EFFECTIVE POTENTIAL AND OFF-SHELL TWO-BODY SCATTERING AMPLITUDES IN THE EIKONAL APPROXIMATION

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An effective potential is computed for 2-body elastic scattering with the experimental on-shell t-matrices as an input. Nonrelativistic eikonal approximation and locality, together with spherical symmetry of potential, are assumed. The explicit form of potential for pp, π^+p and π^-p in the energy range from 5 to 20 GeV is investigated. The half off-shell scattering amplitude is calculated in the potential model. In the position representation this amplitude is found to be asymmetric along the eikonal direction, and an interesting absorption interpretation of this fact is given.

1. Introduction

Nonrelativistic eikonal approximation, initiated by Moliére [1] and Glauber [2], has been extensively and successfully used for description of high energy hadronic collisions, particularly for the scattering on compound systems. The validity of the original Glauber formulae should be restricted to high energies and small momentum transfers, however they were used far beyond that region, giving quite reasonable results [3]. There have been attempts to explain this situation by performing theoretical or numerical estimates of corrections to the Glauber model [4–13]. Some of the corrections include the eikonal two-body amplitudes which are off energy shell or at least half off-shell. It is generally assumed that they have the same form as the on-shell amplitudes. Our aim, in this paper, is to express the off-shell amplitudes trough the on-shell amplitude *via* potential model [7], and to investigate numerically their shape as well as that of the potential in the high energy region.

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A concept of an optical potential [14] is often used in high energy physics. One of the methods adopted in the optical potential investigations is to assume some general shape of the potential and obtain the values of free parameters from a fit to the experimental data [15, 16]. We shall proceed more directly. Assuming locality and spherical symmetry of the potential, as it is usually done in the eikonal approximation if the spin interactions are neglected, we can derive potential uniquely from the two-body on-shell elastic scattering amplitudes [2, 10].

We are aware of the fact that at high energies the concept of the potential in the conventional sense may seem dubious. However, one should look upon our potential as a mathematically equivalent to the on-shell amplitude effective potential which enables us to construct the off-shell amplitudes. Thus, in spite of its energy dependence and complexity which make the physical interpretation more difficult, it proves to be a powerful device to the investigation of the off-shell effects.

In Section 2 we derive the general formulae for our potential, and review the numerical results for the pp, π^+p and π^-p elastic scattering in the energy range from 5 GeV to 20 GeV. Section 3 is devoted to the investigation of the half off-shell eikonal amplitude in the potential model. As an example, the numerical calculations are carried out for the case of π^-p elastic scattering at 9 GeV. It turns out that the half off-shell amplitude is slightly asymmetric along the eikonal direction. We give the possible physical interpretation of this effect by connecting it to the absorption.

2. *Effective eikonal potential*

We shall use the notation similar to that of Harrington [7], although our formulae will not be limited to any specific reference frame or eikonal direction. Assuming that the centre of mass motion has already been separated we can fully describe the two-particle state by its relative motion variables — energy E, momentum $\vec{p}$, and position $\vec{r}$. The full Hamiltonian of this systems is

$$h = h_0 + V = \frac{p^2}{2\mu} + V, \tag{2.1}$$

where μ is the reduced mass and V-interaction potential. By the eikonal approximation we shall understand a linearization procedure of Hamiltonian through expanding it around some vector $\vec{p}_j$ and leaving terms linear in difference $(\vec{p}-\vec{p}_j)$:

$$\frac{p^2}{2\mu} \to \frac{p_j^2}{2\mu} + \frac{\vec{p}_j(\vec{p}-\vec{p}_j)}{\mu}. \tag{2.2}$$

The choice of $\vec{p}_j$ is free here with the only restriction that for physical values of $\vec{p}$ the difference $(\vec{p}-\vec{p}_j)$ should be small. In general $\vec{p}_j$ is taken equal to $\vec{p}_i$, $\vec{p}_f$, $\frac{1}{2}(\vec{p}_i+\vec{p}_f)$, or $|\vec{p}_i|(\vec{p}_i+\vec{p}_f)/|\vec{p}_i+\vec{p}_f|$ [16], where indices i and f correspond to the initial and final state, respectively. $\vec{p}_j$ will be called the eikonal momentum and its direction — the eikonal direction. We shall always choose the z-axis of a reference frame in the eikonal direction and denote the component of any vector in a plane perpendicular to $\vec{p}_j$ by an extra index $\perp$.

