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SINGLE PHOTON EMISSION IN ELECTRON-POSITRON COLLIDING BEAM REACTION $e^+e^- \to \mu^+\mu^-$

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We evaluate the energy spectrum of the photons emitted in the reaction $e^+e^- \to \mu^+\mu^-\gamma$, and the hard photon correction to the total cross-section of the reaction $e^+e^- \to \mu^+\mu^-$. We develop a simple technique based on the analytical QED formulae, in particular, on the current conservation.

1. *Introduction*

The study of the muon pair production in the reaction $e^+e^- \to \mu^+\mu^-$ by means of colliding electron-positron beams at high energies (See Fig. 1) is interesting from the point of view of checking quantum electrodynamics at small distances [1]. The experimental analysis of this reaction, however, requires the detailed knowledge of the radiative corrections to the lowest-order in α cross-section [2] and the hard photon correction due to emission of undetected real photons of relatively high energy [3, 4].

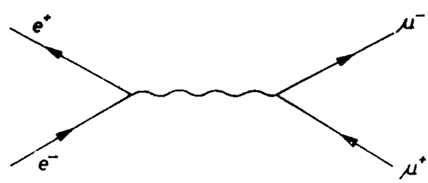

Fig. 1. Feynman diagram for $e^+e^- \to \mu^+\mu^-$

It is not difficult to evaluate the radiative corrections to the differential cross-section for symmetrical detection of muon pairs or to the total cross-section by applying the usual technique [5]. On the other hand, the evaluation of the hard photon correction appears to be rather complicated.

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In this work, we discuss the process

$$e^+ + e^- \to \mu^+ + \mu^- + \gamma \tag{1}$$

on the basis of the lowest-order perturbation theory, assuming a purely electromagnetic electron-muon interaction. This reaction, if accurately measured, can be used to test the validity of quantum electrodynamics at small distances. We will obtain 1) the energy spectrum of the emitted photons, 2) the hard photon correction to the total cross-section of the reaction (1) for emitted photons of c. m. s. energy higher than ε. The main purpose of the work is to present the simple technique based on the analytical QED formulae, in particular, on the current conservation to evaluate the above mentioned quantities. Our approach is close to the one employed in the Ref. [6].

We will restrict our considerations to the case in which the muons' charge is not measured in the experiment. In this case, the total hard photon correction is obtained by summing the contributions of initial and final bremsstrahlungs. We call the process in which the photons are emitted from the initial (final) particles the initial (final) bremsstrahlung.

The plan of the paper is as follows: In Section 2, we describe our method of evaluation in some detail and derive the analytical formulae for the energy spectrum of the photons and report some numerical values. In Section 3, we discuss the hard photon corrections. Some numerical results are also given. The asymptotic formulae are derived in Section 4.

2. *The energy spectrum of the photons*

a. Initial bremsstrahlung

We evaluate the energy spectrum for the initial bremsstrahlung. The two lowest-order Feynman diagrams due to the initial bremsstrahlung are shown in Fig. 2.

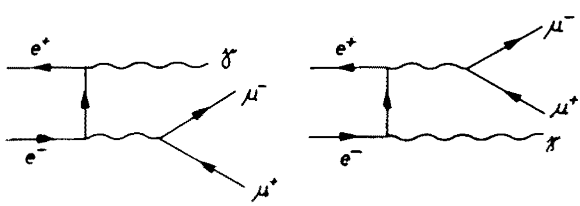

Fig. 2. Feynman diagrams for the initial bremsstrahlung in $e^+e^- \to \mu^+\mu^-\gamma$

