Vol. B5 (1974) ACTA PHYSICA POLONICA No 6

HIGH TRANSVERSE MOMENTUM HADRON-HADRON COLLISIONS*

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(Presented at the XIV Cracow School of Theoretical Physics, Zakopane, June 15–28, 1974)

Some theoretical aspects of the subject of high-$p_\perp$ hadron production in strong interactions are discussed. This includes properties of inclusive spectra, the parent-child relation and its applications to direct lepton and γ-ray production. Also discussed are hypotheses regarding the phase-space populations of associated particles, correlations and production of jets.

Dynamical models are only very briefly discussed, and suggestions for future experimentation are listed.

These notes are a quite subjective and incomplete look at a rapidly developing field. I do not try to review all contributions to it, either experimental or theoretical, and apologize in advance to those whose work is not mentioned here [1].

I. Inclusive Spectra

A large yield of inclusive production of hadrons of high $p_\perp$ (up to $p_\perp \approx$ 8–9 GeV) in pp collisions has been discovered at the CERN ISR, and explored at NAL as well. The inclusive pion spectrum is, at least at large CM angles, now fairly well understood for $10 \lesssim \sqrt{s} \lesssim 50$ GeV. Three major sources of information are:

1) The CERN-Columbia-Rockefeller (CCR) group [2] measure the process $\mathrm{pp} \to \pi^0 +$ all at the ISR. Their results, for roughly $23 < \sqrt{s} < 53$ GeV, $45° < \theta_{\mathrm{CM}} < 135°$, and $2\ \mathrm{GeV} \lesssim p_\perp \lesssim 8$ GeV are summarized in the formula

$$E\frac{d\sigma}{d^3p} = Ap_\perp^{-n}e^{-bp_\perp/\sqrt{s}}\ \mathrm{cm}^2/\mathrm{GeV}^2, \tag{1}$$

with $A = (1.54\pm0.1)\times10^{-26}$, $n = 8.24\pm0.7$, $b = 26.1\pm0.6$.

* Work supported in part by the U. S. Atomic Energy Commission.

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2) A Chicago-Princeton group at NAL measure [3] charged particle spectra at $\theta_{CM} \approx 90°$, $1 \text{ GeV} < p_\perp < 8 \text{ GeV}$, and $E_{lab} = 200\text{–}300$ GeV. For pions they quote a scaling-law, valid for $p_\perp > 0.2\sqrt{s}$:

$$E\frac{d\sigma}{d^3p} \propto \frac{1}{s^{5.5}}\, e^{-72p_\perp/\sqrt{s}}. \tag{2}$$

The spectrum is especially rich in heavy particles: $p/\pi^+ \approx 1$; $K^+/\pi^+ \approx \frac{1}{2}$, while $\bar{p}/\pi^-$ and K^-/π^- are smaller, but strong functions of $p_\perp$ and E_{inc}.

3) An NAL group [4] has measured $pp \to \pi^0 + \text{all}$ in the internal target area ($30 < E_{inc} < 30$ GeV) at fixed laboratory angle of 100 mrad, corresponding *roughly* to $\theta_{CM} \approx 90°$. Their fit, for $1 \text{ GeV} < p_\perp < 4$ GeV and a quite large range of incident energies, is

$$E\frac{d\sigma}{d^3p} \propto (p_\perp^2 + 0.95)^{-4.5} g(x), \tag{3}$$

with $x = p/p_{max}$, no dependence on θ_{CM} (which at fixed θ_{Lab} evidently varies with incident energy), and $g(x)$ having roughly the same shape as the electroproduction structure function νW_2.

These three experiments span quite different kinematical regions, and to see it all at once, I find it useful to plot on log-log paper $E\frac{d\sigma}{d^3p}$ vs $\left(1 - \frac{p}{p_{max}}\right)$ at constant values of $p_\perp$. This is done in Fig. 1, where I have taken some liberties with normalizations in order to produce smooth interpolations. The reasons for this peculiar way of plotting things is that a threshold dependence $(1 - p/p_{max})^m$ can be easily read off, as well as many other qualitative features. Notice that there is little overlap between the three experiments, and that from this perspective it would appear that an especially important area for further measurements is at *lower* E_{inc} ($E_{inc} \approx 50\text{–}200$ GeV) and at $p_\perp \gtrsim 3$ GeV. There seems to be a possibility of a qualitative change in the spectra for $p_\perp \gtrsim 3$ GeV, and the threshold dependence (p/p_{max} large) of such a change might teach us much.

Various theoretical models suggest that there should exist at sufficiently high $\sqrt{s}$ and $p_\perp$ a scaling law of the general form

$$E\frac{d\sigma}{d^3p} \propto \frac{1}{p_\perp^n} f\left(\frac{p}{p_{max}}, \theta_{CM}\right). \tag{4}$$

This form can be motivated in quite a few ways, but we mention only a few:

1) The relevance of θ_{CM} (or equivalently θ_{Lab} or the rapidity y, which equals log tan $(\theta_{CM}/2)$ at high $p_\perp$) is that it is approximately conserved in "soft" internal processes involving the evolution of a system of *high* $p_\perp$ moving in the direction of θ_{CM}. That is, if the Lorentz factor γ of the high-$p_\perp$ system is large, the system is likely to maintain its direction of motion as it evolves.

