DISINTEGRATION OF DEUTERON BY HIGH ENERGY ELECTRON

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(Received April 24, 1973; Revised paper received May 28, 1973)

The semi-relativistic approach, widely applied to the two-body problem, is generalized onto the three-body disintegration process of deuteron by a high energy electron. The proposed way of description of fast moving, Lorentz contracted, composite systems leads us to small anisotropy of the angular distribution of the "spectator" nucleon in the lab-system. This effect increases with energy, but it saturates for electron energy of the order of 20 GeV.

1. Introduction

The relativistic many-body problem, hence in particular the description of the structure of fast moving composite systems like deuteron, seems to be of great importance both for theoretical and experimental reasons. The disintegration of deuteron by a high energy electron seems to be a process most suitable for the investigation of this structure for the following reasons.

1. It is reasonable to assume — as one usually does [1] — that the interaction of an electron with deuteron is dominated by the additive interaction of the electron with constituent nucleons as if they were free. *E. g.*, the interaction of an electron with the mesonic current inside the deuteron is rather negligible [2], particularly for collisions with large momentum transfers to only one of the constituent nucleons, which we are mainly interested in. 2. As the interaction of the electron with nucleons is well known, one can sepa-

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rate (contrary to strong interactions) the structure of the system from that of the interaction, and thus to test the hypothesis concerning the former. 3. Large momentum transfer to one of the nucleons justifies neglecting of the final state interaction between nucleons [3] and an application of the Born approximation to their motion [4]. 4. Small coupling constant and relatively large momentum transfer of the electron justify the suppression of the multiple scattering of the electron [5]. 5. The disintegration process, in contrast to elastic scattering with large momentum, does not require one to use the deuteron wave function at small distances, where the very notion of the wave function becomes vague. The major of the cross-section arises from the kinematically accepted region where the wave function is large or, in other words, where the momenta of the spectator nucleon are of the order of the Fermi momentum.

At the present stage of the theory of high energy processes one has to deal with two different approaches to the relativistic composite systems. The first one maintains covariant symmetry, and in the two-body problem it is represented *e.g.* by the Bethe-Salpeter equation. The second approach, the so-called semi-relativistic one [6], distinguishes dynamically the overall centre-of-mass system (CMS), preserving only the relativistic kinematics. Several phenomenological models such as those based on the quasipotential approach [7] belong rather to the second approach. It is tempting to extend the semi--relativistic approach to the three-body problem of disintegration of the deuteron by an electron. Let us emphasize that the semi-relativistic description, as non-manifestly covariant, tells us nothing about the transformation properties of the wave function under the boost to a reference frame which moves with regard to the centre-of-mass system where this wave function is determined. Another situation occurs within the manifestly covariant theory. The so-called covariant, equal-time wave functions [8] in a moving system can be, in principle, determined from the transformation properties of the corresponding equation of motion; *e.g.* from the Bethe-Salpeter equation. In the semi-relativistic picture of the discussed three-body process we must know the deuteron (two-body) wave function in the overall CMS of the colliding deuteron and electron. Therefore an additional hypothesis is required concerning the transformation properties of the deuteron wave function, known in the rest system of the deuteron, to the CMS. Our assumption is that this wave function represents a static shape, and hence (apart from spins) the Lorentz contraction is the only relativistic distortion of this shape [9]. Consequently, there are no effects such as *e.g.* distortion of the phase of internal motion of the constituents, characteristic for the covariant wave function [10]. Within the framework of a fully covariant theory, our assumption would mean the impulse approximation, where one neglects the internal (relative) motion of the constituents [11]. Meanwhile, according to the proposed hypothesis, the static character of the wave function is a quite general feature independent of the internal motion. Let us illustrate this on an example which supports our point of view. Let $\Psi(t, \mathbf{x}_J, \mathbf{x}_e)$ denote the wave function of the whole atom, $\mathbf{x}_J$, $\mathbf{x}_e$ are the coordinates of the nucleus and electron, respectively. Then the current density $\varrho(t, \mathbf{x})$ is equal to

$$\varrho(t, \mathbf{x}) = Ze \int d^3x_J |\Psi(t, \mathbf{x}_J, \mathbf{x})|^2 - e \int d^3x_e |\Psi(t, \mathbf{x}, \mathbf{x}_e)|^2.$$

