INDEPENDENT PRODUCTION OF PARTICLE CLUSTERS: A THIRD GENERAL FEATURE OF HIGH ENERGY HADRON COLLISIONS?

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The experimental facts on two-particle correlations and charge transfer in high energy proton-proton collisions suggest that the secondaries in the central rapidity region are produced in clusters of dominantly neutral charge and low multiplicity, these central clusters being independently emitted. Since fragmentation of either incident hadron is in effect formation of a leading cluster, one is led to a unified picture of independent cluster production (ICP) for all common inelastic collisions. The paper first reviews the evidence for ICP and gives an estimate of the mass and multiplicity distributions for central clusters. Following Stodolsky, it then uses the bremsstrahlung analogy to express the leading particle spectrum $d\sigma/dx$ in terms of the density ϱ of central clusters. On the basis of ISR data, we find agreement between the approximate constancy of $d\sigma/dx$ for $0.2 \lesssim x \lesssim 0.8$ and our estimate of $\varrho \simeq 0.9$–1.2. We also discuss the p_T properties of clusters by using the condition that the overlap function should agree with elastic data. The paper ends with a possible interpretation of ICP in terms of the present view on hadron structure based on deep inelastic lepton-nucleon scattering experiments, which suggests that a hadron is mainly composed of its "valence quarks" and of neutral "glue" (there is in addition a probably infinite number of quark-antiquark pairs, but the total amount of four-momentum they carry is very small). High energy hadron collisions can then be pictured as follows. The valence quarks fly through and give rise to the leading particles or clusters, while some glue can be radiated away in bremsstrahlung-like fashion and gives rise to the central clusters.

1. Introduction

The small transverse momentum property and the leading particle effect have long been known as basic features of particle production in high energy hadron collisions. A third property which, although much more difficult to recognize experimentally, might have the same degree of generality, seems to emerge from recent work on two-particle correlations in rapidity and on the distribution of charge transfer between c.m. hemispheres. The experimental facts on two-particle correlations and charge transfer at the highest

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available energies are readily understandable if the secondaries occurring in the central rapidity region are produced in clusters of dominantly neutral charge and low multiplicity, these clusters being emitted in an approximately uncorrelated way. Since fragmentation of an incident hadron (in particular diffraction dissociation) can be regarded as dissociation of the hadron into a cluster of equal charge, the concept of particle cluster leads to a possible unified picture of particle production at high energies.

The proposed picture is that common collisions would proceed through the production of clusters, this production obeying the small transverse momentum property and the leading particle effect. The latter can be stated as follows: each incident hadron either goes through unchanged or fragments into a leading cluster, the forward (backward) leading cluster being expected to carry the same internal quantum numbers as the forward (backward) incident hadron. Regarding the centrally produced clusters, it is natural to expect that they would tend to have not only neutral charge, but more generally the internal quantum numbers of the vacuum. If this is the case, conservation of internal quantum numbers imposes no correlation between central clusters, allowing for independent production.

Although the above picture of independent cluster production (ICP) is likely to be oversimplified and is not yet firmly established, it does emerge in many analyses of recent data (Refs [1–9] for central clusters, Refs [10, 11] for leading clusters), and it relates to earlier proposals [12–19]. It also leads to various interesting physical consequences. In the present paper we first recall the experimental evidence for ICP (Section 2) and give a rough estimate of the expected mass and multiplicity distributions for central clusters (Section 3). We then discuss two further aspects of the ICP model. The first, based on an argument due to Stodolsky [20], concerns the relation between leading particle spectrum and density of central clusters in rapidity (Section 4). The second discusses the restrictions imposed on transverse momenta of clusters by the condition that the overlap function should be compatible with the elastic scattering data (Section 5). The analogy of central cluster emission with bremsstrahlung plays an important role in both considerations. The last section gives a brief summary of the previous analysis and discusses a possible interpretation of the ICP model in terms of the picture of hadron structure resulting from the recent deep inelastic lepton scattering experiments.

2. *Experimental evidence*

The ISR data [21] for the two charged particle correlation

$$R(\eta_1, \eta_2) \equiv \left[\sigma_{\text{inel}} \frac{d^2\sigma}{d\eta_1 d\eta_2} \Big/ \frac{d\sigma}{d\eta_1} \frac{d\sigma}{d\eta_2}\right] - 1$$

(with $\eta = -\ln\tan(\Theta/2) \simeq y, \Theta$ = c.m. production angle, y = c.m. longitudinal rapidity) can be expressed [1–9, 22, 23] as a superposition of a long-range effect due to single diffraction dissociation and a short-range effect of attractive sign between particles in the central region ($-2 \lesssim y \lesssim 2$). This short-range correlation can be interpreted as due to a clustering effect; one then assumes that the central secondaries are produced in clusters (the central clusters mentioned in the introduction), these clusters being themselves uncorrelated in rapidity. The mean charged multiplicity of a cluster is estimated by various authors to be around 2 to 2.5 and the data are compatible with isotropic cluster decay [1–9]. The correlations observed at NAL [24] are very similar and compatible with the above picture. The combined NAL-ISR data suggest a weak dependence of the main cluster properties on incident energy.

The main evidence leading to the conjecture that the central clusters are dominantly neutral comes from the study of the charge transfer u between c.m. hemispheres. The variable u is defined as $Q_h - \langle Q_h \rangle$ where Q_h is the total charge of the final state particles in one c.m. hemisphere (say the hemisphere of positive rapidity). The average $\langle Q_h \rangle$ is taken over all inelastic collisions: it is one for pp collisions. The u distribution has been measured in the $p_{lab} \sim$ 10–24 GeV/c range for pp [25] and Kp collisions [26] as well as for pp at NAL [24]. It turns out to be remarkably universal [27] and shows strong suppression of large charge transfers. For pp collisions the probabilities $P(u = 0)$ and $P(|u| = 1)$ are about equal, the $P(|u| \geqslant 2)$ being very small. All this is qualitatively understandable if the central clusters are mainly neutral and have properties varying only little with energy down to the 20 GeV range. The absence of large u values is related both to the dominant neutrality and to the low charged multiplicity of the clusters. A first discussion of this problem is given in [8, 9]. A quantitative statement cannot be made for the moment, but it is safe to expect that the ratio of charged to neutral clusters should be much smaller than the charged to neutral pion ratio which is about 2.