2) Provided θ_{CM} is approximately conserved, as mentioned above, then p/p_{max} plays the role of Feynman's scaling variable used for low $p_\perp$ processes.

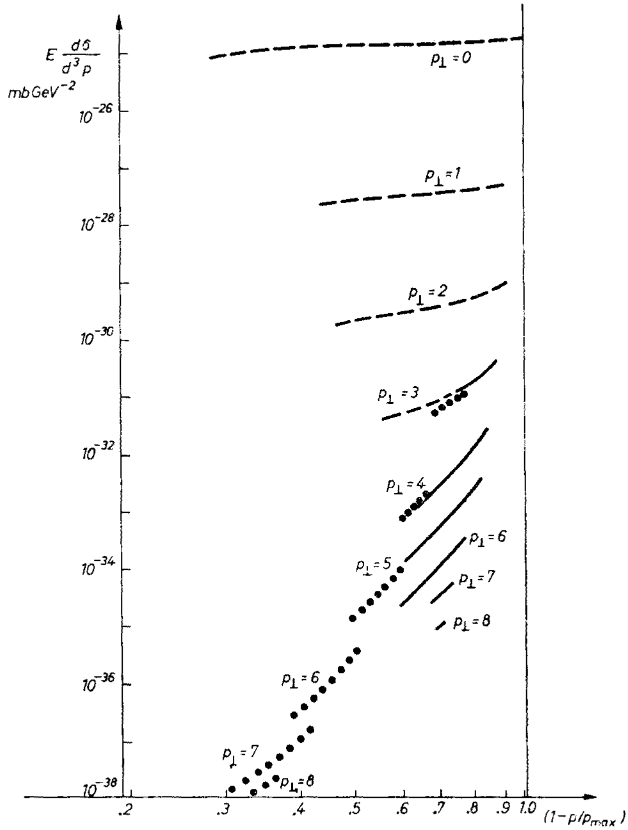

Fig. 1. Sketch of inclusive data measured at NAL and ISR. These curves are only eyeball fits and interpolations of their published data and should not be used for quantitative purposes. The solid line is from reference [2], the dashed line from reference [4], and the dotted line from reference [3]

3) The form of Eq. (4) is invariant with respect to decay processes: e. g. if Eq. (4) is true for inclusive production of ϱ^0, and then one computes the spectrum of decay pions, then the pion spectrum will have the same form. The analogue of this fact at low $p_\perp$ provided some motivation for Feynman's conjectures of scaling and of the existence of the "central plateau", which also share this characteristic of "invariance".

4) The power-law dependence on $p_\perp$ is motivated by the power-law behaviour observed in high-momentum transfer electromagnetic processes (e. g. elastic form factors or threshold behaviours of deep-inelastic structure functions), as well as the success of fitting elastic scattering processes at fixed angle with cross-sections having power-law behaviour [5].

II. Parent-Child Relation

To illustrate point 3) made above, consider the inclusive production of a high-$p_\perp$ parent, such as isobar (or even parton) which decays into secondary hadrons ("children") with limited $p_\perp$ relative to the direction of motion of the parent. For isobar decay (and, if parton, from the hypothesis [6] of "parton fragmentation") one has a scaling law

$$x \frac{dN_c}{dx} = g_{cp}(x), \tag{5}$$

where

$$x \approx E_{child}/E_{parent} \approx |\vec{p}|_{child}/|\vec{p}|_{parent} \approx p_\perp^{child}/p_\perp^{parent},$$

at sufficiently high $p_\perp$. Evidently, because of the limited $p_\perp$ in the decay, the angle of child and parent will be the same at sufficiently high $p_\perp$ (what this means in practice must be in any specific case critically examined). It is then a simple matter to fold Eq. (5) into Eq. (4) and find the same form for the distribution of children, Eq. (4), where

$$f_{child}(x, \theta_{CM}) = \int_x^1 dz z^{n-3} f_{parent}\left(\frac{x}{z}, \theta_{CM}\right) g_{cp}(x). \tag{6}$$

This kind of relation is part of the folklore of cosmic-ray physics. Also useful for semi-quantitative estimations is the notion of local exponent. Namely, one parametrizes

$$E \frac{d\sigma}{d^3p} \propto p_\perp^{-n_{eff}\left(\frac{p}{p_{max}}, \theta_{CM}\right)}, \tag{7}$$

where n_{eff} is just the slope of the experimental distribution when plotted on log-log paper. It is typically a slowly varying function. Furthermore, because the convolution integral involved in Eq. (6) usually involves a sharply peaked function, one is safe in simply taking the local value of n_{eff} when comparing the inclusive distribution for parents and children. That is, at the same momentum

$$\frac{\left(E \frac{d\sigma}{d^3p}\right)_{child}}{\left(E \frac{d\sigma}{d^3p}\right)_{parent}} \approx \int_0^1 dz z^{n_{eff}-3} g_{cp}(z). \tag{8}$$

The function $g_{cp}(z)$ is typically simple; hence (8) is easy to evaluate. Typical values of n_{eff} are shown in Fig. 2; for practical high-$p_\perp$ cases ($p_\perp \gg 2$ GeV), we can put $n_{eff} \approx 12$ to 20% accuracy or so.