The Green functions and amplitudes in the eikonal approximation will be denoted with a tilde.

In the eikonal approximation, a state $|\vec{p}_0\rangle$ will be said to be on energy shell if

$$E - \frac{p_j^2}{2\mu} - \frac{\vec{p}_j(\vec{p}_0 - \vec{p}_j)}{\mu} = 0. \tag{2.3}$$

It is clear from the definition that the on-shell momentum is equal to the eikonal momentum if, and only if

$$E = \frac{p_j^2}{2\mu}. \tag{2.4}$$

Otherwise, denoting

$$\eta_j = \frac{\mu E}{2p_j} - \frac{p_j}{2} \tag{2.5}$$

we get the on-shell condition in the general form

$$p_{0z} = p_j + \eta_j. \tag{2.6}$$

This means that except for $\vec{p}_j = \vec{p}_i$ or $\vec{p}_j = \vec{p}_f$ in the centre of mass system the initial and final states will always be slightly off-shell.[1]

Depending on states sandwiching the scattering matrix t, its matrix elements can be full off-shell, left off-shell, right off-shell or on-shell. We shall denote them by $'\tilde{t}'$, $'\tilde{t}$, $\tilde{t}'$, and $\tilde{t}$, respectively.

The eikonal approximation has two very important advantages. First, due to the Green function linearization, it is possible to find an exact solution of the Lippmann--Schwinger equation in a simple and compact form. Secondly, if the potential is spherically symmetric, the relation between the on-shell amplitude and potential is easily convertible, and allows us to express the potential in terms of the amplitude. This enables us to investigate the shape of the potential and then to calculate the off-shell amplitude with the help of this potential.

Now we proceed to the derivation of the formula for the eikonal amplitude. Neglecting the spin interactions and assuming that the potential, is local we get the solution of the eikonal Lippmann-Schwinger equation in the form

$$\tilde{t} = V + V\tilde{g}V, \tag{2.7}$$

where $\tilde{g}$ is the eikonal Green function and in the position representation equals

$$\langle \vec{r}'|\tilde{g}|\vec{r}\rangle = -\frac{i\mu}{p_j}\delta^2(\vec{r}'_\perp - \vec{r}_\perp)e^{i(p_j+\eta_j)(z'-z)}\theta(z'-z)e^{-\frac{i\mu}{p_j}\int_z^{z'} V(\vec{r}_\perp,\xi)d\xi}. \tag{2.8}$$

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[1] In the eikonal approximation momentum is on-shell if its z-component is determined; other components are arbitrary (see Eq. (2.6)). See also Ref. [20].

Substituting Eq. (2.8) into Eq. (2.7) and going to the momentum representation we get the full off-shell amplitude

$$\langle\vec{p}'|'\tilde{t}'|\vec{p}\rangle = \frac{1}{(2\pi)^3}\int d^2\vec{r}_\perp e^{-i\vec{r}_\perp(\vec{p}'_\perp-\vec{p}_\perp)}\left\{\int_{-\infty}^{\infty} dz e^{-iz(p'_z-p_z)}V(\vec{r}_\perp, z)-\right.$$

$$\left.-\frac{i\mu}{p_j}\int_{-\infty}^{\infty} dz\int_{-\infty}^{\infty} dz'\theta(z'-z)e^{-iz'(p'_z-p_j-\eta_j)-iz(p_j+\eta_j-p_z)}V(\vec{r}_\perp, z)V(\vec{r}_\perp, z')e^{-\frac{i\mu}{p_j}\int_z^{z'}V(\vec{r}_\perp,\xi)d\xi}\right\}. \tag{2.9}$$