We use the following notation for the energy momentum four vectors:

$$e^-(p) + e^+(\bar{p}) \to \mu^-(q) + \mu^+(\bar{q}) + \gamma(K)$$

$$(\text{or } \mu^-\mu^+(Q) + \gamma(K)). \tag{2}$$

The scattering amplitude, in this case, is of the form [7]

$$\mathcal{M} = i\varepsilon_\mu(K)\mathcal{M}^{\mu\alpha}(p, \bar{p}, Q, K)J_\alpha(Q), \tag{3}$$

where $J_\alpha(Q)$ is the electromagnetic current provided by the muon pair, $\varepsilon_\mu(K)$ is the polarization vector of the photon and $\mathscr{M}^{\mu\alpha}(p, \bar{p}, Q, K)$ is the scattering amplitude for the process

$$e^-(p)+e^+(\bar{p}) \to \gamma(K)+\langle\!\langle\gamma\rangle\!\rangle(Q). \tag{4}$$

We write the absolute square of the matrix element as

$$|M|^2 = \sum_{\text{spins}} |\mathscr{M}|^2 = \sum_{\text{spins}} (-g_{\mu\nu})\mathscr{M}^{\mu\alpha}\mathscr{M}^{\nu\beta+}J_\alpha(Q)J_\beta^+(Q). \tag{5}$$

For the product of the muon pair currents we have

$$T_{\alpha\beta} = \sum_{\text{spins}} J_\alpha(Q)J_\beta^+(Q) = 2\{(Q_\alpha Q_\beta - Q^2 g_{\alpha\beta}) - Q'_\alpha Q'_\beta\}, \tag{6}$$

where $Q = q+\bar{q}$, $Q' = q-\bar{q}$. Next we recall the relation

$$\frac{d^3q}{2q_0}\frac{d^3\bar{q}}{2\bar{q}_0} = \frac{\bar{v}}{8}\frac{d^3Q}{2Q_0}dwd\Omega, \tag{7}$$

where $w = Q^2$ = invariant mass squared of the muon pair, $d\Omega$ = angular variable of the muon pair defined at the rest system of the pair, $\bar{v} = \sqrt{1-\dfrac{4\mu^2}{w}}$ = velocity of the muons with the mass μ in the rest system of the muon pair.

In Eq. (6) we perform the angular integration over $d\Omega$ which has no kinematical constraints and, therefore, covers the entire 4π angle. We define

$$\bar{T}_{\alpha\beta} = \frac{1}{4\pi}\int T_{\alpha\beta}d\Omega = \frac{4}{3}\left(1+\frac{2\mu^2}{w}\right)(Q_\alpha Q_\beta - wg_{\alpha\beta}). \tag{8}$$

From Eq. (5) together with the current conservation condition

$$\mathscr{M}^{\mu\alpha}Q_\alpha = 0 \tag{9}$$

and Eq. (8) we get

$$|M|^2 = \sum_{\text{spins}} \mathscr{M}^{\mu\alpha}\mathscr{M}^+_{\mu\alpha}. \tag{10}$$

The calculation of the trace is now straightforward and the result is given in the following manner:

$$\frac{1}{8}|M|^2 = \left(\frac{B}{A}+\frac{A}{B}\right)+(w+2m^2)\left\{-2m^2\left(\frac{1}{A^2}+\frac{1}{B^2}\right)+\frac{2}{AB}(s-2m^2)\right\}, \tag{11}$$

where m is the electron mass, $s = (p+\bar{p})^2$, $t = -(p-K)^2 = -(Q-\bar{p})^2$, $A = (p-K)^2-m^2 = -t-m^2$ and $B = (\bar{p}-K)^2-m^2 = w+t+m^2-s$.

In order to take into account kinematics and phase space, we combine $d^3Q/2Q_0$ in Eq. (7) with the final state photon phase space as

$$\frac{d^3Q}{2Q_0}\frac{d^3K}{2K_0}\delta^4(p+\bar{p}-K-Q) \simeq \frac{4\pi}{8}\frac{dt}{s}. \tag{12}$$