Thus only the dependence of $g_{cp}(z)$ near $z = 1$ is crucial. For an isotropic 2-body decay into particles of negligible mass ($\varrho \to \pi\pi$, $\pi^0 \to \gamma\gamma$, etc.), $g_{cp} = 2z$ and

$$\frac{\text{child}}{\text{parent}} = \frac{2}{n_{eff}-1} \tag{9}$$

and if $g(z) \approx (1-z)^p, p \ll n_{eff}$

$$\frac{\text{child}}{\text{parent}} \approx \frac{1}{(n_{eff})^{p+1}}. \tag{10}$$

Thus parents, or children emering from two-body decays of parents, are highly favored objects to be found in an inclusive spectrum.

What is the parent of the inclusive hadrons we observe? We can note here, without further discussion, various possibilities

1) *No parents*; i. e. parent = child.

2) *Low-mass isobars.* These are not likely to predominate because of Eq. (8). If $\varrho/\pi = 3$, then the ratio of inclusive pions from ϱ-decay to direct π's is

$$\frac{\pi_{\varrho\text{-decay}}}{\pi_{\text{direct}}} \approx \frac{3\times 2}{n_{\text{eff}}-1} \approx 0.6. \tag{11}$$

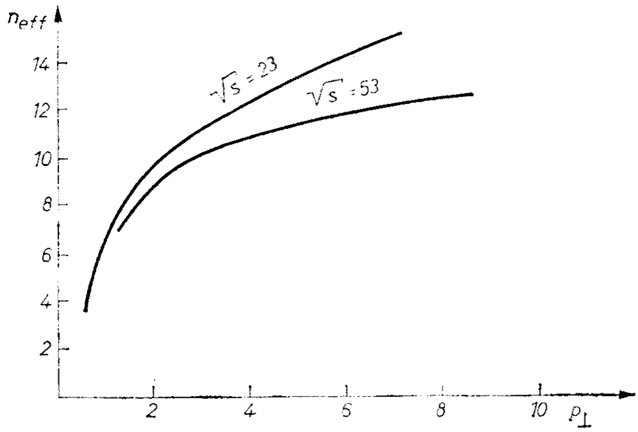

Fig. 2. Effective exponents $n_{\text{eff}} = -p_\perp \frac{d}{dp_\perp}\left[\log E\frac{d\sigma}{d^3p}\right]$ for inclusive distribution as measured at NAL and the CERN ISR

While significant, and not to be totally neglected, they do not dominate. The same is true for nucleons from Δ-decay, even assuming $\Delta/\text{N} \approx 4$, and taking into account the fact that $g_{\Delta n}(x) \approx 2$ for $x \approx 1$ (because of the finite nucleon mass)

$$\frac{\text{N}_{\Delta\text{-decay}}}{\text{N}_{\text{direct}}} \approx \frac{4\times 2}{n_{\text{eff}}-2} \approx 0.8. \tag{12}$$

Of course any isobar (such as ω) with 3-body decays gives a negligible contribution because of Eq. (10), as well as kaons from φ-decay (because $g(x) \approx \delta(x-\frac{1}{2})$), etc.

3) *Partons.* If the basic dynamical process is production of a high $p_\perp$ parton, as conjectured for deep-inelastic weak and electromagnetic processes [6], followed by its evolution into a hadron system according to "parton fragmentation", then the ratio of partons to hadrons depends on the dependence of $g(x)$ near $x = 1$. If

$$g(x) \approx (m+1)\,(1-x)^m, \tag{13}$$

(where $1 \lesssim m \lesssim 2$ is a typical kind of guess) we again get from Eq. (8):

$$\frac{\text{child}}{\text{parton}} = \frac{\Gamma(n_{\text{eff}}-2)\Gamma(m+2)}{\Gamma(n_{\text{eff}}+m-1)} \approx (m+1)!\left(\frac{1}{n_{\text{eff}}}\right)^{m+1}. \tag{14}$$

Even for $m = 1$, we get

$$\frac{\text{child}}{\text{parton}} \approx \frac{1}{50}. \tag{15}$$

4) *New particles* (e. g. charmed hadrons).