Now, taking the left or right state on energy shell, *i. e.* for $p'_z = p_j+\eta_j$ or $p_z = p_j+\eta_j$ we get the half off-shell amplitudes

$$\langle\vec{p}'|\tilde{t}'|\vec{p}\rangle = \frac{1}{(2\pi)^3}\int d^3\vec{r}e^{-i\vec{r}(\vec{p}'-\vec{p})}V(\vec{r})e^{-\frac{i\mu}{p_j}\int_z^{\infty}V(\vec{r}_\perp,\xi)d\xi}, \tag{2.10}$$

$$\langle\vec{p}'|'\tilde{t}|\vec{p}\rangle = \frac{1}{(2\pi)^3}\int d^3\vec{r}e^{-i\vec{r}(\vec{p}'-\vec{p})}V(\vec{r})e^{-\frac{i\mu}{p_j}\int_{-\infty}^{z}V(\vec{r}_\perp,\xi)d\xi}. \tag{2.11}$$

Finally, the on-shell amplitude is

$$\langle\vec{p}'|\tilde{t}|\vec{p}\rangle = \frac{1}{(2\pi)^3}\frac{ip_j}{\mu}\int d^2\vec{r}_\perp e^{-i\vec{r}_\perp(\vec{p}'_\perp-\vec{p}_\perp)}[e^{-\frac{i\mu}{p_j}\int_{-\infty}^{\infty}V(\vec{r}_\perp,\xi)d\xi}-1]. \tag{2.12}$$

All the above formulae are valid in any reference frame in which symmetry condition is satisfied and for any choice of eikonal momentum. However, for the sake of simplicity, we take $\vec{p}_j = \vec{p}_i \equiv \vec{k}$ and continue our considerations in the cms frame. Introducing

$$f = -(2\pi)^2\mu t, \tag{2.13}$$

one can easily recognize in Eq. (2.12) the well known Glauber formula [2] for the eikonal amplitude

$$\tilde{f}(\vec{\Delta}_\perp) = \frac{ik}{2\pi}\int d^2\vec{b}e^{-i\vec{b}\vec{\Delta}_\perp}(1-e^{i\chi(\vec{b})}), \tag{2.14}$$

where

$$\chi(\vec{b}) = -\frac{\mu}{k}\int_{-\infty}^{\infty}V(\vec{b}, z)dz, \tag{2.15}$$

$$\vec{\Delta} = \vec{p}'-\vec{p}, \quad \vec{b} = \vec{r}_\perp. \tag{2.16}$$

Eq. (2.14) will be our starting point to calculate the potential. With one additional assumption about the spherical symmetry of the potential, the relation between the phase function $\chi(\vec{b})$ and potential becomes an ordinary Abel integral equation

$$\chi(b^2) = -\frac{\mu}{k}\int_{-\infty}^{\infty} V(\sqrt{b^2+z^2})dz \tag{2.17}$$

which can be easily solved to give

$$V(r) = \frac{k}{\pi\mu}\int_0^{\infty} \frac{d}{dr^2}\chi(y+r^2)\frac{dy}{\sqrt{y}}. \tag{2.18}$$

Now, to obtain an explicit form of potential, we have to assume some parametrization of $\tilde{f}(\vec{\Delta}_\perp)$. In the high energy domain the simplest and the most popular parametrization for small momentum transfer is

$$\tilde{f}(\vec{\Delta}_\perp) = \frac{i\sigma k}{4\pi}(1-i\varrho)e^{-\alpha\Delta_\perp^2}, \tag{2.19}$$

where

$$\varrho = \frac{\mathrm{Re}\tilde{f}(0)}{\mathrm{Im}\tilde{f}(0)}, \tag{2.20}$$