Further support for the ICP comes from the recently reported NAL data on correlations between two pions of equal charge [28]. The strength of the correlations, $R^{--}(y_1 \approx y_2 \approx 0) \approx 0.4$, and their $y_1 - y_2$ dependence cannot be easily explained in terms of the long-range effects alone (this explanation is suggested by the Mueller-Regge approach combined with duality arguments). Rather a superposition of about equally strong long--range and short-range effects is required. Short-range correlations between like charge pions appear naturally in the ICP model if some of the central clusters contain several pions of equal charge. Assuming that only neutral clusters are centrally produced, one can predict the strength of the short-range correlations between particles of equal charge in terms of those between all charged particles [3]. In this way one expects the former to be about half of the latter and this result is in very good agreement with the NAL data (the long-range component is naturally expected to be charge independent). The correlations between like charge particles come out larger when central clusters are partly neutral, partly charged.

The correlation between a charged particle and a π^0 (in fact a gamma ray) has been measured at ISR [29]. It is similar to that between charged particles, which leads one to incorporate the neutral pions in the clusters discussed above. The ratio $\langle n_{ch} \rangle / \langle n_0 \rangle$ for charged particles to neutral pions per central cluster should not be far from 2 [3].

So far we have reviewed the evidence for central clusters. The concept of leading cluster is much more straightforward, because our ideas on diffraction dissociation or fragmentation lead quite naturally to it. Recently, the cluster concept for diffraction dissociation has been used with a dynamical contents going beyond correlation effects [10, 11], the cluster production and its decay being treated as separate dynamical steps.

For mass and multiplicity distributions of leading clusters, their charged and neutral particle contents, *etc.*, one can assume that useful indications are obtained from the simplest case of single diffraction dissociation. This matter will not be reviewed here. We only mention two qualitative features:

i) although the cluster mass distribution has only weak resonance signals, there are strong resonance effects between cluster decay particles (ϱ in the decay of the "A bump" or "A cluster" resulting from pion dissociation, Δ^{++} in the decay of the N^* cluster resulting from proton dissociation, *etc.*);

ii) from NAL to ISR energies, the mass and multiplicity distributions vary slowly, the low mass bump developing a growing tail of higher masses, and similarly for multiplicities.

It would not be surprising if similar features would hold also for central clusters, but no experimental evidence is yet available. Regarding feature *ii*) above, the appearance of growing tails in the mass and multiplicity distributions calls for two remarks. Firstly, the mean number of clusters per collision will grow more slowly than the average multiplicity of all observed secondaries; if the dispersion of the multiplicity of secondaries per cluster is rather small, this helps to understand the success of KNO scaling from Serpukhov to NAL energies [30]. Secondly, one should realize that the superposition of two light clusters (a leading and a central one, or two central ones) may look part of the time as a heavy cluster; a sharp separation is probably impossible without adopting a specific model with a specific parametrization.

3. *Further discussion of central clusters*

To give a more concrete idea of what central clusters might look like, we now make a few special assumptions. They are:

i) uniform, uncorrelated rapidity distribution of central clusters in the central plateau region (interval $-2 \lesssim y \lesssim 2$ at ISR energies);

ii) small transverse momenta of the clusters, so that the p_T of the particles in the collision c.m. frame is approximately equal to their p_T in the cluster rest frame;

iii) cluster mass spectrum of form $\varrho(M) \sim M \exp[-(M/M_0)^\alpha]$; for simplicity no long tail is here included;

iv) neglect of baryon-antibaryon and $K-\bar{K}$ pairs, and pion distribution in a cluster given at each cluster mass M by independent emission and relativistic phase space.

From *iv*, one obtains for the probability of a cluster of mass M to contain n pions the formula

$$P_n(M) = c(M)\,(\lambda^n/n!) \int \delta^4(k_c - \sum_1^n k_i) \prod_1^n d_3\vec{k}_i/2E_i, \tag{3.1}$$

where $k_i = (\vec{k}_i, E_i)$ = 4-momentum of pion, k_c = 4-momentum of cluster. The function $c(M)$ is to be determined from the normalization condition $\sum_n P_n(M) = 1$. The problem is then to see whether the free parameters λ, M_0 and α can be chosen to reproduce the experimental data on two-particle correlation and single particle p_T distribution in the