Heavy particles $X(M \gg 1 \text{ GeV})$ may be expected to decay predominantly into modes containing 3 or more particles. If this is true, the inclusive spectrum of the parent X must be much larger than the spectrum of any of its children, i. e. of any ordinary hadron. Furthermore, if semileptonic decays of X have an appreciable branching ratio, we can expect $g_{X\mu}(x)$ and $g_{Xe}(x)$ near $x = 1$ to be at least as large as $g_{Xh}(x)$. This would imply a μ/h or e/h ratio comparable to unity, not the observed 10^{-4}. Thus it is necessary that any such X should decay overwhelmingly into hadrons.

III. Direct Production of Leptons and Photons

As an example of a parent-child relation consider the process $pp \to \mu$ + hadrons via decay of mesons into muon pairs, as suggested by Chicago-Princeton group at NAL who observed [7] the direct muon production process. Assume the inclusive production of ϱ^0, ω^0, η and φ^0 is approximately a constant multiple of the pion and kaon production. Then from the parent-child relation (4)

$$\frac{\mu^-}{\pi^-} = \frac{\mu^-}{\pi^0} = \frac{1}{n_{\text{eff}}-1}\left[(6.7\times 10^{-5})\frac{\varrho^0}{\pi^0} + (7.6\times 10^{-5})\frac{\omega^0}{\pi^0} + \right.$$
$$\left. + (2.5\times 10^{-4})\left(\frac{\varphi^0}{K^+}\right)\left(\frac{K^+}{\pi^+}\right) + (2\times 10^{-5})\frac{\eta}{\pi^0}\right], \tag{16}$$

where the numerical factors are the branching ratios of the vector mesons into lepton pairs.

What should we take for ϱ^0/π^0, ω^0/π^0, η/π^0 and φ^0/K? The simplest choice would seem to be the quark-model value [8] of 3, for the vector mesons, and 1 for the η/π^0 ratio, which follows simply from assuming that dynamical a s-wave $q\bar{q}$ pair is produced in any spin configuration (at high $p_\perp$) with equal probability. This gives (using the experimental $K^+/\pi^+ \approx \frac{1}{2}$ and $n_{\text{eff}} \approx 12$)

$$\frac{\mu^-}{\pi^-} = 0.1(2.0\times 10^{-4} + 2.3\times 10^{-4} + 3.7\times 10^{-4} + 0.2\times 10^{-4}) \approx 8.2\times 10^{-5}, \tag{17}$$

which agrees very well [9] with the observed ratio, which is in fact nearly independent of $p_\perp$ ($2 < p_\perp < 6$ GeV), and of atomic number of the target.

The ratio μ^-/e^- is evidently expected to be ≈ 1 inasmuch as the branching ratios of the mesons into e^+e^- and $\mu^+\mu^-$ are very nearly equal.

The same idea can be used to estimate the yield of "direct" photons, produced via decays of $\omega(\pi^0\gamma)$ and $\eta(\gamma\gamma)$. Repeating the previous considerations gives

$$\frac{\gamma}{\pi^0} = \frac{1}{n_{\text{eff}}-1}\left[0.09\left(\frac{\omega}{\pi^0}\right) + 0.76\left(\frac{\eta}{\pi^0}\right)\right]. \tag{18}$$

Using $n_{\text{eff}} \approx 12$, $\omega/\pi^0 = 3$, and $\eta/\pi^0 = 1$ as before gives

$$\left(\frac{\gamma}{\pi^0}\right) \approx 0.10. \tag{19}$$

Inasmuch as the γ-rays from π^0 (call them γ_{π^0}) give an inclusive ratio

$$\frac{\gamma_{\pi^0}}{\pi^0} = \frac{2}{n_{\text{eff}}-1} \approx 0.2, \tag{20}$$

this gives

$$\frac{\gamma_{\text{direct}}}{\gamma_{\pi^0}} \approx 0.5, \tag{21}$$

and there should be an $\approx 50\%$ discrepancy in the inclusive γ-spectrum from what is expected from π^0 decays alone, with most of this to be attributed to η-decays.

IV. Populations in Phase Space

The most safe prediction regarding the theories of the inclusive spectra is that many different approaches will succeed in fitting the data. (However, already one sees even this is not trivial; most published work fits pion spectra and does not attempt to understand the K/π, p/π, $\bar{p}/\pi$ ratios and their energy dependence. Inasmuch as these numbers are large, this is a serious deficiency). But it is clear that observation of the particles produced in association with a high-$p_\perp$ hadron will be of great help in focusing our ideas regarding the underlying dynamics. An immediate problem is to try to determine the regions of phase-space in which other associated particles are most likely to be found. Useful variables in this regard are $p_\perp$ and rapidity y. One certainly expects particles emitted with limited $p_\perp$ and any value of y. The interest focuses more on the other high-$p_\perp$ hadrons. For this, the following hypotheses [10] are natural, and shared by most models of high-$p_\perp$ dynamics:

1. Jet on the Same Side:

All high-$p_\perp$ particles (if any) emitted in the same hemisphere as the highest-$p_\perp$ particle emerge essentially in the same direction. More quantitatively, consider the collinear Lorentz frame for which the initial beam momenta are in the z-direction, and the highest-$p_\perp$ particle emerges in the $+x$ direction (at 90° to the beam axis). Then this hypothesis states that for particles emitted with p_x positive, and $p_x \gg 0.5$ GeV, the distribution in $\sqrt{p_y^2+p_z^2}$ falls steeply, with perhaps $\sqrt{\langle p_y^2+p_z^2\rangle} \lesssim 0.4$ GeV, as for ordinary processes.