σ is the total cross-section and 2α — the slope of the elastic differential cross-section. Substituting Eq. (2.19) into Eq. (2.14) and calculating the phase function $\chi(b^2)$, one obtains from Eq. (2.18)

$$V(r) = -\frac{2ik}{\pi\mu}\frac{d}{dr^2}\int_0^{\infty}\ln\left[1-De^{-\frac{r^2+u^2}{4\alpha}}\right]du \tag{2.21}$$

or, equivalently,

$$V(r) = \frac{ik}{2\pi\alpha\mu}\int_0^{\infty} du\left[1-\frac{1}{D}e^{(u^2+r^2)/4\alpha}\right]^{-1}, \tag{2.21a}$$

where

$$D = \frac{\sigma(1-i\varrho)}{8\pi\alpha}. \tag{2.22}$$

The remaining integration in Eq. (2.21a) can be done numerically. However, for some values of parameters σ, α, and ϱ it is possible to express $V(r)$ in an analytic form. Namely, if σ, α, and ϱ satisfy the inequality

$$|D| \equiv \frac{\sigma}{8\pi\alpha}\sqrt{1+\varrho^2} < 1, \tag{2.23}$$

then for any r we can expand the logarithm in Eq. (2.21) and after carrying out integration and differentiation term by term we get

$$V(r) = -\frac{ik}{2\mu\sqrt{\pi\alpha}} \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} [De^{-\frac{r^2}{4\alpha}}]^n. \tag{2.24}$$

The condition (2.23) is satisfied for $\pi^{\pm}p \to \pi^{\pm}p$ in the whole high energy domain above 5 GeV, and for pp → pp at energies higher than 10 GeV. With our parametrization of the amplitude, the elastic cross-section is

$$\sigma^{el} = \frac{\sigma^2(1+\varrho^2)}{32\pi\alpha}, \tag{2.25}$$

which means that to satisfy Eq. (2.23) it is sufficient that $\sigma^{el} < \frac{1}{4}\sigma$. It is interesting that the strong violation of the inequality (2.23) induces a positive imaginary part of the potential. An example of such a case is shown in Fig. 1a, the curve corresponding to $p_{lab} = 5$ GeV/c. This eff.ct will appear in any reaction in the sufficiently small energy region since $|D|$ decreases with increasing energy. It is probably caused by the fact that the eikonal approximation and our parametrization in Eq. (2.19) are not valid there, especially at large momentum transfers which correspond to small r.