We assume that the centre of gravity of the atom at a given time t is well localized in the origin of its rest frame R_0 and the internal wave function $\psi_0(x_{0e}-x_{0J})$ of this atom is real. Then

$$|\Psi_0(t_0, x_{0J}, x_{0e})|^2 \approx \delta[(m_e x_{0e}+m_J x_{0J})/m]\, |\psi_0(x_{0e}-x_{0J})|^2,$$

where $m = m_e+m_J$, and hence

$$j_0(t_0, x_0) = 0, \qquad \varrho_0(t_0, x_0) = \varrho_0(x_0) =$$

$$= Ze\left(\frac{m}{m_e}\right)^3 \left|\psi_0\left(\frac{m}{m_e}x_0\right)\right|^2 - e\left(\frac{m}{m_J}\right)^3 \left|\psi_0\left(\frac{m}{m_J}x_0\right)\right|^2,$$

where x_0 is the space coordinate of R_0. Consequently, the four-potential (A, Φ) takes in R_0 the form

$$A_0 = 0, \qquad \Phi_0(x_0) = \frac{1}{4\pi}\int d^3x\, \frac{\varrho_0(x)}{|x_0-x|}. \tag{1.1}$$

Since (A, Φ) is a four-vector, it determines the transformation properties of the density current, while the static character of Φ_0 implies that ϱ_0, and hence ψ_0 both transform under the boost like static shapes (with corresponding normalization factors) thus distorted only by the Lorentz contraction. Therefore the wave function in a moving system R takes the form

$$\psi(y) = \gamma^{\frac{1}{2}}\psi_0[L(v)y], \tag{1.2}$$

where y denotes the equal-time relative coordinate between the constituents in R. Here $L(v)$ is the 3×3 matrix which in the case of the deuteron motion parallel to z-axis is equal to

$$L(v) = \begin{pmatrix} 1, & 0, & 0 \\ 0, & 1, & 0 \\ 0, & 0, & \gamma \end{pmatrix},$$

where $\gamma = (1-v^2)^{-1/2}$ and v is the velocity between R_0 and R. In further considerations v will always denote the velocity between the lab-system, where deuteron before the collision is at rest, and the overall CMS of the colliding electron and deuteron. Thus

$$v = \frac{p}{E+M_d}, \tag{1.3}$$

where p, E are the momentum and energy of electron in the lab-system, and M_d is the deuteron mass. We see then that for electron energy E less than about 1 GeV the deuteron motion is "nonrelativistic", and hence its internal wave function remains practically undistorted. This is much as if the deuteron were an infinitely heavy body. In this approximation the deuteron disintegration by electron was evaluated by Jankus [1]. The boost of spins will be clarified further on when we construct the scattering amplitude.

2. *Disintegration amplitude*

The deuteron wave function in its rest frame R_0 is taken from low-energy physics, and it is equal to

$$\psi_{0m}(\boldsymbol{y}_0) = \frac{1}{\sqrt{4\pi}\, r}\left[u(r) + \frac{w(r)}{\sqrt{8}} S_{\mathrm{np}}(\boldsymbol{n}_0)\right]\chi_m, \qquad m = 0, \pm 1, \tag{2.1}$$

where $\boldsymbol{y}_0$ is the relative coordinate between nucleons in R_0,

$$r = |\boldsymbol{y}_0|, \qquad \boldsymbol{n}_0 = \frac{\boldsymbol{y}_0}{r}, \qquad S_{\mathrm{np}}(\boldsymbol{n}_0) = 3(\boldsymbol{\sigma}_{\mathrm{p}}\boldsymbol{n}_0)(\boldsymbol{\sigma}_{\mathrm{n}}\boldsymbol{n}_0) - (\boldsymbol{\sigma}_{\mathrm{p}}\boldsymbol{\sigma}_{\mathrm{n}}).$$

χ_m are the triplet wave functions, and normalization of the wave function is such that

$$\int_0^\infty dr[u^2(r) + w^2(r)] = 1, \qquad (\psi_{0m}, \psi_{0m'}) = \delta_{mm'}. \tag{2.2}$$

The structure of the wave function of deuteron suggests a formulation of the interaction between the electron and nucleons in terms of the Pauli spinors. The boost of the nucleon's spin is realized in the following way. Let u denote the Dirac bispinor and φ the corresponding Pauli spinor of a free fermion. Then

$$u = \sqrt{\frac{E+M}{2E}}\begin{pmatrix} \varphi \\ \dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{P}}{E+M}\varphi \end{pmatrix}, \qquad u^\dagger u = \varphi^\dagger\varphi = 1,$$