2. Coplanarity:

For any particles emitted, the mean of the momentum component in the direction normal to the plane defined by beam momenta and the momentum of the highest $p_\perp$ particles is small. More precisely, using the coordinate system defined in hypothesis 1, the distribution in p_y of any subgroup (such as defined by large positive p_x, small p_x, or large negative p_x) of particles emitted falls steeply, with perhaps $\langle p_y \rangle \lesssim 0.5$ GeV.

There is already some evidence [11] from the CCR group at the ISR that Hypothesis 2 is wrong. They look at π^0's emitted in opposite hemispheres each with $p_\perp > 2$ GeV. Again using the z-axis as beam direction and defining φ as in Fig. 3, they fit the φ distribution as

$$\frac{dN}{d\varphi} \propto e^{-B \sin \varphi} \tag{22}$$

and find $B = 1.5 \pm 0.2$. This means

$$\langle \sin \varphi \rangle \approx \langle \varphi \rangle \approx \frac{1}{1.5}. \tag{23}$$

$$\text{But } \langle p_y \rangle \gtrsim (2 \text{ GeV}) \langle \sin \varphi \rangle \gtrsim \frac{2}{1.5} \approx 1.3 \text{ GeV!} \tag{24}$$

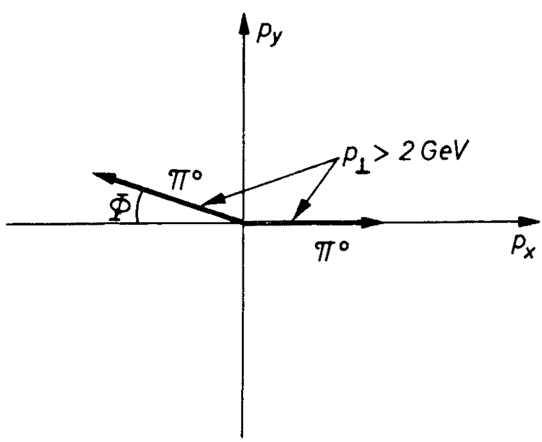

Fig. 3. Kinematics of $\pi^0 - \pi^0$ correlation measurements

This observation was not discussed in much detail by CCR, but deserves the most careful attention. If generally confirmed, it is a very fundamental result which impacts on many models of high-$p_\perp$ hadron production. It should, if possible, be checked for $p_\perp \gtrsim 4$ GeV, beyond possible transition region indicated by the inclusive data.

3. Jet on the Opposite Side:

The first part of this hypothesis is that *in the opposite hemisphere there exists with high probability at least one high-$p_\perp$ particle.* (To avoid this requires a linearly rising associated multiplicity $\Delta\bar{n} \gtrsim (2\text{–}3)\left(\frac{p_\perp}{1 \text{ GeV}}\right)$). The rest of the hypothesis is that *other high-$p_\perp$ particles in that hemisphere all emerge in the same direction, just as in Hypothesis 1.*

These hypotheses give the result that the important regions in phase space lie in a plane and that in $p_\perp - y$ space the main high-$p_\perp$ regions containing particle are "needles" with opposite signs of $p_\perp$. The regions for a typical configuration are shown in Fig. 4. We do not mean to imply that all shaded regions contain particles, but only that it is very unlikely to find particles elsewhere.

Just as for ordinary processes, we may identify fragmentation regions and plateaux within the shaded areas. A general discussion has been given by Savit [12], and by me in

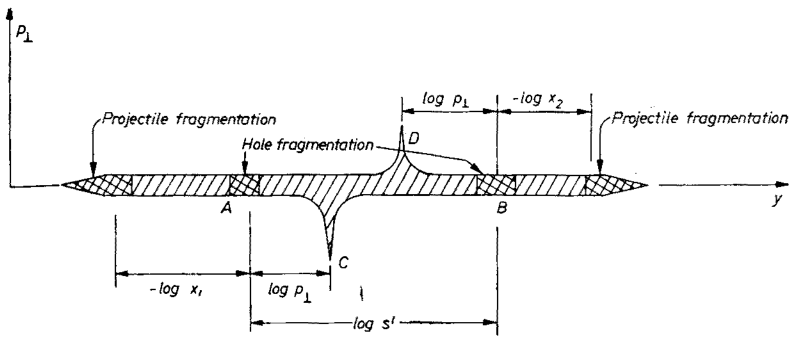

Fig. 4. Populations of secondary hadrons in a typical high-$p_\perp$ event, according to the three hypotheses made in Section IV. We have taken $p_\perp^2 \ll s$, and illustrated the hole fragmentation and projectile fragmentation regions as well

the Aix-en-Provence Conference proceedings [1]. In general, as many as 15 different regions can be identified. However, in practice these will not all exist because there is not enough cms energy available. The most important regions are the high-$p_\perp$ regions and the hole-fragmentation regions. These may be identified as shown in Fig. 4 as low-$p_\perp$