From Eqs. (8), (11) and (12) we obtain the cross-section for the initial bremsstrahlung as

$$d\sigma_{\mathrm{i}} = \frac{\alpha^3}{12}\frac{1}{vs^2}|M|^2\left(1+\frac{2\mu^2}{w}\right)\frac{\bar{v}}{w}\,dwdt, \tag{13}$$

where the invariant flux is expressed as $sv/2$. v is the velocity of the initial particles in the c. m. s., *i. e.*,

$$v = \sqrt{1-\frac{4m^2}{s}}. \tag{14}$$

Eq. (13) is rewritten as

$$d\sigma_{\mathrm{i}} = \tilde{\sigma}_0\frac{1}{8}|M|^2\frac{2\alpha}{\pi}\frac{dwdt}{4vs^2}, \tag{15}$$

where

$$\tilde{\sigma}_0 = \frac{2\pi\alpha^2}{3}(3-\bar{v}^2)\frac{\bar{v}}{w}. \tag{16}$$

The kinematical bound for t is

$$\frac{s-w}{2}(1-v)-m^2 \leqslant t \leqslant \frac{s-w}{2}(1+v)-m^2. \tag{17}$$

We integrate Eq. (11) over t and obtain

$$\frac{1}{8}|\overline{M}|^2 = \frac{\pi}{\alpha}s^{3/2}v\left\{\frac{\gamma_0}{K_0}\left(\frac{w}{s}+\frac{1-v^2}{2}\right)+\frac{2K_0}{s}\gamma_1\right\}, \tag{18}$$

where

$$\gamma_0 = \frac{2\alpha}{\pi}\left(-1+\frac{v^2+1}{v}u\right),$$

$$\gamma_1 = \frac{2\alpha}{\pi}\left(-1+\frac{2u}{v}\right),$$

$$u = \frac{1}{2}\ln\frac{1+v}{1-v}.$$

The result is

$$d\sigma_{\mathrm{i}} = \tilde{\sigma}_0\frac{dw}{2\sqrt{s}}\left\{\frac{\gamma_0}{K_0}\left(\frac{w}{s}+\frac{1-v^2}{2}\right)+\frac{2K_0}{s}\gamma_1\right\}. \tag{19}$$

Taking into account the relation which expresses the dependence of w on the photon energy K_0 in the c. m. s. as

$$w = s-2\sqrt{s}\,K_0 \tag{20}$$

we get the energy spectrum of the photons emitted from the initial electron pair

$$\frac{d\sigma_i}{dK_0} = \tilde{\sigma}_0 \left\{ \frac{\gamma_0}{K_0} \left(\frac{w}{s} + \frac{1-v^2}{2} \right) + \frac{2K_0}{s} \gamma_1 \right\}. \tag{21}$$

This formula coincides with the one derived by Mosco who used a different approach [3].

b. Final bremsstrahlung

We derive the energy spectrum for the final bremsstrahlung. The two lowest-order Feynman diagrams under consideration are shown in Fig. 3.

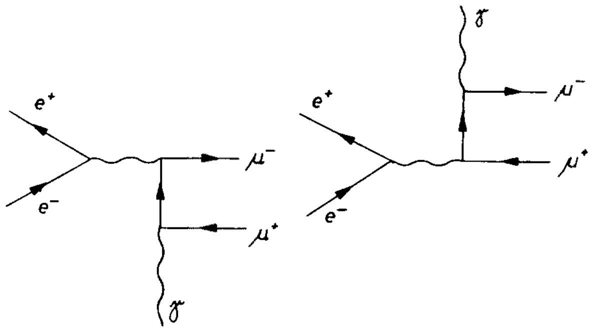

Fig. 3. Feynman diagrams for the final bremsstrahlung in $e^+e^- \to \mu^+\mu^-\gamma$

The scattering amplitude, in this case, is of the form

$$\mathcal{M} = i\varepsilon_\alpha(K)\mathcal{M}^{\alpha\mu}(P, q, \bar{q}, K)J_\mu(P), \tag{22}$$

where $J_\mu(P)$ is the electromagnetic current of the colliding electron-positron pair, and $\varepsilon_\alpha(K)$ is the polarization vector of the photon, and $\mathcal{M}^{\alpha\mu}(P, q, \bar{q}, K)$ is the scattering amplitude for the process

$$\langle\langle\gamma\rangle\rangle(P)+\gamma(-K) \to \mu^+(\bar{q})+\mu^-(q). \tag{23}$$