determines the relation between the Dirac bispinor u and the Pauli spinor φ. Since the initial nucleons bound in the deuteron are (in R_0) described by the Pauli spinors φ_p, φ_n, which determine χ_m, the quantities E, $\boldsymbol{P}$, M will be identified with the energy, momentum, and mass (E_d, $\boldsymbol{p}_d$, M_d), respectively, of the initial deuteron. In other words, the Dirac spinors of the initial nucleons have vanishing "small" components in R_0. This is consistent with the four-component wave function (2.1) assumed as an adequate characteristic of the internal structure of deuteron in R_0. Consequently, the identity

$$(\bar{u}'\alpha u) = (\varphi'^\dagger\tilde{\alpha}\varphi) \tag{2.3}$$

(where primed quantities denote the final state) determines the Pauli (2×2) operator $\tilde{\alpha}$ through the corresponding (4×4) Dirac operator α, and *vice versa*. According to the assumption 1, the interaction between the electron and nucleon takes in the momentum space the well-known form resulting in the Rosenbluth elastic cross-section, namely $(1/t)j_\mu J_\mu^{(\mathrm{N})}$ ($\mathrm{N} = \mathrm{p}, \mathrm{n}$). Here t is the invariant momentum transfer, and j_μ, $J_\mu^{(\mathrm{N})}$ are the current operators of electron and nucleon, respectively, in the momentum space, *i.e.*:

$$j_\mu = e\gamma_\mu, \qquad J_\mu^{(\mathrm{N})} = F_1^{(\mathrm{N})}(t)\gamma_\mu + F_2^{(\mathrm{N})}(t)\sigma_{\mu\nu}\Delta p_\nu, \tag{2.4}$$

where $(\Delta p_\nu)^2 = t$, $F_{1,2}^{(\mathrm{N})}(t)$ are the nucleon form factors. Let us denote by $\boldsymbol{y}$ the relative coordinate between nucleons, $\boldsymbol{y} = \boldsymbol{y}_p - \boldsymbol{y}_n$, and by $\boldsymbol{Y}$ the relative coordinate between the centre of gravity of the deuteron and electron *i.e.* $\boldsymbol{Y} = \frac{1}{2}(\boldsymbol{y}_p - \boldsymbol{y}_n)$, all represented in the overall CMS. Of course, y_p, y_n are the relative coordinates of proton and neutron, correspondingly, with respect to electron, in the same CMS. According to the semi-relativistic approach combined with the assumptions 1–5 the absolute amplitude $M_{\rm if}$ for the disintegration process takes the following form

$$M_{\rm if} = \int d^3y_{\rm p} d^3y_{\rm n} \varphi'^{\dagger} \varphi_{\rm p}'^{\dagger} \varphi_{\rm n}'^{\dagger} \exp\left[-i(\boldsymbol{q}_{\rm p}'\boldsymbol{y}_{\rm p}' + \boldsymbol{q}_{\rm n}'\boldsymbol{y}_{\rm n}')\right] \times \\ \times \left[V^{(\rm p)}(\boldsymbol{y}_{\rm p}) + V^{(\rm n)}(\boldsymbol{y}_{\rm n})\right] \exp(i\boldsymbol{q}\boldsymbol{Y})\gamma^{\frac{1}{2}}\psi_{0m}[L(\boldsymbol{v})\boldsymbol{y}]\varphi \tag{2.5}$$

which manifestly distinguishes the overall CMS. Here $\boldsymbol{q}_{\rm p,n}'$ are the final momenta of proton and neutron, $\boldsymbol{q}$ is the initial momentum of the deuteron, $V^{(\rm p,n)}$ are the "potentials" between nucleons and electron which will be determined from the previously assumed interaction between electron and nucleons and $\boldsymbol{v}$ is defined in (1.3). Let w, w' with corresponding indices (we do not attach any index to the electron only) denote the initial and final CMS energies of the particles. Then the cross-section for the deuteron electro-disintegration takes the form

$$d^9\sigma = \frac{(2\pi)^4\delta^4(P_\mu' - P_\mu)}{2(s-4M^2)} |M_{\rm if}|^2 (2w2w_d2w'2w_{\rm p}'2w_{\rm n}') \times \\ \times \frac{d^3p'}{2E'(2\pi)^3} \frac{d^3p_{\rm p}'}{2E_{\rm p}'(2\pi)^3} \frac{d^3p_{\rm n}'}{2E_{\rm n}'(2\pi)^3}, \tag{2.6}$$