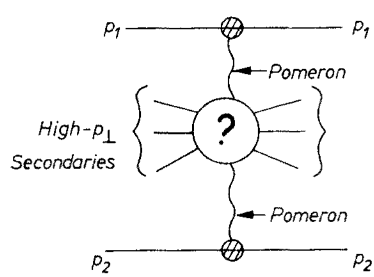

Fig. 5. Mueller-Regge diagram for high-$p_\perp$ inclusive production

regions $\propto \log p_\perp$ to right and left of the high-$p_\perp$ "needles". They may be motivated by appeal to Mueller-Regge ideas. For fixed $p_\perp$ and sufficiently high s, we may expect to be able, according to conventional ideas embodying concepts of short-range correlations in rapidity, to have the leading hadrons and projectiles connected to the "interesting" high-$p_\perp$ regions by hadronic plateaux, or equivalently, by Mueller-Regge Pomerons (Fig. 5). We wish to determine the length of the hadronic plateaux. The interior boundaries of those plateaux mark the hole-fragmentation regions. The boundary can be determined by keeping $p_\perp$ fixed and lowering the beam energy until we arrive at a minimum-energy configuration. When this has occurred the Pomerons will have disappeared, since a large fraction of the incident cms energy goes into production of high-$p_\perp$ secondaries. This happens when $s \approx 4p_\perp^2 e^{\Delta y}$, with Δy the separation in rapidity of the two high-$p_\perp$ needles. In terms of the phase-space distributions this evidently occurs when the projectile fragmentation regions merge with the hole-fragmentation regions, as defined in Fig. 4.

V. *Binary-Collision Models*

The structure of the phase-space populations for which we have argued is very suggestive of a picture in which constituents of the initial hadrons (in either the parton or multiperipheral sense) suffer a high-$p_\perp$ *2-body* collision and then possibly fragment into a multiparticle system. In other words the dynamics consists of left-mover A colliding with right-mover B producing systems C and D in the final state, in associated with debris (or "bremsstrahlung") along the beam directions. The initial objects A and B have momenta, or rapidity, which locate them in the appropriate "hole-fragmentation" regions, and C and D are of course found in the high-$p_\perp$ "needles". If A and B carry internal quantum numbers such as Q, B, Y, then these can be transported into the high-$p_\perp$ regions C and D of phase-space; there will consequently be a deficiency in those quantum-numbers in the regions A and B (hence the name hole-fragmentation). The hole-fragmentation regions are, therefore, interesting regions of phase-space to explore experimentally. In practical circumstances they somewhat overlap with the projectile fragmentation regions. An implicit assumption made here is that only the underlying two-body high-$p_\perp$ collision $A+B \to C+D$ transports momentum and internal quantum numbers Q, B, Y, etc. over large intervals of phase-space and all the other dynamical elements responsible for the production of the many final-state hadrons have the property of short-range correlation in rapidity familiar from multiperipheral Mueller-Regge, and parton-model concepts.

By summing all the 3-momenta $\vec{p}$ in each "needle" C and D we can estimate the total $p_\perp$ and θ of the objects C and D produced in the collision. These evidently are the "parents" discussed in Section I. Whether hadron, isobar or parton we shall suppose that at sufficiently high-$p_\perp$ the "mass" of C and D can be safely neglected. Thus from $p_\perp$ and θ (or y) we can construct null 4-vectors p_C^μ and p_D^μ, which provide sufficient information to reconstruct p_A and p_B, using energy-momentum conservation for the process $A+B \to C+D$. If experimentally one could determine the total $p_\perp$ emitted into the "needle" regions C and D, as well as the rapidities of the needles (much easier), one could have a chance at studying fairly directly the actual 2-body process $A+B \to C+D$. The $p_\perp$ and θ determination can be done experimentally by use of calorimeters which measure the total energy deposition in a given region of solid angle. However, there are (at present energies), in addition to practical difficulties of attaining high precision, an in *principle* difficulty, associated with the fluctuations in energy deposited into even a quite large solid angle surrounding the jest axis [13]. Such fluctuations render precise measurement of the $p_\perp$ of a jet impossible, even in principle. I estimate the uncertainty $\Delta p_\perp$ in determining the total $p_\perp$ of a jet is $\gtrsim 1$ GeV (independent of the $p_\perp$ of the jet, if a scaling law such as Eq. (4) controls the hadron distribution). Because of the steep fall of the inclusive jet spectrum with $p_\perp$, this may make quantitative study quite difficult. For $p_\perp \gg 10$ GeV (and $\sqrt{s} \gg 100$ GeV!!), however, this problem should be less severe.