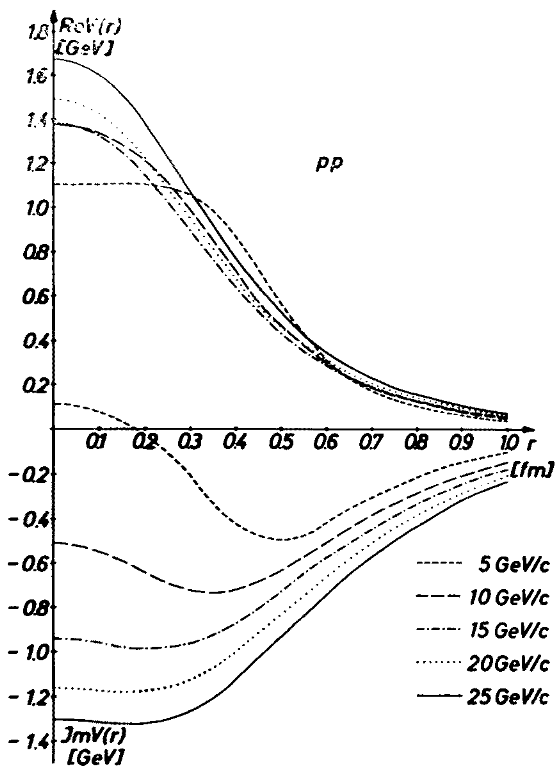

Fig. 1a

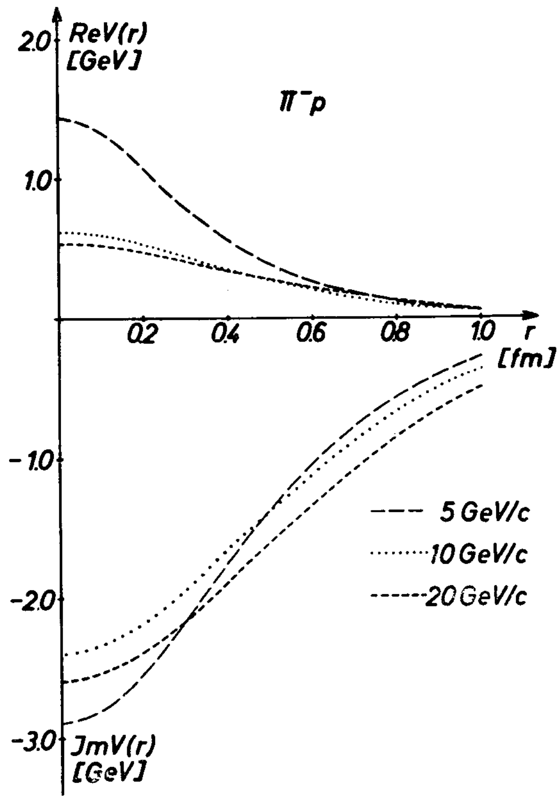

Fig. 1b

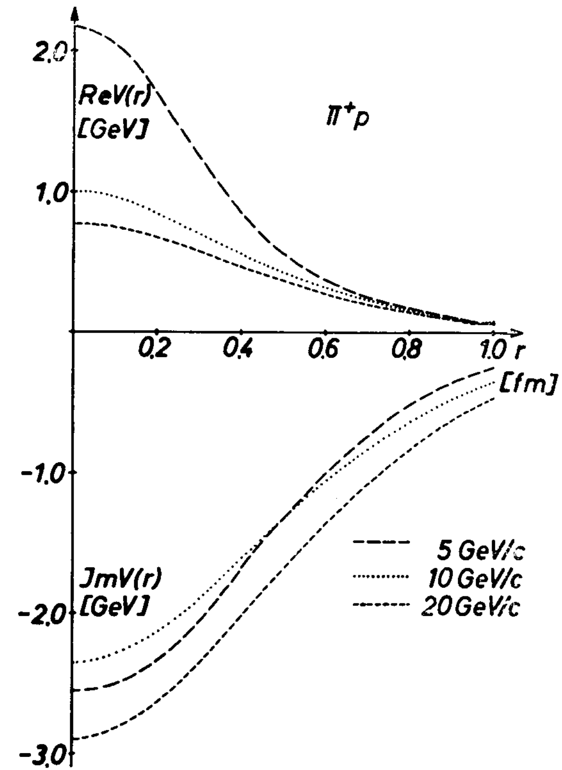

Fig. 1c

Fig. 1. Real and imaginary parts of the effective eikonal potential for a) pp → pp, b) $\pi^+ p \to \pi^+ p$, c) $\pi^- p \to \pi^- p$. The values of the parameters for which the calculations were done are given in Table I

Our results are presented in Figs 1a, b, c. We consider the following scattering processes

$$pp \to pp, \pi^+p \to \pi^+p, \pi^-p \to \pi^-p$$

as illustrative examples. Parameters σ, α, and ϱ used in our numerical calculations are listed in Table I.