where $s = P_\mu^2 = P_\mu'^2$ is the invariant energy square in the CMS, M is the nucleon mass, while $\boldsymbol{p}'$, E' (with corresponding indices) denote the momenta and energies of final particles in an arbitrary reference frame. As we shall see further, the only reason that the cross--section $d^9\sigma$ depends on the CMS will be due to the deuteron wave function, as all kinematic factors, like w, w' will cancel with the corresponding ones resulting from $|M_{\rm if}|^2$ averaged over the polarization states. The integration over $\boldsymbol{y}_{\rm p,n}$ leads to the amplitude $M_{\rm if}$ represented in the momentum space which is equal to

$$M_{\rm if} = \left(\frac{2M}{w_d}\right) \varphi'^{\dagger}\varphi_{\rm p}'^{\dagger}\varphi_{\rm n}'^{\dagger} \left\{ V^{(\rm p)}(\boldsymbol{q}-\boldsymbol{q}')\psi_{0m}\left[L^{-1}(\boldsymbol{v})\left(\boldsymbol{q}_{\rm n}' - \frac{1}{2}\boldsymbol{q}\right)\right] + \\ + V^{(\rm n)}(\boldsymbol{q}-\boldsymbol{q}')\psi_{0m}\left[L^{-1}(\boldsymbol{v})\left(\frac{1}{2}\boldsymbol{q} - \boldsymbol{q}_{\rm p}'\right)\right]\right\} \varphi, \tag{2.7}$$

where $f(\boldsymbol{q}) = \int d^3y f(\boldsymbol{y}) \exp[i\boldsymbol{q}\boldsymbol{y}]$ means always the Fourier transform of $f(\boldsymbol{y})$. According to (2.1) the deuteron wave function in momentum space is equal to

$$\psi_{0m}(\boldsymbol{p}) = \sqrt{4\pi}\left[U(p) - \frac{1}{\sqrt{8}} W(p) S_{\rm pn}(\boldsymbol{v})\right],$$

where $p = |\boldsymbol{p}|$, $\boldsymbol{v} = \boldsymbol{p}/p$, $U(p) = \int_0^\infty dr\, r u(r) j_0(pr)$, $W(p) = \int_0^\infty dr\, r w(r) j_2(pr)$, and $j_k(z)$ are the spherical Bessel functions. Finally, the "potentials" $V^{(\rm p,n)}$ in momentum space are the operators in the Pauli spinor space which correspond, in the meaning of the identity (2.3), to the operators $(1/t) j_\mu J_\mu^{(\rm p,n)}$ in the Dirac spinor space.

3. Cross-section

Let us assume that all particles are unpolarized, and hence we have to evaluate the quantity

$$A = \tfrac{1}{6} \sum_{\text{pol.}} |M_{\text{if}}|^2, \tag{3.1}$$

where "pol" denotes the polarization of all, initial and final particles, and the factor 1/6 is due to three polarizations of deuteron and two of electron. The one-photon exchange mechanism allows A to be expressed as the contraction of two current tensors, namely $s_{\mu\nu}$ of electron and $S_{\mu\nu}$ of deuteron. Thus we can write

$$A = \frac{1}{6t^2} S_{\mu\nu} s_{\mu\nu}. \tag{3.2}$$

We confine ourselves to the "nonrelativistic" momenta of the spectator nucleon (in the lab-system), when the cross-section is relatively large. Thus the behaviour of the deuteron wave function at very small distances becomes irrelevant which makes the use of the wave function safe. It is justified to put then $w_s' = w_d/2$ ("s" means spectator). Moreover, we neglect the D-state admixture *i.e.* $w(r) = 0$ as, in contrast to the elastic scattering, it does not influence essentially the investigated cross-section. Then the current tensor of deuteron takes the following form

$$S_{\mu\nu} = \left(\frac{8\pi M}{w_d}\right) \sum_{\substack{\alpha_p, \alpha_n \\ m}} \left\{ \varphi'^{\dagger}_{\alpha_p} \varphi'^{\dagger}_{\alpha_n} \left[\tilde{J}^{(p)}_{\mu}(t) U\left(L^{-1}(v)\left(q_n' - \frac{1}{2}q\right)\right) + \right.\right.$$

$$\left.\left. + \tilde{J}^{(n)}_{\mu}(t) U\left(L^{-1}(v)\left(q_p' - \frac{1}{2}q\right)\right)\right] \chi_m \right\} \{\mu \to \nu\}^*,$$

where $\tilde{J}^{(p,n)}(t)$ are the nucleon current operators defined in (2.4) written in the Pauli spinor space, and α_p, α_n label the polarization states of final nucleons.