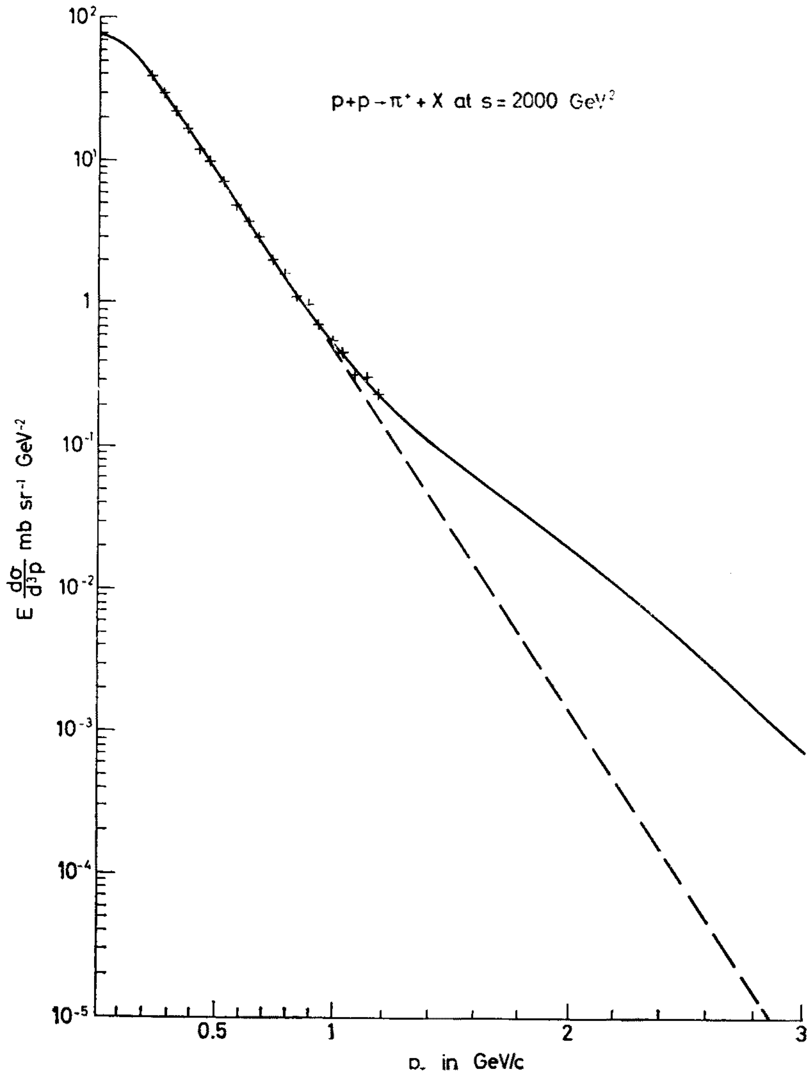

Fig. 1. Experimental p_T distribution of π^+ in the reaction p+p $\rightarrow \pi^+ +$ anything at $p_L^{cm} = 0$ and $s = 2000$ [31] compared with the model calculation (solid curve) described in the text (the model curve has been normalized to the data). The quantity plotted is the invariant cross-section $Ed\sigma/d^3p$. The dashed line is an extrapolation of the low p_T behavior

central region. This turns out to be the case for $\lambda = 20\ \mathrm{GeV}^{-2}$, $M_0 = 1$ GeV, $\alpha = 1.3$. The resulting multiplicity distribution per cluster approximately follows the Poisson law with average $\langle n \rangle = 3$ (corresponding to mean charged multiplicity 2 per cluster). The mean cluster mass is $\langle M \rangle \simeq 1.3$ GeV.

The p_T distribution is given in Fig. 1. For $p_T \lesssim 1$ GeV it agrees well with the familiar experimental shape in $\exp(-6p_T)$. At larger p_T it is much larger than the extrapolation of this exponential, an effect resulting from high mass clusters [11, 32]. This effect will increase with the incident energy if the cluster mass distribution develops a growing tail at high masses as is the case for the leading clusters (see end of previous section). It can account for at least part of the observed large p_T secondaries [33], other possible mechanisms being large momentum transfer collisions of partons or hadrons [34].

The close agreement of the calculated p_T distribution with the data in Fig. 1 should not be given too much significance, only the qualitative trend is of importance. Indeed, several complications are bound to affect the p_T distribution: resonance effects between particles in the cluster, non-isotropy of the cluster, effect of the p_T of the cluster itself (this effect comes in through the ratio p_T/M which is small even at considerable p_T because the cluster mass M is $\gtrsim 1$ GeV). What is interesting is that an isotropic cluster model based on phase space [see *iv*) above] gives a pion p_T distribution dropping as fast as shown in Fig. 1; this effect results from the competition between various multiplicities of the cluster, as discussed in detail for leading clusters in Ref. [11]. Only large mass clusters give pions of large p_T, and such clusters are rare.

We end this section with a remark concerning the possible physical nature of the central clusters. The physical nature of the leading clusters is by now rather familiar; they presumably are excited states of ordinary hadrons as occur in single diffraction dissociation. One still lacks a theory of the excitation mechanism, but the general properties of the resulting states have been extensively studied. As argued in the Introduction, the simplest possibility for central clusters is to be hadronic states with the internal quantum numbers of the vacuum, *i.e.*, hadronic excitations of the vacuum. The $I = 0$ states of two pions (in particular the f^0) are candidates, but higher multiplicity states are more likely ($\langle n \rangle \simeq 3$ to 4 corresponding to $\langle n_{ch} \rangle \simeq 2 \langle n_0 \rangle \simeq 2$ to 2.5). An interesting possibility is to connect the central clusters with the gluons, *i.e.*, the vector particles postulated in some quark models of hadrons [35]. The quark model interpretation of deep inelastic lepton-nucleon experiments indeed suggests that a fraction of the nucleon (carrying about half of its momentum) does not interact with electrons or neutrinos [36], and one can speculate that this "chargeless" fraction is composed of bound gluons having the internal quantum numbers of the vacuum.

If gluons can exist free, they must be sufficiently heavy not to have been detected as resonances, and a broad mass spectrum peaking between 1 and 2 GeV is in the realm of possibilities. They could then be produced in high energy hadron collisions by a bremsstrahlung-like process and would appear through their decay products with properties rather similar to those discussed above for the central clusters. If, on the other hand, gluons and quarks would have an additional "colour" quantum number preventing them from being freely created (a popular speculation recently, connected with asymptotic freedom), the candidates for central clusters would be gluon pairs rather than single gluons.

4. Central clusters and leading particle spectrum

Returning to the independent cluster production model as presented in the introduction, we now remark that one can directly apply to it an argument of Stodolsky [20] which relates the leading particle spectrum to the density of centrally produced particles.

Following Heisenberg [37] and Feynman [38] Stodolsky's discussion uses the bremsstrahlung analogy to treat the independent emission of Bose quanta (mesonic quanta instead of photons) in high energy collisions of two leading particles (hadrons instead of electrons). Here we apply it to the independent emission of central clusters by the colliding hadrons, where the latter go through as leading hadrons (leading clusters if excited, leading particles if unexcited). In our terminology, the assumptions needed for the argument are:

i) the leading hadrons and central clusters have small p_T;

ii) except at the outer edges of the kinematically allowed interval of longitudinal rapidity y, the central clusters follow a Poisson distribution with constant density ϱ in rapidity.