TABLE I

The values of parameters taken for numerical calculations of the effective eikonal potential at different energies

Reaction	p_{lab} [GeV/c]	σ [mb]	α [GeV/c]$^{-2}$	ϱ
$pp \to pp$	5	41.2	3.5	−0.33
	10	40.0	4.2	−0.30
	15	38.8	4.5	−0.27
	20	38.6	4.6	−0.26
	25	38.6	4.6	−0.26
$\pi^+p \to \pi^+p$	5	26.6	3.5	−0.25
	10	24.8	4.3	−0.21
	20	23.5	4.5	−0.15
$\pi^-p \to \pi^-p$	5	29.0	3.9	−0.16
	10	26.5	4.6	−0.13
	20	25.0	5.0	−0.12

Our effective eikonal potential has the following properties:

1. It is energy dependent through momentum k and parameters σ, α, and ϱ. For very high energies the last three parameters vary very slowly, and nearly all energy dependence is due to the variation of k. Therefore, asymptotically the imaginary part of $V(r)$ should increase monotonically with energy, as can be seen in Figs 1a, b, c. This might not be true for Re $V(r)$, if ϱ changes sign in the high energy region.

2. It is asymptotically of the Gaussian shape, as can be seen from the leading term in Eq. (2.24). This is just the result of the Gaussian parametrization assumed for the on--shell amplitude. The range of the potential is roughly 0.3 to 0.6 fm.

3. It is not very sensitive to the variation of σ and ϱ in the limits of experimental errors. Under such a change our curves may be shifted by about 5–10%. The parameter α, which is known with rather high uncertainty, has a stronger influence on our potential. Changing it by 20% we can get a shift of $V(0)$ up to 40%, with a slight change of the shape.

4. From our considerations and the figures it follows that neither such simple parametrizations of potential as Yukawa's or exponential, nor any kind of real potential can be used for description of experimental data.

Now, we shall proceed to one of the main applications of the potential, namely, to the calculation of the off-shell eikonal amplitudes.

3. *Off-shell eikonal amplitudes*

From the formula (2.9) it is clear that the full off-shell amplitude is very complicated and depends on the four independent scalar variables — $\vec{\Delta}_{\perp}$, p_z, and p'_z. Since in various applications amplitudes are usually only half off-shell, we shall restrict our considerations to the much simpler formulae (2.10) and (2.11). As in the previous Section, all our considerations will refer to the centre of mass system for the eikonal momentum $\vec{p}_j = \vec{p}_i \equiv \vec{k}$.

Let us introduce the two phase functions [10]

$$\chi^{(+)}(b, z) = -\frac{\mu}{k}\int_{-\infty}^{z} V(b, \xi)d\xi, \tag{3.1}$$

$$\chi^{(-)}(b, z) = -\frac{\mu}{k}\int_{z}^{\infty} V(b, \xi)d\xi, \tag{3.2}$$

and the Fourier transforms of the half off-shell amplitudes

$$'\tilde{t}(b, z) = V(b, z)e^{i\chi^{(+)}(b,z)}, \tag{3.3}$$

$$\tilde{t}'(b, z) = V(b, z)e^{i\chi^{(-)}(b,z)}. \tag{3.4}$$

It is clear from the definition that with the spherically symmetric potential we have

$$\chi^{(-)}(b, z) = \chi^{(+)}(b, -z), \tag{3.5}$$

and

$$\tilde{t}'(b, z) = \tilde{t}'(b, -z). \tag{3.6}$$

Therefore, it is sufficient to calculate explicitly one of the two amplitudes. We shall choose the left off-shell amplitude. Suppose that the inequality (2.23) is satisfied. Then, substituting Eq. (2.24) for potential, we get

$$'\tilde{t}(b, z) = -\frac{ik}{2\mu\sqrt{\pi\alpha}}\left[\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}D^n e^{-\frac{n(b^2+z^2)}{4\alpha}}\right]\times$$