We define

$$\sum_{\text{spins}} \varepsilon_\alpha \mathcal{M}^{\alpha\mu} \varepsilon_\beta^* \mathcal{M}^{\beta\nu *} = - \sum_{\text{spins}} \mathcal{M}^{\alpha\mu} \mathcal{M}_\alpha^{\nu *} = -M^{\mu\nu}. \tag{24}$$

We first perform angular integration over $d\Omega$ of the muon pair defined in the c. m. s. We define

$$\overline{M}^{\mu\nu} = \frac{1}{4\pi} \int M^{\mu\nu} d\Omega = f_1 \cdot \left(g^{\mu\nu} - \frac{P_\mu P_\nu}{P^2} \right). \tag{25}$$

We determined this form by taking into account the current conservation condition

$$P_\mu \overline{M}^{\mu\nu} = \overline{M}^{\mu\nu} P_\nu = 0. \tag{26}$$

We obtain from Eqs. (22), (24) and (25)

$$|M|^2 = \sum_{\text{spins}} J_\mu J_\nu^+(-\overline{M^{\mu\nu}}) =$$

$$= -2\{(P_\mu P_\nu - P^2 g_{\mu\nu}) - P'_\mu P'_\nu\} f_1 \left(g^{\mu\nu} - \frac{P_\mu P_\nu}{P^2}\right) = 4 f_1 s \left(1 + \frac{2m^2}{s}\right), \tag{27}$$

where $P^2 = (p+\bar{p})^2 = s$, $P'^2 = (p-\bar{p})^2$. On the other hand, we get from Eq. (25)

$$f_1 = \frac{1}{3} g_{\mu\nu} \overline{M^{\mu\nu}} = \frac{1}{3} \frac{1}{4\pi} \int g_{\mu\nu} M^{\mu\nu} d\Omega. \tag{28}$$

Thus, it is enough to evaluate $g_{\mu\nu} M^{\mu\nu}$. After a straightforward calculation of the trace, we get

$$\frac{1}{4} g_{\mu\nu} M^{\mu\nu} = (s+2\mu^2)\left[-\mu^2 \left\{\frac{1}{(qK)^2} + \frac{1}{(\bar{q}K)^2}\right\} + \frac{2}{s+2\mu^2}\left(\frac{\bar{q}K}{qK} + \frac{qK}{\bar{q}K}\right) + \right.$$

$$\left. + \frac{1}{(qK)(\bar{q}K)}(w-2\mu^2)\right]. \tag{29}$$

Substituting Eq. (29) into Eq. (28) we obtain

$$f_1 = \frac{8}{3} \frac{\pi}{\alpha} \left\{\frac{2(s+2\mu^2)w}{(s-w)^2} \bar{\gamma}_0 + \bar{\gamma}_1\right\}, \tag{30}$$

where

$$\bar{\gamma}_0 = \frac{2\alpha}{\pi}\left(-1 + \frac{\bar{v}^2+1}{\bar{v}} \bar{u}\right),$$

$$\bar{\gamma}_1 = \frac{2\alpha}{\pi}\left(-1 + \frac{2\bar{u}}{\bar{v}}\right),$$

$$\bar{u} = \frac{1}{2} \ln \frac{1+\bar{v}}{1-\bar{v}}.$$

We take into account kinematics and phase space in a similar way as in Eqs. (7) and (12). Then the cross-section for the final bremsstrahlung is obtained from Eqs. (27) and (30). It reads as

$$d\sigma_f = \sigma_0 \cdot \frac{3\alpha}{8\pi} f_1 \frac{\bar{v}}{v'} \frac{dt}{s^2} \frac{dw}{2\left(1 + \frac{2\mu^2}{s}\right)}, \tag{31}$$

where σ_0 is the 《elastic》 cross-section to the lowest order for the diagram shown in Fig. 1

$$\sigma_0 = \pi\alpha^2 \frac{v'}{v} \frac{4}{3}\left(1 + \frac{2m^2}{s}\right)\left(1 + \frac{2\mu^2}{s}\right)\frac{1}{s}. \tag{32}$$

v' is the 《elastic》 velocity of the muon in the c. m. s., when $K_0 = 0$, *i. e.*,

$$v' = \sqrt{1 - \frac{4\mu^2}{s}}. \tag{33}$$

The integration over t in the kinematical bound for t (see Eq. (17)) is straightforward and we obtain

$$d\sigma_f = \sigma_0 \frac{3\alpha}{16\pi} \frac{\bar{v}}{v'} (s-w) \frac{f_1}{s} \frac{dw}{s+2\mu^2}. \tag{34}$$