The latter condition means that there are no rapidity correlations between central clusters and that the probability of finding n central clusters in a rapidity interval $y, y+\Delta y$ is

$$P_n = \exp(-\varrho\Delta y)(\varrho\Delta y)^n/n!. \tag{4.1}$$

If all clusters have small p_T, the configuration of the set of central clusters determines that of the two leading hadrons through the conservation laws of energy and longitudinal momentum. A solution will exist if the set of central clusters does not carry too much energy and longitudinal momentum. It is this condition which sets the allowed y interval over which *ii*) can hold. As $s \to \infty$ for finite cluster masses, the allowed y interval grows like $\ln s$.

Stodolsky's calculation consists in deriving the statistical distribution of energy loss of the leading hadrons, using assumptions *i*), *ii*) and energy-momentum conservation. In the c.m. frame (or in any frame where both incident hadrons are ultra-relativistic in opposite directions), one obtains for each of the two leading hadrons the distribution

$$(E_0-E)\frac{d\sigma}{dE} = A\left(\frac{E_0-E}{E_0}\right)^{\varrho}. \tag{4.2}$$

E is the energy of the leading hadron, E_0 the energy of the corresponding incident particle. This equation holds asymptotically for both E_0 and $E < E_0$ very large, with a non-vanishing ratio E/E_0. Whereas ϱ is the central cluster density occurring in *ii*) and (4.1), the normalization coefficient A is determined by the condition

$$\sigma_c \simeq \int_{E_{min}}^{E_0} (d\sigma/dE) = \frac{A}{\varrho}\left(\frac{E_0-E_{min}}{E_0}\right)^{\varrho} \simeq \frac{A}{\varrho}$$

with σ_c the total cross-section for central cluster emission. E_{min} is a minimum energy of the leading hadron about which we have only to assume $E_{min} \ll E_0$; A and the spectrum (4.2) are then independent of its value. Note that the above equations suppose $\varrho > 0$. Equation (4.2) leads to the following distribution of leading hadrons in c.m. rapidity:

$$d\sigma/dy = A\exp(-|y-y_0|)\,[1-\exp(-|y-y_0|)]^{\varrho-1}, \tag{4.3}$$

where y_0 is the c.m. rapidity of the corresponding incident particle. Equation (4.3) holds asymptotically for $|y_0|$ large and $|y-y_0|$ finite. One sees that $d\sigma/dy$ tends exponentially to zero with increasing value of $|y-y_0|$.

We now proceed to evaluate ϱ from the data. It is given by

$$\varrho = \sigma_c^{-1}(d\sigma_c/dy), \tag{4.4}$$

where $d\sigma_c/dy$ is the differential cross-section for central cluster emission. We first have

$$d\sigma_c/dy = \langle n_{\mathrm{ch}}\rangle^{-1} d\sigma_{\mathrm{ch}}/dy, \tag{4.5}$$

where $\langle n_{\mathrm{ch}}\rangle = 2$ to 2.5 is the mean charged multiplicity of a central cluster, and $d\sigma_{\mathrm{ch}}/dy$ is the differential cross-section for charge particles in the central plateau region. The measured value of $d\sigma_{\mathrm{ch}}/dy$ is ~ 60 mb at ISR energies [39], where we neglect a small increase recently observed over the ISR energy range. As to σ_c, it is that part of the inelastic cross-section which corresponds to central cluster production, and it is expected to be $\sigma_c = \sigma_{\mathrm{inel}} - \sigma_{\mathrm{sd}} - \sigma_{\mathrm{dd}}$, where σ_{sd} refers to single diffraction dissociation (final states with one leading cluster and one unexcited particle only) and σ_{dd} to double diffraction dissociation (final states with two leading clusters only). The value of σ_{sd} is estimated at NAL to be ~ 7 mb [40]. As to σ_{dd}, it is not measured but is believed to be small, perhaps of order 1 or 2 mb. Our estimate is therefore $\sigma_c \sim 25$ mb. The resulting value of ϱ is in the range 0.9 to 1.2. This is to be compared with the case of ordinary photon bremsstrahlung where ϱ is of order of the fine structure constant.

Since ϱ is found close to one, Eq. (4.2) predicts for each leading hadron the approximate constancy of $d\sigma/dE$ for all large $E < E_0$ with nonvanishing value of the ratio E/E_0, *i.e.*, over the whole fragmentation region of the corresponding incident particle. Expressing these properties in terms of $d\sigma/dx$ ($x = p_{\mathrm{L}}/p_{\mathrm{L}}^{\max}$ in the c.m. frame), one finds that the $d\sigma/dx$ of a leading hadron is approximately constant over most of the c.m. hemisphere containing the corresponding incident particle (see below for more details). In case of dissociation, this is supposed to hold for the leading cluster, but will then also hold for the heaviest particle in the cluster, especially if it is a nucleon.

Comparison of this prediction can best be made for the outgoing proton in the inclusive reaction $p+p \to p +$ anything. In the 15–30 GeV range, the $d\sigma/dx$ is known to be flat, at least at small p_{T} [41], but this should perhaps not yet be taken as a confirmation since one can doubt the validity of the above assumptions at such low energies. We shall therefore consider $p+p \to p+$anything at ISR energies, but must also take into account the presence of further contributions to the proton spectrum.