$$\times\left[1-De^{-\frac{b^2}{4\alpha}}\right]^{1/2}\exp\left\{-\frac{1}{2}\sum_{m=1}^{\infty}\frac{1}{m}(De^{-b^2/4\alpha})^m P\left(\sqrt{\frac{m}{4\alpha}}\,z\right)\right\}, \tag{3.7}$$

where

$$P(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt. \tag{3.8}$$

One can see that the procedure of continuation off energy shell introduces some dependence of the ratio of real to imaginary part of amplitude on momentum transfer. In practical calculations we shall assume $\sigma = 0$ for simplicity. With this assumption the parameter D in Eq. (3.7) must be replaced by

$$D_0 = \frac{\sigma}{8\pi\alpha} \tag{3.9}$$

and both on-shell and off-shell amplitudes are purely imaginary.

We have carried out the numerical calculations for the π^-p and π^-n elastic scattering at $p_{lab} = 9$ GeV/c using the values of parameters listed in Table II. As an example, the shape of the π^-p left off-shell amplitude is presented in Figs 2a, b (dashed curves). The

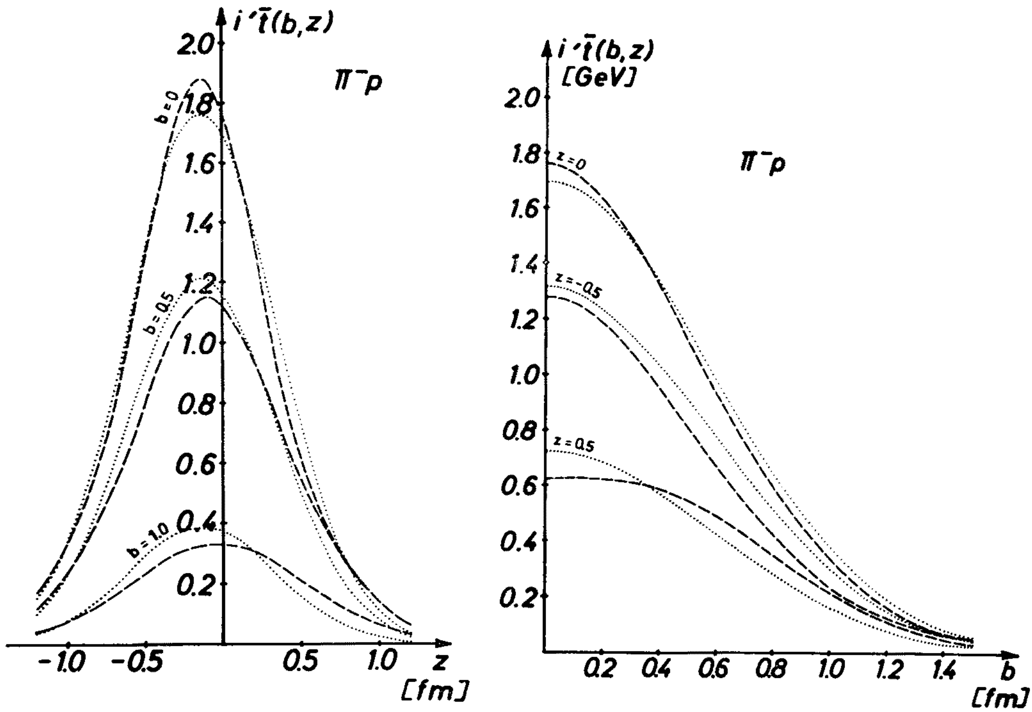

Fig. 2. The left off-shell scattering amplitude $'\tilde{t}(b, z)$ for π^- p $\to \pi^-$ p at 9 GeV/c. a) for fixed b equal to 0.0, 0.5, and 1.0 fm; b) for fixed z equal to 0.0, -0.5, and 0.5 fm. The dashed curves correspond to the actual shape of amplitude as given by Eq. (3.7) with parameters from Table II. The dotted curves are the result of the two-parameter fit with the formula (3.10) and constraints (3.15) and (3.16). The corresponding values of parameters are listed in Table III