If measurements of jets turn out somehow to be feasible, then it is straightforward to write down the inclusive production cross-section in terms of the differential cross--section $d\sigma/dt'$ for the process $A+B \to C+D$ and of assumed Feynman-like distribution functions $f_A(x_1)$ and $f_B(x_2)$ for the momenta $p_A = x_1 p_1$ and $p_B = x_2 p_2$

$$x_1 \frac{dN_A}{dx_1} = f_A(x_1), \qquad x_2 \frac{dN_B}{dx_B} = f_B(x_2). \tag{25}$$

The cross-section is, after some careful calculation,

$$\frac{d\sigma}{dp_\perp^2 dy_C dy_D} = \sum_{A,B} f_A(x_1) f_B(x_2) \frac{d\sigma}{dt'}(s', t')_{AB\to CD}. \tag{26}$$

From measurements of θ_C, θ_D and $p_\perp$ one can reconstruct x_1, x_2 and the 2-body scattering variables s' and t'. In the CM frame

$$s' = p_\perp^2 \left(1+\tan\frac{\theta_C}{2}\cot\frac{\theta_D}{2}\right)\left(1+\cot\frac{\theta_C}{2}\tan\frac{\theta_D}{2}\right), \tag{27}$$

$$t' = -p_\perp^2 \left(1+\tan\frac{\theta_C}{2}\cot\frac{\theta_D}{2}\right) \tag{28}$$

and

$$x_1 = \frac{p_\perp}{\sqrt{s}}\left(\cot\frac{\theta_C}{2}+\cot\frac{\theta_D}{2}\right), \tag{29}$$

$$x_2 = \frac{p_\perp}{\sqrt{s}}\left(\tan\frac{\theta_C}{2}+\tan\frac{\theta_D}{2}\right). \tag{30}$$

Eq. (26) is the basic formula expressing the ideas underlying Hypotheses 1, 2 and 3. If correct, or at least relevant, it raises three central issues:

1) What are the important constituents A and B? This will be best examined studying the dependence of the production spectra on projectile type (e. g. n, p, $\bar{\text{p}}$, π, K); the flux of constituents A and B should depend strongly on the nature of the projectile.

2) How do the constituents interact? To study $d\sigma/dt'$, a jet experiment using calorimeters as detectors would be ideal. But even if it is too difficult, then the scaling behaviour of inclusive and double inclusive cross-sections will reflect directly the scaling behaviour of $d\sigma/dt'$ (provided the functions $f(x)$ and $g(x)$ exist and have no intrinsic scale associated with them).

3) What are the high-$p_\perp$ fragments of the "parents" C and D (and what are the parents)? We have already discussed the relationship between high-$p_\perp$ par-

ents and high-$p_\perp$ children. A good way to study the structure of the fragmentation of D is to trigger on C, either by observing a single high-$p_\perp$ particle, or with a calorimeter, and then observe the particle or particles emitted from D on the opposite side. Some experimental information on such $\pi^0-\pi^0$ correlations from the ISR exists [11]. With the trigger being a 5 GeV π^0 from C, the inclusive distribution of π^0's emitted by D has been observed in a limited angular regions. The data has not been presented in a convenient form for this application, but my own rough estimates [14] of the inclusive spectrum indicate that the parent D, whatever it may be, fragments into a multiparticle ($n \gtrsim 3$) system, because there are very few single π^0's in D with $p_\perp \gtrsim 2.5$ GeV (as would be expected if $D = \pi^0$, or if D decays into only two bodies). This conclusion is supported also by the measurements of associated charged multiplicity [15] which also indicate fragmentation, for $p_\perp \gtrsim 5$ GeV, of system D into at least 2–3 additional charged particles.

Thus a typical high-$p_\perp$ event consists of at least one high-$p_\perp$ "needle" C or D containing a multiparticle system. It is not clear whether both sides C and D predominantly tcontain a multiparticle system (jet). To determine this, one may have to measure inclusive production of jets. For if one triggers inclusively on a single high-$p_\perp$ hadron from jet C, one obtains a biased selection of events for which (according again to the parent-child relations in Eq. (4)),

$$\frac{\langle p_\perp \rangle_{\text{hadron}}}{(p_\perp)_{\text{jet}}} \approx \int_0^1 dx x^{n_{\text{eff}}-3} g(x) \gtrsim 0.7-0.8. \tag{31}$$

Hence for $(p_\perp)_{\text{jet}} \approx$ 5–10 GeV, the energy imparted to the remaining members of the jet should on the average be not much more than 1–2 GeV. The remaining hadrons configuration could be mistaken for a single particle, because may be very poorly correlated in angle with the trigger hadron. In any case they will be much fewer in number than the hadrons produced on the opposite side. At least quantitatively, this does not disagree with the observations.