Firstly, at large x, say $|x| \gtrsim 0.8$, single diffraction dissociation gives an important contribution ($\sigma_{\mathrm{sd}} \sim 7$ mb at NAL) which is additional to the processes here discussed. It is true that the latter processes include some collisions without central clusters, but from the Poisson law their cross-section is only $\sim \sigma_c \exp(-\langle N_c\rangle_c)$ with

$$\langle N_c\rangle_c = (\sigma_{\mathrm{inel}}/\sigma_c)\,\langle N_c\rangle. \tag{4.6}$$

$\langle N_c\rangle$ is the mean number of central clusters per inelastic collision, and $\langle N_c\rangle_c$ the mean number of central clusters per collision of the family here discussed. We know that

$\sigma_c \sim 25$ mb. A rough estimate of $\langle N_c \rangle$ is given by

$$\langle n_{\rm ch}^{\rm tot} \rangle \simeq 2\langle n_{\rm ch}^{\rm lc} \rangle + \langle N_c \rangle \langle n_{\rm ch}^{\rm cc} \rangle, \qquad (4.7)$$

where $n_{\rm ch}^{\rm tot}$, $n_{\rm ch}^{\rm lc}$ and $n_{\rm ch}^{\rm cc}$ refer to charged multiplicities of the whole collision, of one leading cluster and of one central cluster, respectively (the average $\langle n_{\rm ch}^{\rm lc} \rangle$ includes the case where the leading cluster reduces to a leading particle). At ISR one has $\langle n_{\rm ch}^{\rm tot} \rangle \sim 12$. Using our estimate $\langle n_{\rm ch}^{\rm cc} \rangle \sim 2-2.5$ and adopting $\langle n_{\rm ch}^{\rm lc} \rangle \sim 3$, we obtain

$$\langle N_c \rangle \sim 2.5-3, \quad \langle N_c \rangle_c \sim 3-4, \quad \sigma_c \exp(-\langle N_c \rangle_c) \sim 0.5-1 \text{ mb}. \qquad (4.8)$$

Hence, the protons produced in single diffraction dissociation must be subtracted from $d\sigma/dx$ before the latter is compared with Eqs (4.2)–(4.3). These protons include the undis-

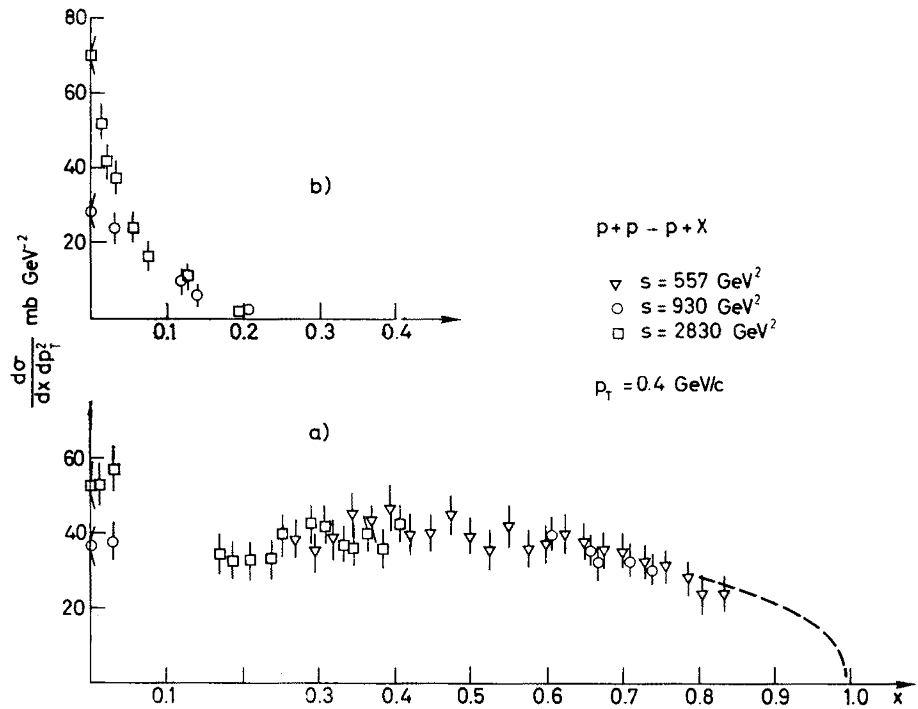

Fig. 2.a). Experimental proton cross-section $d\sigma/dxdp_T^2$ at $p_T = 0.4$ GeV/c. in the fragmentation region (Ref. [43]) and the difference of proton and antiproton cross-sections at $x \approx 0$ (Ref. [42]). The dashed curve shows a typical extrapolation of the proton spectrum into the region $x \simeq 1$ using the triple Regge (Reggeon-Reggeon-Pomeron) formula (see Ref. [44]). b) Experimental antiproton cross-section $d\sigma/dxdp_T^2$ at $p_T = 0.4$ GeV/c (Ref. [42])

sociated proton as well as the proton possibly contained in the dissociated cluster. The undissociated proton has $|x| \gtrsim 0.8$. and produces in $d\sigma/dx$ a peak near $|x| = 1$. Its subtraction is essential. As to the proton which may be contained in the dissociated cluster, its contribution to $d\sigma/dx$ is smaller (the nucleon in the dissociated cluster will be a neutron in about half of the single diffraction dissociation events) and has a shape expected to be rather similar to the case of collisions with central cluster emission. Its subtraction is therefore inessential for our purpose.

Secondly, the proton spectrum must contain in the central region ($x \sim 0$) contributions not covered by Eq. (4.3). The application of (4.3) at $y = 0$ gives an $s^{-\frac{1}{2}}$ decrease of $d\sigma/dy$, but such an extrapolation seems unjustified in view of the assumptions underlying the derivation of (4.2)–(4.3); indeed the hypothesis of independent emission is unlikely to hold when a leading hadron is so strongly slowed down. There is also an appreciable antinucleon production at small x, say $|x| \lesssim 0.2$, the ratio $\bar{p}/p$ being of order 0.5 [42]. This means that additional protons are produced through pair creation (one can assume them to have the same distribution as $\bar{p}$), and they must be subtracted for our present discussion.

