \approx 10^5$ GeV^2, with $\nu = 3/4$, $(-s_{\text{pole}}) \approx$ $\approx 4\times 10^2$ GeV^2 and as $\nu \to 1$ the pole moves rapidly to the origin.

Finally, for the propagator (27) it is clear that the pole is not on the real axis, since for $s < 0$ there is no cancellation while for $s > 0$ there is an imaginary part, so that the zero must be complex. To find the pole of (27) we have to solve:

$$1 = \frac{\alpha}{3\pi}(ks)\,[\ln(-ks)+a],$$

$$e^a = \frac{\alpha}{3\pi}\,e^a(ks)\ln(-kse^a),$$

let

$$x = -kse^a, \text{ or } s = -\frac{xe^{-a}}{k},$$

and

$$\Lambda = \frac{3\pi}{\alpha}\,e^a,$$

then our equation is $-\Lambda = x\ln x$, with $x = \varrho e^{i\varphi}$ this is equivalent to two real equations:

$$0 = \varphi\cos\varphi + (\ln\varrho)\sin\varphi$$

and

$$-\Lambda = \varrho((\ln\varrho)\cos\varphi - \varphi\sin\varphi),$$

with solution:

$$s_{\text{pole}} = \left(\frac{3\pi}{\alpha k}\right)\left(\frac{-e^{i\varphi}\sin\varphi}{\varphi}\right),$$

$$e^a = \frac{\alpha}{3\pi}\,\frac{\varphi}{\sin\varphi}\exp\left[-\frac{\varphi\cos\varphi}{\sin\varphi}\right],$$

or

$$a = \ln\left(\frac{\alpha}{3\pi}\right) + \ln\left(\frac{\varphi}{\sin\varphi}\right) - \frac{\varphi\cos\varphi}{\sin\varphi}.$$

Therefore, given φ we can compute a and s_{pole}, which amounts to computing s_{pole} for the corresponding value of a.

E.g.
$$\varphi = \frac{\pi}{2}, \quad s_{\text{pole}} = -\frac{i6}{\alpha k} \quad \text{and} \quad a = \ln\left(\frac{\alpha}{6}\right) = -6.71$$

$$\varphi = 0, \quad s_{\text{pole}} = -\frac{3\pi}{\alpha k} \quad \text{and} \quad a = \ln\left(\frac{\alpha}{3\pi}\right) - 1 = -8.16,$$

etc. In this way we can trace the motion of the complex pole as a function of a. We should remember that when the pole is complex ($\varphi \neq 0$) there is a pair of poles, necessary to guarantee the reality of (27) on the negative s-axis. The motion of the complex poles is limited to a curve resembling a cardioid. For $a > \ln(\alpha/6)$ the real part of s_{pole} is positive and can be as large as $\text{Re}(s_{\text{pole}}) \approx 1100\ \text{GeV}^2$, taking $k \approx 0.25\ \text{GeV}^{-2}$ from the present data; the value of a is about -4. For larger values of a the pole moves slowly into the origin.

The possibility of the photon propagator having complex poles, in the physical sheet, was raised by Lee and Wick in an attempt to modify Quantum Electrodynamics. The purpose of the modifications was to ensure finite mass differences in isospin multiplets. It is known that as a consequence of these poles the theory (Lee-Wick Electrodynamics) suffers from acausality. It is interesting that here we get the same kind of photon propagator (with complex poles) only by enforcing the requirements that $R(s) \propto s$ and unitarity. It follows in particular that the propagator (27) will give rise to finite electromagnetic mass differences. The net effect of the assumption $R(s) \propto s$ is to give electrodynamics a cut-off mechanism due to the hadrons which couple more and more to time like photons.