Substituting Eq. (30), we get

$$d\sigma_f = \sigma_0 \frac{\bar{v}}{v'} \frac{dw}{2\sqrt{s}} \left(\frac{w}{s} \frac{\bar{\gamma}_0}{K_0} + \frac{2K_0}{s+2\mu^2} \bar{\gamma}_1 \right). \tag{35}$$

Using Eq. (20), the energy spectrum of the photons emitted from the final muons is expressed as

$$\frac{d\sigma_f}{dK_0} = \sigma_0 \frac{\bar{v}}{v'} \left(\frac{w}{s} \frac{\bar{\gamma}_0}{K_0} + \frac{2K_0}{s+2\mu^2} \bar{\gamma}_1 \right). \tag{36}$$

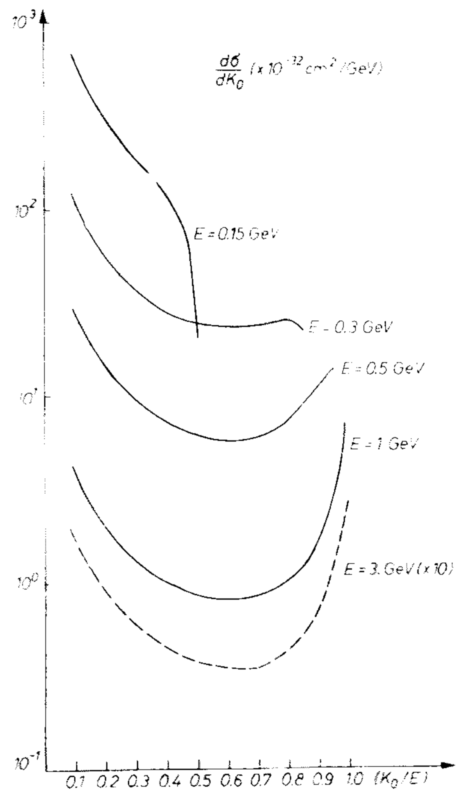

Fig. 4. Energy spectrum of the photons emitted in $e^+e^- \to \mu^+\mu^-\gamma$

This expression coincides with the one derived by Longhi who used a different approach [4].

c. Total energy spectrum of the photons

The total energy spectrum of the photons emitted in the reaction (1) is given by summing the energy spectrum of the initial and final bremsstrahlungs, *i. e.*,

$$\frac{d\sigma}{dK_0} = 2\left(\frac{d\sigma_{\rm i}}{dK_0} + \frac{d\sigma_{\rm f}}{dK_0}\right). \tag{37}$$

The factor 2 in the right-hand side comes from the non-identification of the muons' charge. In this case, the interference term between the diagrams of Figs (2) and (3) does not contribute. In Fig. (4), we report the energy spectrum $d\sigma/dK_0$ for $E = 0.15, 0.3, 0.5, 1.0, 3.0$ GeV, E being the energy of the colliding electrons.