brief analysis suggests that $'\tilde{t}(b, z)$ is of a Gaussian-like shape in both variables but with the maximum slightly shifted in the negative z direction. Moreover, the shift depends on the b parameter and tends to zero with increasing b. This effect has an interesting physical interpretation [17]. It can be connected with absorption. When the flux of particles approaches a spherical potential, one could expect the interaction region to be also symmetric. But on their way along the positive z direction, the particles get partially absorbed and the interaction becomes weaker. This means that the effective maximum of interaction must be at some $z < 0$. The dependence of this shift on b seems to confirm this interpretation since the absorption must decrease with the increasing impact parameter, that is for more peripheral collisions.

Since normally one needs the amplitude in the momentum representation, we should take now a Fourier transform of Eq. (3.7). As this would be rather complicated calculation, let us proceed in a much simpler, although more approximate, way. One can assume that the amplitude is of the shape

$$'\tilde{t}(b, z) = Ae^{-Bb^2 - C(z-d)^2}, \tag{3.10}$$

and then make a four parameter fit to our curves. The assumption that d does not depend on b is important if we want to get an easily Fourier-transformable function. Then, from (2.11) and (3.10) we get

$$\langle \vec{p}' | '\tilde{t} | \vec{p} \rangle = \frac{A}{8\pi^{3/2} B \sqrt{C}} e^{-\left(\frac{\Delta_\perp^2}{4B} + \frac{\Delta_z^2}{4C} + id\Delta_z\right)} = '\tilde{t}(\vec{\Delta}), \tag{3.11}$$

where

$$\vec{\Delta} = \vec{p}' - \vec{p} \tag{3.12}$$

also for the unphysical momenta. However, when going on-shell, which means putting (see Eq. (2.6))

$$\Delta_z = 0, \tag{3.13}$$

we would not reproduce exactly the input amplitude, although the difference between the correct and fitted A and B parameters is less than 10%. Therefore, we shall use the constraint that on-shell Eq. (3.11) becomes

$$\tilde{t}(\vec{\Delta}) = -\frac{i\sigma k}{16\pi^3 \mu} e^{-\alpha \Delta_\perp^2}. \tag{3.14}$$

Hence, the following conditions should be satisfied

$$A = -\frac{ik\sigma\sqrt{C}}{8\pi^{3/2}\alpha\mu}, \tag{3.15}$$

$$B = \frac{1}{4\alpha}. \tag{3.16}$$

We have carried out the two-parameter fit and its results are shown in the Figs 2a, b (dotted curves). The values of the parameters obtained from the fit are given in Table III. It follows from the symmetry relation (3.6) that the right off-shell amplitude will have the same values of parameters with the opposite sign of d.

It is clear from Figs 2a, b that our fit is not very good. However, for the small momentum transfer the Fourier transform is not sensitive to the detailed shape of $'\tilde{t}(b, z)$

and, therefore, our amplitude should be correct in the eikonal approximation region. Besides, the above fit has an important advantage of providing us with a very simple formula for the half off-shell amplitude which can be applied for further calculations [18]. Since the slope in the variable Δ_z^2 is smaller than that in $\Delta_\perp^2$ and parameter d is rather small, the off-shell effect may be expected to be negligible in the small momentum transfer region.

TABLE II

The values of parameters taken from Ref. [19], used for calculation of $'t(b, z)$ at the laboratory momentum 9 GeV/c

Reaction	σ [mb]	α [GeV/c]$^{-2}$	ϱ
$\pi^- p$	26.9	4.5	0
$\pi^- n$	25.3	4.5	0

TABLE III

The values of parameters obtained from the two-parameter fit to the left off-shell scattering amplitudes at 9 GeV/c

Reaction	C [fm]$^{-2}$	d [fm]
$\pi^- p$	2.20	−0.136
$\pi^- n$	2.12	−0.127

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