The summation over the deuteron polarizations (m) leads to $S_{\mu\nu}$ equal to

$$S_{\mu\nu} = \left(\frac{8\pi M}{w_d}\right) \sum_{\alpha_p, \alpha_n} \varphi'^{\dagger}_{\alpha_p} \varphi'^{\dagger}_{\alpha_n} [\tilde{J}^{(p)}_{\mu} U(l_n) + \tilde{J}^{(n)}_{\mu} U(l_p)] \times$$

$$\times (\tfrac{3}{4} + \tfrac{1}{4} \sigma_p \sigma_n) [U(l_n) \tilde{J}^{(p)\dagger}_{\nu} + U(l_p) \tilde{J}^{(n)\dagger}_{\nu}] \varphi'_{\alpha_p} \varphi'_{\alpha_n},$$

where $l_{p,n} = |L^{-1}(v)(q_{p,n}' - \frac{1}{2}q)|$, and it will be expressed through the lab-momenta in the next section. If we assume that the momentum transfer $|t|^{1/2}$ (of the electron) is considerably larger than the Fermi momentum of nucleons in the deuteron ($|t| \gg 0.04\,(\text{GeV}/c)^2$) then the interference terms in $S_{\mu\nu}$ can be neglected and

$$S_{\mu\nu} = \frac{6\pi M}{w_d} \{|U(l_n)|^2 \, \text{Sp}\, (\tilde{J}^{(p)}_{\mu} \tilde{J}^{(p)\dagger}_{\nu}) + |U(l_p)|^2 \, \text{Sp}\, (\tilde{J}^{(n)}_{\mu} \tilde{J}^{(n)\dagger}_{\nu})\}. \tag{3.3}$$

From the definition of $\tilde{J}^{(\mathrm{p,n})}$ — *cf.* (2.3) — one easily obtains

$$\mathrm{Sp}\,(J_\mu^{(\mathrm{p})}\tilde{J}_\nu^{(\mathrm{p})\dagger}) = \frac{1}{w_\mathrm{d} w'_\mathrm{p}}\,\mathrm{Sp}\left[(M - i\not{q}'_\mathrm{p})J_\mu^{(\mathrm{p})}\left(M - \frac{i}{2}\not{q}_\mathrm{d}\right)\tilde{J}_\nu^{(\mathrm{p})}\right], \qquad \tilde{J}_\mu = \gamma_4 J_\mu^\dagger \gamma_4,$$

where on the right-hand side one deals already with the Dirac operators. Of course, the same concerns the second term $\mathrm{Sp}\,(\tilde{J}_\mu^{(\mathrm{n})}\tilde{J}_\nu^{(\mathrm{n})\dagger})$. It should be remembered that with regard to the boost of spins, the momenta $\boldsymbol{p}_{\mathrm{p,n}}$ and energies $E_{\mathrm{p,n}}$ of initial nucleons are equal to one half of the momentum p_d and energy E_d of the initial deuteron, hence $q/2$ replaces the initial four-momenta of both nucleons. Thus

$$S_{\mu\nu} = \frac{6\pi M}{w_d^2}\left\{|U(l_\mathrm{n})|^2\,\mathrm{Sp}\left[(M - i\not{q}'_\mathrm{p})J_\mu^{(\mathrm{p})}\left(M - \frac{i}{2}\not{q}_\mathrm{d}\right)\tilde{J}_\nu^{(\mathrm{p})}\right]\frac{1}{w'_\mathrm{p}} + \right.$$

$$\left. + |U(l_\mathrm{p})|^2\,\mathrm{Sp}\left[(M - i\not{q}'_\mathrm{n})J_\mu^{(\mathrm{n})}\left(M - \frac{i}{2}\not{q}_\mathrm{d}\right)\tilde{J}_\nu^{(\mathrm{n})}\right]\frac{1}{w'_\mathrm{n}}\right\}. \tag{3.4}$$

Combining this with the well known electron current tensor $s_{\mu\nu}$,

$$s_{\mu\nu} = \frac{e}{4ww'}\,\mathrm{Sp}\,(-i\not{q}'\gamma_\mu \not{q}\tilde{\gamma}_\nu), \qquad (m_\mathrm{e} = 0),$$

the cross-section $d^9\sigma$ for unpolarized particles can be written in the manifestly invariant form

$$d^9\sigma = \frac{(2\pi)^5\delta^{(4)}(P'_\mu - P_\mu)}{s - 4M^2}\,\frac{d^3p'}{2E'(2\pi)^3}\,\frac{d^3p'_\mathrm{p}}{2E'_\mathrm{p}(2\pi)^3}\,\frac{d^3p'_\mathrm{n}}{2E'_\mathrm{n}(2\pi)^3}\,\times$$