3. *Hard photon correction*

The correction due to emission of real photons with the c. m. s. energy K_0 higher than ε (which is a cut-off determined by the experimental condition) is called hard photon correction. The hard photon correction is obtained by integrating the energy spectrum of the photons over K_0 which may vary from a lower limit ε to an upper limit $\frac{s-4\mu^2}{2\sqrt{s}}$, *i. e.*,

$$\varepsilon \lesssim K_0 \leqslant K_{0\,\max} = \frac{\sqrt{s}}{2} v'^2. \tag{38}$$

In the case of the final bremsstrahlung, the hard photon correction reads as follows:

$$\sigma_{\rm f} = \sigma_0 \delta_{\rm H}^f(\varepsilon), \tag{39}$$

where

$$\delta_{\rm H}^f(\varepsilon) = \frac{1}{v'}\frac{2\alpha}{\pi}\left[\frac{1-v'^2}{4}\left[2\left(\eta-\frac{1}{\eta}\right)-\left(\eta+\frac{1}{\eta}\right)\ln\eta + c\ln\eta + \right.\right.$$

$$+\frac{c}{2}\ln^2\eta + \frac{4v'}{1-v'^2}\ln\left|\frac{(1-v')\eta-1}{(1+v')\eta-1}\right| + c\left\{\int\limits_{1-\frac{\eta}{a}}^{1-\frac{1}{a}} \frac{\ln(1-x)}{x}dx + \right.$$

$$\left.\left.\left. + \int\limits_{1-a\eta}^{1-a} \frac{\ln(1-x)}{x}dx + \ln a \ln\left|\frac{a\eta-1}{a-\eta}\right|\right\}\right] + $$

$$+\frac{(1-v'^2)^2}{16(3-v'^2)}\left[\left\{c(\eta+1)^2/\eta+1-\left(\eta-\frac{1}{\eta}\right)^2\Big/2\right\}\ln\eta+\right.$$

$$\left.+\frac{3}{4}\left(\eta^2-\frac{1}{\eta^2}\right)-2(c+1)\left(\eta-\frac{1}{\eta}\right)\right]\Bigg] \tag{40}$$

and

$$c=\frac{2(1+v'^2)}{1-v'^2},$$

$$a=\frac{1+v'}{1-v'},$$

$$\eta=\frac{1+\sqrt{1-\dfrac{4\mu^2}{s-2\sqrt{s}\,\varepsilon}}}{1-\sqrt{1-\dfrac{4\mu^2}{s-2\sqrt{s}\,\varepsilon}}}.$$

The hard photon correction due to the initial bremsstrahlung is similarly obtained by integrating Eq. (21) over K_0 in the region (38). The integration, in this case, is performed analytically. The result is

$$\sigma_i=\sigma_0\delta^i_H(\varepsilon), \tag{41}$$

where

$$\delta^i_H(\varepsilon)=\frac{2s}{\mu^2}\frac{(1-v'^2)}{(3-v^2)^2(3-v'^2)}\cdot\frac{v}{v'}\left[\left[\frac{v'(3-v'^2)(3-v^2)}{4}\times\right.\right.$$

$$\times\ln\left|\frac{a\eta-1}{a-\eta}\right|-\ln\eta-\frac{1-v^2}{3}\frac{(\eta-1)(\eta^2+4\eta+1)}{(\eta+1)^3}-$$

$$\left.-\frac{\eta-1}{\eta+1}\left\{\frac{(1-v^2)(2-v'^2)}{2}+(1-v'^2)\right\}\right]\gamma_0-\frac{\gamma_1}{2}\left\{\ln\eta-\right.$$

$$\left.\left.-\frac{1-v'^2}{4}\left(\eta-\frac{1}{\eta}\right)-\frac{2(\eta-1)(\eta^2+4\eta+1)}{3(\eta+1)^3}\right\}\right]. \tag{42}$$

In Table I, we report numerical results for $\delta^i_H(\varepsilon)$ and $\delta^f_H(\varepsilon)$ for some values of the cut-off ε and for various values of the energy E of the colliding electrons.