We give in Fig. 2 the ISR data [43] for protons at $p_T = 0.4$ GeV/c (which is close to the average proton transverse momentum). For $0.15 \lesssim x \lesssim 0.8$, the non-invariant proton cross-section $d\sigma/dx\, dp_T^2$ is indeed approximately constant in agreement with our prediction. In the region $|x| \gtrsim 0.8$ one must subtract from the data the contribution from the single diffraction dissociation, which requires the use of a model. Our estimate of the resulting extrapolation to this region, shown in Fig. 2, is based on a triple Regge fit [44] and it exhibits the type of fall-off which is expected to occur due to phase space effects. Regarding the central region $|x| \lesssim 0.2$, after subtraction of the pair creation contribution (in fact of $d\sigma/dx$ for $\bar{p}$), the data seems to follow a distribution $d\sigma/dy \sim s^{-\frac{1}{4}} \cosh (y/2)$ as predicted by a Regge-Mueller analysis of Chan *et al.* [45]; the mechanism here involved should dominate at $y \sim 0$ above the tail of (4.3) which decreases as $s^{-\frac{1}{2}}$. This trend of the data is also apparent in Fig. 2 where $d\sigma/dx$ at $x \sim 0$ is increasing with energy (the parametrization of Ref. [45] predicts an increase as $s^{\frac{1}{4}}$).

5. *Overlap function*

Up to now, the transverse momentum properties of the clusters have played a minor role in our discussion. They become very important when we consider the overlap function $F(t)$, which is defined as the contribution of inelastic collisions to the unitarity equation for elastic scattering [46]. As was shown by many authors, the condition that a particle production model should give an overlap function compatible with the elastic scattering data is a very severe one, and the popular production models mostly fail to fulfil it, even at the crudest qualitative level unless they are explicitly adjusted to this purpose [47–51].

Uncorrelated jet models without phases give for $F(t)$ at all energies too small a slope to fit the elastic data; inclusion of phases increases the slope (as shown by Koba and Namiki [48], this is a general, model-independent property of the overlap function), but no physically natural choice of phases is known which would fit the data. What happens in the multi-Regge model has been recently clarified by Hamer and Peierls [49] and by Henyey [50, 51]. The slope of $F(t)$ is found to increase very rapidly with increasing p_{lab}, in contradiction with the data. But modifying the model by the assumption of clustering of secondaries strongly reduces the slope, and it may be possible to restore agreement with the $F(t)$ slope derived from elastic data by constructing a multiperipheral model for cluster production. The overlap function contribution of quasi-two-body collisions has also been studied; the effect of higher spin values of the outgoing particles (which may be clusters) is important [52]. A very satisfactory result has been obtained for diffraction dissociation; assuming reasonable spin value and t channel helicity conservation, Ajduk [53] and Sakai and White [54] find for the corresponding part of $F(t)$ a slope of the same magnitude as required by the elastic data for the whole $F(t)$.

Our intention in the present section is to examine the transverse momentum properties of the independent cluster production model in the light of the condition that it should give an overlap function of the correct slope. We first exhibit why the usual ansatz of factorized amplitudes fails badly, and then present a better alternative based on a bremsstrahlung-like picture for the emission of central clusters.

In the most usual form adopted for an independent emission model, the amplitude factorizes in the particles emitted. Here we deal with emission of clusters instead of particles, and it is factorization in the transverse momenta which matters. We therefore consider for an n cluster final state an amplitude of the form

$$M_n(x_1, \ldots, x_n, \vec{p}_T^{\,1}, \ldots, \vec{p}_T^{\,n}) = f(x_1, \ldots, x_n) \prod_1^n g_i(x_i, \vec{p}_T^{\,i}), \tag{5.1}$$

where the $\vec{p}_T^{\,i}$ are the transverse momenta and the $x_i = p_L^i/p_{cm}$ the Feynman variables of the clusters. Its contribution to the overlap function is

$$F_n(-\Delta^2) = \int [dx]\, |f(x)|^2 \int \delta(\sum_1^n \vec{p}_T^{\,i}) \prod_1^n g_i^*(x_i, \vec{p}_T^{\,i}) g_i(x_i, \vec{p}_T^{\,i} + x_i\vec{\Delta})\, d\vec{p}_T^{\,i}. \tag{5.2}$$

$\vec{\Delta}$ is the c. m. momentum transfer in elastic scattering ($t = -\Delta^2$). The volume element $[dx]$ stands for the longitudinal phase space element $s^{-\frac{1}{2}} p_{cm}^{-1} \delta(\ldots)\ \Pi_1^n\ E_i^{-1} dp_L^i$, where the normalization is so chosen that $F_n(0)$ is a cross-section. (5.2) is a high energy approximation, and the energy and p_L conservation constraints contained in $\delta(\ldots)$ take the simple form

$$\sum_1^n x_i = 1, \quad \sum_1^n |x_i| = 2$$

or equivalently

$$\Sigma^{(+)} x_i = -\Sigma^{(-)} x_i = 1, \tag{5.3}$$

where the sums $\Sigma^{(+)}$ and $\Sigma^{(-)}$ contain the positive and negative x_i, respectively.

The p_T distribution of the clusters is essentially given by $|g_i|^2$, and it is unreasonable to expect it to be narrower than typical p_T distributions of secondaries. The occurrence of $\vec{p}_T^{\,i} + x_i\vec{\Delta}$ in the last g_i factor of (5.2) is then responsible for a strong reduction of the slope whenever the $|x_i|$ are appreciably smaller than one, *i. e.*, in all but the diffractive collisions. Only a rapid variation of the phase of g_i with $\vec{p}_T^{\,i}$ may produce a larger slope, but this is a highly artificial assumption. To see the effect more quantitatively, we take the simple choice

$$g_i(x_i, \vec{p}_T^{\,i}) = \exp(-2a_i|\vec{p}_T^{\,i}|^2), \quad a_i > 0 \tag{5.4}$$

and find by explicit calculation

$$\int \delta(\sum_1^n \vec{p}_T^{\,i}) \prod_1^n g_i^*(x_i, \vec{p}_T^{\,i}) g_i(x_i, \vec{p}_T^{\,i} + x_i \vec{\Delta}) d\vec{p}_T^{\,i} = K \exp(-\Delta^2 \sum_1^n a_i x_i^2), \tag{5.5}$$

where K is independent of Δ. In a diffractive process, one has $n = 2$ and both $|x_i| = 1$, hence (taking a_i independent of i)

$$F_2(-\Delta^2) \propto \exp(-2a\Delta^2). \tag{5.6}$$

In a non-diffractive collision producing central clusters we expect $|x_i| \simeq 0.5$ for the leading hadrons (particles or clusters) and $|x_i| \ll 1$ for the central clusters, so that we get for $n > 2$

$$F_n(-\Delta^2) \propto \exp(-ca\Delta^2),\ c \simeq 0.5.$$

The slope is smaller by a factor ~ 4 than in F_2, Eq. (5.6).