$$\times\left(\frac{e^2}{t^2}\right)\left\{|U(l_\mathrm{n})|^2\,\mathrm{Sp}\left[(M - i\not{p}'_\mathrm{p})J_\mu^{(\mathrm{p})}\left(M - \frac{i}{2}\not{p}_\mathrm{d}\right)\tilde{J}_\nu^{(\mathrm{p})}\right] + \right.$$

$$\left. + |U(l_\mathrm{p})|^2\,\mathrm{Sp}\left[(M - i\not{p}'_\mathrm{n})J_\mu^{(\mathrm{n})}\left(M - \frac{i}{2}\not{p}_\mathrm{d}\right)\tilde{J}_\nu^{(\mathrm{n})}\right]\right\}\mathrm{Sp}\,(-\not{p}'\gamma_\mu \not{p}\tilde{\gamma}_\nu). \tag{3.5}$$

This cross-section is manifestly invariant, and — as was said before — only the deuterons structure function $U(l_{\mathrm{p,n}})$ reveals the distinguished position of the CM-system because the invariant $l_{\mathrm{p,n}} = |L^{-1}(\boldsymbol{v})(\boldsymbol{q}'_{\mathrm{p,n}} - \boldsymbol{q}_\mathrm{d}/2)|$ are defined through the CMS momenta $\boldsymbol{q}$, and the velocity $\boldsymbol{v}$. In the lab-system, where the cross-section (3.5) will be evaluated, we obtain

$$l_{\mathrm{p,n}} = \{[\boldsymbol{p}'_{\mathrm{p,n}} - \boldsymbol{v}(\boldsymbol{p}'^2_{\mathrm{p,n}} + M^2)^{1/2} - M_\mathrm{d}/2]^2\}^{1/2}, \tag{3.6}$$

where $\boldsymbol{v}$ is the velocity between the lab- and the overall CM-system — *cf.* (1.3).

It is remarkable that the deviation of l from $p' = |\boldsymbol{p}'|$ as given in (3.6) vanishes only in two particular cases: (*i*) when $\boldsymbol{v}$ vanishes, which takes place for infinitely heavy target particle or, realistically for momenta p of the impinging particle much less than the target mass M_d, and (*ii*) when the binding energy of the composite target particle tends to zero.

In our case the possibility (*ii*) should mean that $M_d \to 2M$ which automatically implies that $U(l) \to 0$ for $l \neq 0$. The vanishing of the Fermi momenta of composite nucleons results in $l = p' = 0$ for arbitrary value of v. It must be so, as for unbound "composite" target particle, the distinguished position of the overall CM-system should lead to inconsistencies. We see then that the relation (3.6) exhibits the dynamical aspect of the wave function. The wave function gives us, namely, not only the momentum distribution of nucleons inside the deuteron (in R_0), but at the same time it accounts for the fact that these constituent nucleons are off mass shell. This is the reason for the difference between the energy $(\boldsymbol{p}'^2 + M^2)^{1/2}$ of the free final spectator nucleon and the half of the rest-energy $M_d/2$ of deuteron (both in lab-system) which together with nonvanishing v determines the deviation of l from its non-relativistic value equal to p'. The consequence of this fact will be discussed in the next section.

4. *Sum rule and angular distribution of spectator*

On integrating the differential cross-section (3.5) over the momenta of recoiled nucleon and the energy E' of electron one obtains the following expression for the angular distribution cross-section of the electron in the lab-system

$$\frac{d\sigma}{d\Omega'} = \frac{1}{(2\pi)} \int d^3 p_s' |U(l_s)|^2 \left[\frac{d\sigma_p}{d\Omega'}(t, \boldsymbol{p}_s') + \frac{d\sigma_n}{d\Omega'}(t, \boldsymbol{p}_s') \right]. \tag{4.1}$$

Here "s" denotes the spectator nucleon, $\boldsymbol{p}_s'$ is its momentum in lab-system and

$$\frac{d\sigma_{p,n}}{d\Omega'}(t, \boldsymbol{p}_s') = \frac{1}{2\pi} \frac{e^2}{16(s-4M^2)t^2} \frac{E'}{E'_{p,n}} \left| \frac{\partial f}{\partial E'}(\boldsymbol{p}_s') \right|^{-1} \times$$

$$\times \mathrm{Sp}\,(-p'\gamma_\mu p\gamma_\nu)\,\mathrm{Sp}\left[(M - i p'_{p,n}) J_\mu^{(p,n)} \left(M - \frac{i}{2} p_d \right) \bar{J}_\nu^{(p,n)} \right],$$