How could we tell whether such a mechanism is indeed taking place? We should watch the behaviour of the cross section σ_h. Note that due to the way we defined $R(s)$ the condition $\nu = 1$ is not saying that the cross section σ_h is constant, but that at large s, $\sigma_h \propto s^{-2}$, as was mentioned before. The transition from $\sigma_h \propto$ const. to $\sigma_h \propto s^{-2}$ takes place in the region of the pole and, in general, is accompanied by a peak which looks like a wide resonance in the total cross section. On the low energy side of the "resonance" $\sigma_h \propto$ const. and on the high energy side $\sigma_h \propto s^{-2}$. The peak does *not* correspond to a resonance since it is due to two poles on the physical sheet rather than one pole on the second sheet. However, when the ($\gamma \to \gamma$) amplitude has maximum modulus, its phase is $\pi/2$. In contradistinction from resonant behaviour, the phase of the amplitude moves clockwise (in an Argand diagram) rather than anticlockwise as the energy increases. At the energy of the peak the two contributions to Bhabha scattering in lowest order are out of phase and the angular distribution is markedly different, from that at low energies. The position of the peak, as noted before, depends on the value of the constant a. It can also be checked by numerical computations that the modifications of the photon propagator from s^{-1} in the neighbourhood of $s = 0$, i.e. say between $-50\ \text{GeV}^2 \leqslant s \leqslant +50\ \text{GeV}^2$ are at most 10% for $-4 \leqslant a \leqslant 3$ for the choice $k \approx 0.25\ \text{GeV}^{-2}$.

One may wonder about other low energy tests of Q.E.D. such as the gyromagnetic ratio of the muon. It turns out (N. Cabibbo, private communication) that the contribution to $(g-2)$ from a propagator like (27) is of the form:

$$\Delta g_\mu \cong \frac{2\alpha}{3\pi} m_\mu^2 \Pi'(0) = -\frac{2}{9}\left(\frac{\alpha}{\pi}\right)^2 (km_\mu^2)(\ln ks_0 + a + 1), \tag{30}$$

where s_0 is the value of s at which the cut in the logarithm begins (in (27) we chose $s_0 = 0$). If we choose $s_0 \approx 10\ \text{GeV}^2$, $k \approx 0.25\ \text{GeV}^{-2}$, we obtain $\Delta g_\mu \approx 3 \times 10^{-9}\,(a+2)$ a contribution which is rather small, and would have escaped determination for "reasonable" values of a.

To conclude, the low energy tests of Δg and the photon propagator up to $s = \pm 25\ \text{GeV}^2$ have rather small corrections to Q.E.D. values, and therefore we have to measure the photon propagator at larger values of s or more accurately if we want to discard (27) as untrue.

E. Conclusions

We have examined in detail the possibility that $e\bar{e}$ annihilation, though electromagnetic does not scale, even at high energies, The main theoretical consequence of this assumption is to lead, via unitarity, to a cut-off of Q.E.D. by damping the photon propagator at high energies. As a result the photon propagator acquires complex poles in the physical sheet and resembles somewhat the propagator in Lee-Wick types of theories. On the experimental side, this assumption does not lead to contradiction with available tests of Q.E.D. at low values of s, nor with the observed scaling, in deep inelastic electron scattering, at low values of s. Therefore this possibility should be taken seriously when contemplating what might happen at higher energies. I will now optimistically assume that there is some truth in this hypothesis, and discuss some questions that are raised by it.

(*i*) In the first place, one can and should attempt to compute electromagnetic mass differences, and in principle, this could lead to a determination of the subtraction constant a.

(*ii*) On a more "elevated" level, one can ask whether a similar mechanism could lead to a cut-off of weak interactions. A rough estimate shows that the cut-off in the case of weak interactions would take place at the same C. of M. energies. This raises the interesting possibility of a "joint" cut-off of electromagnetic and weak interactions.

(*iii*) Another question, which may be related to the previous one, is of the value of the cross section σ_h at $s \approx 10$–$25\ \text{GeV}^2$. What determines it (or the constant k, equivalently). In a recent paper, Greenberg and Yodh noted that $\sigma_h \propto G$ where G is the Fermi constant of weak interactions. Could such a coincidence be explained in the context of our considerations?

(*iv*) In a different direction $R \propto ks$ would imply also some constraints on the spectrum of hadronic states and their form factors. Are there simple models for hadronic states, which satisfy this constraint? Clearly one could go on asking such questions, and hopefully one could even answer some of them. The most interesting one is whether the possibility of no scaling is in fact realized in nature. It is nice that we shall be able to know the answer to it, perhaps within the next five years, when the next generation of e^+e^- storage rings will start working.

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