To obtain a more satisfactory overlap function in the ICP model, we therefore try a different ansatz for the transverse momenta. The aim is to ensure that the slope of F_n for $n > 2$ should now be larger than that of F_2. This can be achieved if we regard the central clusters to be shaken off from one or the other leading hadron, in a fashion similar to the emission of bremsstrahlung photons in the collision between two charged particles (we shall return in Section 6 to this type of dynamical picture and to its relation to the quark-parton model). Let us then assign the indices $i = 1, \ldots, n$ as follows:

$i = 1$	: forward leading hadron (f. l. h.)
$i = 2, \ldots, \nu$	: central clusters (c. c.) shaken off f. l. h.
$i = \nu+1, \ldots, n-1$	: c. c. shaken off backward l. h.
$i = n$	: backward leading hadron (b. l. h.)

We have $x_1 > 0 > x_n$ and assume that:

$$x_2, \ldots, x_\nu \geqslant 0,\ x_{\nu+1}, \ldots, x_{n-1} \leqslant 0. \tag{5.7}$$

One cannot exclude that some of the $x_2, \ldots, x_{n-1}$ might occasionally have the opposite sign, but then they should be very close to zero and our further considerations would remain valid up to a very small correction. Clusters 1 to ν form what we shall call the forward jet (f. j.), clusters $\nu+1$ to n the backward jet (b. j.).

We introduce new transverse momentum variables

$$\left.\begin{aligned} \vec{k} &= \vec{p}_T^{\,1} + \ldots + \vec{p}_T^{\,\nu} = -(\vec{p}_T^{\,\nu+1} + \ldots + \vec{p}_T^{\,n}), \\ \vec{\kappa}_i &= \vec{p}_T^{\,i} - x_i \vec{k}. \end{aligned}\right\} \tag{5.8}$$

Equation (5.3) reads

$$x_1 + \ldots + x_\nu = -(x_{\nu-1} + \ldots + x_n) = 1 \tag{5.9}$$

and hence implies

$$\vec{\kappa}_1 + \ldots + \vec{\kappa}_\nu = \vec{\kappa}_{\nu+1} + \ldots + \vec{\kappa}_n = 0. \tag{5.10}$$

$\vec{k}$ is the transverse momentum of the f. j., and $-k$ that of the b. j. The $\vec{\kappa}_i$ are the transverse momenta of the clusters after over-all rotation of the final state by an angle $\theta = k/p_{\mathrm{cm}} \ll 1$ so as to make the jet transverse momentum vanish. In the spirit of the above picture for c. c. emission, the natural transverse momentum variables are now $\vec{k}, \vec{\kappa}_2, \ldots, \vec{\kappa}_{n-1}$. We therefore write the amplitude as

$$M_n(x_1, \ldots, x_n, \vec{p}_{\mathrm{T}}^1, \ldots, \vec{p}_{\mathrm{T}}^n) = A(k)B(x_2, \ldots, x_{n-1}, \vec{\kappa}_2, \ldots, \vec{\kappa}_{n-1}, \vec{k}) \tag{5.11}$$

with B normalized to

$$\int [dx]d\vec{\kappa}_2 \ldots d\vec{\kappa}_{n-1}|B(x_2, \ldots, x_{n-1}, \vec{\kappa}_2, \ldots, \vec{\kappa}_{n-1}, \vec{k})|^2 = 1. \tag{5.12}$$

The factor A gives the p_{T} distribution of the jets, B gives the distribution inside the jets. The overlap function has the form

$$F_n(-\Delta^2) = \int d\vec{k}A^*(\vec{k})\,A(\vec{k}+\vec{\Delta})\,G(\vec{k}, \vec{\Delta}), \tag{5.13}$$

$$G(\vec{k}, \vec{\Delta}) = \int [dx]d\vec{\kappa}_2 \ldots d\vec{\kappa}_{n-1}B^*(x_2, \ldots, x_{n-1}, \vec{\kappa}_2, \ldots, \vec{\kappa}_{n-1}, \vec{k})\times$$
$$\times B(x_2, \ldots, x_{n-1}, \vec{\kappa}_2, \ldots, \vec{\kappa}_{n-1}, \vec{k}+\vec{\Delta}). \tag{5.14}$$

Note that $G(\vec{k}, 0) = 1$ by Eq. (5.12) and $G(\vec{k}, \vec{\Delta}) \leq 1$ by the Schwarz inequality.

Equation (5.13) shows that there are now two sources of decrease of F_n with $t = -\Delta^2$. The first is the peak in $A(\vec{k})$ at $\vec{k} = 0$, which may be similar to what is found in a diffractive process ($n = 2$). The second is the decrease of G which results from the dependence on $\vec{k}$ of the internal jet properties expressed by B (note that $G = 1$ for all $\vec{\Delta}$ only if B is independent of $\vec{k}$). In the picture just presented, it is therefore natural to have for F_n with $n > 2$ a slope larger than for F_2.