where the density of the final states is equal to

$$\left| \frac{\partial f}{\partial E'}(\boldsymbol{p}_s') \right|^{-1} = \frac{[(\Delta \boldsymbol{p} - \boldsymbol{p}_s')^2 + M^2]^{1/2}}{[(\Delta \boldsymbol{p} - \boldsymbol{p}_s')^2 + M^2]^{1/2} + E\cos\vartheta - E - \boldsymbol{p}_s'\boldsymbol{p}'/E'},$$

where ϑ is the lab-angle of the scattered electron. If $|t| \gg p_s'$ there are no constraints upon the p_s' values, and since, moreover, the spectator momenta are concentrated arround 50 MeV/c, and both functions $d\sigma_{p,n}/d\Omega'$ are smooth functions of $\boldsymbol{p}_s'$, one can, with a very good approximation, neglect their dependence on $\boldsymbol{p}_s'$, and put $\boldsymbol{p}_s' = 0$. Then the angular distribution of electron is given by the following formula

$$\frac{d\sigma}{d\Omega'} = I \left[\frac{d\sigma_p^{(R)}}{d\Omega'}(t) + \frac{d\sigma_n^{(R)}}{d\Omega'}(t) \right], \quad I = \frac{1}{2\pi^2} \int d^3 p_s' |U(l_s)|^2, \tag{4.2}$$

where $d\sigma_{p,n}^{(R)}(t)/d\Omega' = d\sigma_{p,n}(t, 0)/d\Omega'$ denote the Rosenbluth formulae for elastic scattering of electron on free proton and neutron, respectively. As $l_s \neq p_s'$, the sum rule is slightly

TABLE I

E (GeV)	v/c	I
0.5	0.2	1.0004
0.9	0.3	1.0009
1.2	0.4	1.0016
1.8	0.5	1.0025
2.8	0.6	1.0036
4.4	0.7	1.0049
7.5	0.8	1.0064
17.0	0.9	1.0082
	1.0	1.0097

modified in comparison with the one well known from low energies [1], when $l_s = p'_s$, and $I = 1$. The numerical calculations with the Hulthen wave function provide us with the values of I specified in Table I for several values of primary electron energy.

As we see, the deviations of I from the value 1 are very small, and even for ultra high energies do not exeed 1 %. Thus their experimental determination is practically impossible. However, there is another effect due to the same reason, namely, the angular distribution

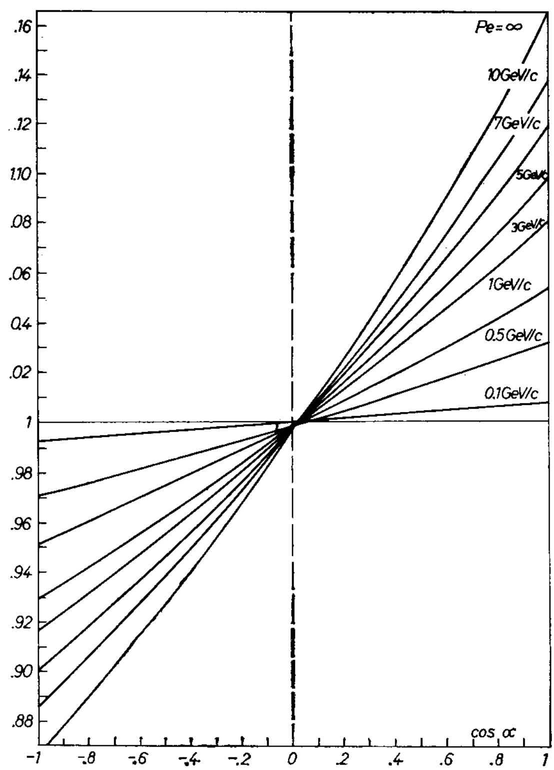

Fig. 1. The laboratory angular distribution of the spectator-nucleon, normalized to isotropic distribution, of momenta less than 140 MeV/c. The angle $\alpha = 0$ indicates the primary direction of electron

of the spectator nucleon, which is more distinct. Let us consider the differential cross--section for the electron scattered into the spherical angle $d\Omega'$, and the spectator nucleon into the momentum space element $d^3p'_s$, both in the lab-system, and for $|t| \gg p_s'^2$. According to (4.1) we have

$$\frac{d\sigma}{d^3p'_s d\Omega'} = G(t)|U(l_s)|^2, \quad G(t) = \frac{1}{2\pi^2}\left[\frac{d\sigma_p^{(R)}}{d\Omega'}(t) + \frac{d\sigma_n^{(R)}}{d\Omega'}(t)\right]. \tag{4.3}$$