TABLE I

Hard photon correction to the reaction $e^+e^- \to \mu^+\mu^-$

E (GeV)	ε/μ	$\delta_H^i(\varepsilon)$	$\delta_H^f(\varepsilon)$
0.15	10^{-2}	0.231	0.030
	10^{-1}	0.106	0.010
0.3	10^{-2}	0.388	0.072
	10^{-1}	0.245	0.036
0.5	10^{-2}	0.494	0.108
	10^{-1}	0.342	0.060
1.0	10^{-2}	0.646	0.157
	10^{-1}	0.479	0.099
1.5	10^{-2}	0.742	0.196
	10^{-1}	0.567	0.124
2.0	10^{-2}	0.812	0.220
	10^{-1}	0.631	0.142
2.5	10^{-2}	0.871	0.240
	10^{-1}	0.685	0.158
3.0	10^{-2}	0.918	0.258
	10^{-1}	0.729	0.172

4. *Asymptotic formulae*

For the sake of completeness, we derive asymptotic formulae for the energy spectrum of the photons and the hard photon correction. In the asymptotic limit when $s \gg m^2$, we get the following formulae for the energy spectrum of the initial bremsstrahlung:

$$\gamma_0 = \gamma_1 = \frac{2\alpha}{\pi}\left(-1+2\ln\frac{\sqrt{s}}{m}\right), \tag{43}$$

$$\frac{d\sigma_i}{dK_0} = \tilde{\sigma}_0 \frac{\gamma_0}{K_0}\left\{1-\frac{2K_0}{\sqrt{s}}+\frac{1}{2}\left(\frac{2K_0}{\sqrt{s}}\right)^2\right\}. \tag{44}$$

In a similar way we have, in the asymptotic limit $w \gg \mu^2$,

$$\bar{\gamma}_0 = \bar{\gamma}_1 = \frac{2\alpha}{\pi}\left(-1+2\ln\frac{\sqrt{w}}{\mu}\right), \tag{45}$$

$$\frac{d\sigma_f}{dK_0} = \sigma_0\bar{\gamma}_0 \frac{1}{K_0}\left\{1-\frac{2K_0}{\sqrt{s}}+\frac{1}{2}\left(\frac{2K_0}{\sqrt{s}}\right)^2\right\}. \tag{46}$$

Under the asymptotic condition $\sqrt{s} \gg \mu$ and $\mu \gg \varepsilon$, the following formulae are easily derived for the hard photon correction:

$$\delta_{\mathrm{H}}^{\mathrm{i}}(\varepsilon) = \frac{2\alpha}{\pi}\left\{2\ln\frac{\sqrt{s}}{m}\ln\frac{\sqrt{s}}{\mu} - 2\ln\frac{\varepsilon}{\sqrt{s}}\ln\frac{\sqrt{s}}{m} + \ln\frac{\varepsilon}{\sqrt{s}} - \left(\frac{11}{3} + 2\ln 2\right)\ln\frac{\sqrt{s}}{m}\right\}, \tag{47}$$

$$\delta_{\mathrm{H}}^{\mathrm{f}}(\varepsilon) = \frac{2\alpha}{\pi}\left\{-2\ln\frac{\varepsilon}{\sqrt{s}}\ln\frac{\sqrt{s}}{\mu} + \ln\frac{\varepsilon}{\sqrt{s}} - \left(\frac{3}{2} + 2\ln 2\right)\ln\frac{\sqrt{s}}{\mu}\right\}. \tag{48}$$

In the ultrarelativistic case, the dominant contributions come from the double logarithmic terms. In this case, we have

$$\delta_{\mathrm{H}}^{\mathrm{i}}(\varepsilon)/\delta_{\mathrm{H}}^{\mathrm{f}}(\varepsilon) = \frac{\ln\sqrt{s}/m}{\ln\sqrt{s}/\mu}\left(1 + \frac{\ln\sqrt{s}/\mu}{\ln\sqrt{s}/\varepsilon}\right) > 1. \tag{49}$$

This shows that the hard photon correction from the lighter particles (electrons) is larger than the one from the heavier particles (muons). Indeed the numerical result shown in Table I supports this fact.

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[7] See, *e.g.*, J. D. Bjorken, S. D. Drell, *Relativistic Quantum Mechanics*, New York 1964. Our metric is such that the scalar product of two four-vectors a anb b is represented as $(ab) = a_0b_0 - \vec{a}\vec{b}$. Throughout this paper, we use the system of units $\hbar = c = 1$. The fine structure constant α is, therefore, $\alpha = e^2/4\pi$.