What are the constituents A, B, C, D? The easiest guess is quark-partons (or neutral gluons) as used in the interpretation of deep-inelastic electroprbduction experiments. It is tempting to conjecture that $d\sigma/dt'$ for the elementary processes

$$\begin{aligned} q+q &\to q+q, \\ q+\bar{q} &\to q+\bar{q} \end{aligned} \tag{32}$$

obey dimensional scaling i. e. $d\sigma/dt' \propto (s')^{-2} f(\theta_{\text{CM}})$. This would lead to dimensional scaling for the inclusive cross-section as well: $E\dfrac{d\sigma}{d^3p} \propto p_\perp^{-4} f\left(\dfrac{p}{p_{\max}}, \theta_{\text{CM}}\right)$. Instead, the exponent is experimentally near 8, suggesting either damping of the qq process or some other mechanism [16]. One such mechanism might be $J = 0$ gluon-gluon scattering via $J = 0$ exchange, giving an $(s')^{-4}$ behaviour to $d\sigma/dt'$ and a $p_\perp^{-8}$ behaviour for the inclusive spectrum [17].

Another guess for the nature of A and B is that they are hadrons, as is the case for the multiperipheral model and variations thereof. Brodsky, Blankenbecler, and Gunion [18], Brodsky and Farrar [19], and Landshoff and Polkinghorne [20] prefer to mix things up, sometimes using partons, and sometimes "hadrons" for initial constituents. Thus for $A+B \to C+D$ one could have

$$\begin{aligned} q+M &\to q+M, \\ q+\bar{q} &\to M+\overline{M}, \\ q+q &\to B+\bar{q}, \\ q+B &\to q+B, \text{ etc.} \end{aligned} \tag{33}$$

where M and B are hadrons of limited mass (say, $\lesssim 2$ GeV). Brodsky and Farrar propose simple scaling laws for these elementary processes depending on the total number N of quarks in the external lines for the scattering amplitude:

$$\frac{d\sigma}{dt'} \propto (s')^{-N+2} f(\theta_{\text{CM}}). \tag{34}$$

This leads in an obvious way to inclusive scaling laws with power $p_\perp^{-2N+4}$. The inclusive spectrum of the associated jet measured by CCR, as well as measurement of associated multiplicity (as we discussed earlier) argue against a large contribution from the process $q+\bar{q} \to M+M$. However if the final system $C+D$ is always hadron $+q$, then the inclusive spectra of jets will be comparable in magnitude to that of single hadrons. Therefore, when triggering on a single particle, one will select overwhelmingly (again because of the parent-child relation) the isolated hadron, not the jet, and the associated system will be the jet. Hence experimentally both the options $C+D =$ hadron + parton and $C+D =$ = parton + parton are open.

Finally an important qualitative feature of the binary-collision mechanism is that internal quantum numbers, in particular B and Y, are transported into the high-$p_\perp$ regions, and in a correlated way. For example, there is high probability of finding a baryon B of high-$p_\perp$ at NAL energies. So if the high-$p_\perp$ system C contains a baryon B, what will be found in D?? If the binary-collision mechanism is $q+q \to q+q$, one would expect another baryon B. If is $q+q \to B+\bar{q}$ one would expect an antibaryon in association. And if one had inclusively produced a gluon in C fragmenting into $B\overline{B}$, then both B and $\overline{B}$ would be found at high-$p_\perp$ on the same side. Thus measurement of the correlations of heavy produced particles could be very useful in untangling some of the dynamics [21].

VI. Concluding Remarks

I hope it is clear that for the present the progress in this field rests mainly with experimentalists, not theorists. From the point of view presented here, obvious but special interest would be

1) Measurements of inclusive spectra with $p_\perp \gtrsim 3$ GeV at low energy ($50 \lesssim E_{\text{inc}} \lesssim$ $\lesssim 200$ GeV) to study threshold behaviour and the possibility of a transition region at $p_\perp \approx 3$ GeV.

2) Measurements of inclusive spectra with π, K, $\bar{K}$ and $\bar{\mathrm{p}}$ incident.

3) Measurement of two-particle correlation functions *including momentum measurement*. This information would suffice to establish or destroy the 3 hypotheses underlying the binary-collision mechanism. Even better, of course, would be inclusion of particle identification. Of special interest, is to measure (with particle identification) the low-$p_\perp$ hadron distributions in the hole-fragmentation regions (in association with a high-secondary).

4) Study of inclusive production of jets to determine whether jet/hadron ≈ 50 or ≈ 1.

5) Study of double-jet production to try to determine $d\sigma/dt'$ as directly as possible. Especially useful would be a study of the dependence upon type of beam particle (π, K, $\bar{K}$, p, $\bar{\mathrm{p}}$).

None of this will be easy, nor will theoretical interpretations become unambiguous and uncontroversial overnight. But it seems to me something of fundamental interest and importance is going on, well worth the efforts required to understand it.

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