The difference between p'_s and l_s specified in (3.6) implies that the angular distributions of the spectator nucleon cease to be isotropic (in the lab-system). We present in Figs 1, 2

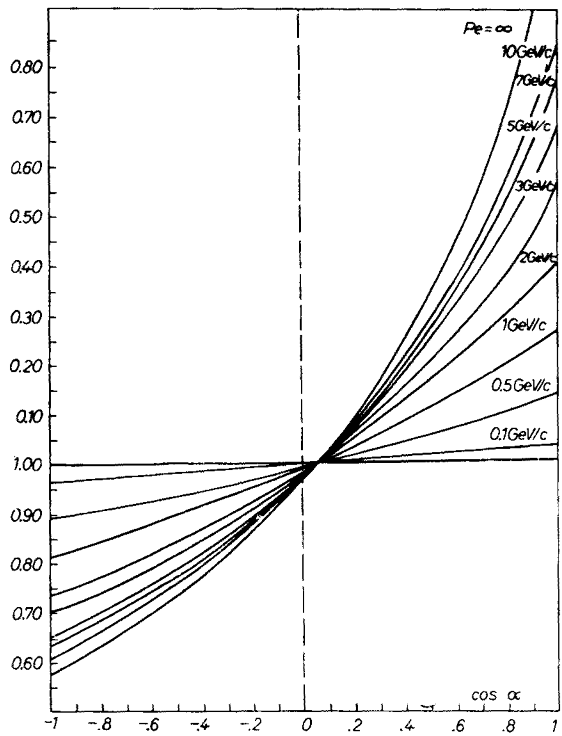

Fig. 2. The laboratory angular distribution of the spectator-nucleon, normalized to isotropic distribution, of momenta larger than 140 MeV/c. The angle α is the same as in Fig. 1

the angular distributions evaluated with the help of the Hulthen function, for spectator momenta smaller — Fig. 1, and larger — Fig. 2 than 140 MeV/c. They are both peaked in the forward direction, the more so, the larger is the primary energy of the electron. Moreover, this anisotropy becomes larger with increasing energy of the spectator nucleons, which is seen in Figures 1 and 2. Note that for the Hulthen function, about 20% of nucleons have momenta larger than 140 MeV/c. [12].

Let us emphasize that this effect should be rather universal in the sense that the mechanism of the disintegration is irrelevant provided it justifies us to deal with the "spectator", *i.e.* the particle which does not participate directly in the collision process. It is true that this situation is best fulfilled in the electromagnetic interaction under discussion, because of the small coupling constant. However, even for nucleon-deuteron collisions one can expect that the disintegration cross-section is due mostly to the single scattering mechanism, and thus the spectator particle has a clear cut meaning. This can be seen in the recently investigated disintegration reaction of 3.3 GeV/c deuterons in the hydrogen bubble chamber in Dubna [13]. Results show the existence of the spectator nucleons, and moreover, they do not contradict some slight anisotropy of these nucleons, of the same character as that discussed here.

5. *Conclusions*

The dynamically privileged position of the overall CMS, together with the "static" character of the deuteron internal wave function imply a nonisotropic angular distribution of the spectator nucleon emerging from the spherically symmetric (in the lab-system) deuteron. The point is that nucleons of Fermi momentum $\boldsymbol{p}$ bound in the deuteron are of indefinite energies, whereas after the collision the momentum $\boldsymbol{p}'_s$ and energy $E'_s = (\boldsymbol{p}'^2_s + M^2)^{1/2}$ of the spectator nucleon create the four-momentum on the mass shell. Since, on the other hand, the four-momentum of the deuteron as a whole is well defined, and the argument of the contracted deuteron wave function (in CMS) is equal to $L^{-1}(\boldsymbol{q}'_s - \frac{1}{2}\boldsymbol{q})$, pure kinematics result in l_s as given in (3.6). The latter means the anisotropy effect under discussion. We see then that this effect should be characteristic for any disintegration mechanism, provided that the single collision assumption is fulfilled when the spectator particle is well defined. The electromagnetic interactions (because of small coupling constant) realize best the aforementioned situation. In other processes we must carefully select these events where the nucleon is really the "spectator" particle.

Finally, let us notice that such an s-dependent effect (*via* v), even if small, is alien to the fully covariant approach when the deuteron structure function depends on t only. It characterizes the semi-relativistic approach distinguishing the overall CM-system, and reminds the s-dependence of the high-energy optical potential based also on the semi--relativistic picture.

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