We end this section with two remarks concerning Eq. (5.11). Firstly, if A is to be (approximately) the same for all n and if A and B have to be such as to give reasonable p_{T} distributions for single clusters, Eq. (5.11) implies correlations between the p_{T}'s of clusters in final states with $n > 2$. This is most readily seen by recasting the uncorrelated case of Eqs (5.1) and (5.4) in the form (5.11)–(5.12); one then sees that the resulting $A(k)$ broadens with increasing n. One should remember, however, that the p_{T} of a cluster influences only weakly the p_{T} of the particles in the cluster (see Section 3), so that the correlations mentioned will not be readily observable.

Secondly, it is interesting to rewrite (5.11) in terms of the impact parameters $b_1, \ldots, b_n$ of the clusters in the final state. This is done by means of the Fourier transformation [55, 49, 50]

$$\tilde{M}(x_1, \ldots, x_n, \vec{b}_1, \ldots, \vec{b}_n) = \int \delta(\sum_1^n \vec{p}_{\mathrm{T}}^i) \prod_1^n d\vec{p}_{\mathrm{T}}^i \exp(i \sum_1^n \vec{b}_i \cdot \vec{p}_{\mathrm{T}}^i) M_n(x_1, \ldots, x_n, \vec{p}_{\mathrm{T}}^1, \ldots, \vec{p}_{\mathrm{T}}^n).$$

Using (5.8), (5.10) and (5.11), one finds

$$\tilde{M}(x_1, \ldots, x_n, \vec{b}_1, \ldots, \vec{b}_n) = \int d\vec{k} \exp(i\vec{k} \cdot \sum_1^n x_i\vec{b}_i)A(k) \int \prod_2^{n-1} d\vec{\kappa}_i \exp[i \sum_2^{\nu} \vec{\kappa}_i \times$$
$$\times (\vec{b}_i - \vec{b}_1) + i \sum_{\nu+1}^{n-1} \vec{\kappa}_i \cdot (\vec{b}_i - \vec{b}_n)]B(x_2, \ldots, x_{n-1}, \vec{\kappa}_2, \ldots, \vec{\kappa}_{n-1}, \vec{k}).$$

This formula has a simple physical interpretation. The integral over the $\vec{\kappa}_i$ is a function of $\vec{k}$, of the relative impact parameters $\vec{b}_2-\vec{b}_1, \ldots, \vec{b}_\nu-\vec{b}_1$ in the forward jet, and of those $\vec{b}_{\nu+1}-\vec{b}_n, \ldots, \vec{b}_{n-1}-\vec{b}_n$ in the backward jet. The overall integral over $\vec{k}$ replaces this jet transverse momentum by the conjugate impact vector $\sum_1^n x_i\vec{b}_i = \vec{b}$; the latter is equal to the impact parameter of the incident particles (as discussed in detail by Henyey [50], this can be seen either by angular momentum conservation or by Fourier transformation of the elastic unitarity condition with respect to $\vec{p}_T$).

6. *Dynamical considerations and concluding remarks*

In this concluding section we first summarize the simple picture of inelastic hadron collisions at NAL-ISR energies which is suggested by the arguments reviewed above.

a) Each inelastic collision produces two leading particles or clusters, originating from the two incident particles through small transfers of energy-momentum. An incident particle can remain unexcited (leading particle) or get dissociated through excitation (leading cluster), mostly with conservation of internal quantum numbers.

b) In some 75% of all inelastic collisions, there is additional production of central clusters. These have small p_T and have a flat rapidity distribution in the central region. They tend to be mostly neutral and can be assumed to be emitted independently of each other in rapidity.

c) In most collisions, the clusters are relatively light (mass $\lesssim$ 2 GeV, perhaps somewhat heavier for leading baryonic clusters) and have rather low multiplicity (mean charged multiplicity of $\sim$2–2.5 per central cluster, presumably somewhat more per leading cluster). At ISR energies, the mean number of central clusters is $\sim$3. The mass and multiplicity distributions of clusters are expected to have growing tails as the incident energy increases. The single particle distribution in a cluster is probably not very far from isotropic in the cluster rest-frame.

d) For collisions producing central clusters, the rapidity distribution of a leading hadron (particle or cluster) is given by

$$d\sigma/dy = \varrho\sigma_c \exp(-|y-y_0|)\,[1-\exp(-|y-y_0|)]^{\varrho-1}$$

in terms of the central cluster density $\varrho = \sigma_c^{-1}\, d\sigma_c/dy$ in the central region (σ_c, $d\sigma_c/dy$ = = total and differential cross-sections for central cluster emission, y_0 = rapidity of incident particle). The value of ϱ is of order one.

e) The transverse momentum properties of the clusters are strongly constrained by the condition that the overlap function resulting from the inelastic collisions should be compatible with the elastic scattering data. Whereas a production amplitude factorizing in individual p_T's cannot fulfil this condition under reasonable assumptions, it can be satisfied if the p_T of the forward (backward) jet formed of all forward (backward) flying clusters is roughly distributed as in two-body collisions.

As stressed in the Introduction, this picture of independent cluster production is for the moment only suggested by the experimental evidence, and many aspects will have to be tested in order to decide on its validity. Its great simplicity makes it attractive, however, and it is natural to reflect on its possible dynamical origin. It is likely that a multiperipheral production mechanism with dominance of vacuum exchange may be so parametrized as to reproduce the ICP picture, but it is clear this would need some *ad hoc* assumptions. As mentioned repeatedly in the previous sections, the ICP picture fits more naturally with a bremsstrahlung-like emission mechanism. We now briefly discuss the latter possibility and we do this in the spirit of Feynman's parton ideas [38], taking into account the recent results of deep inelastic electron and neutrino scattering experiments.

The deep inelastic lepton-nucleon reactions can be interpreted by assuming that a nucleon of large four-momentum P_μ is composed of:

i) three point-like "valence quarks" carrying together about half of P_μ;

ii) a large (infinite?) number of charge carrying (quark-antiquark?) pairs carrying together a small fraction of P_μ;

iii) neutral hadronic stuff carrying about half of P_μ and deprived of interactions with photons and leptons (this stuff being often regarded as the "glue" which binds the quarks).

In a high energy hadron-hadron collision, the valence quarks *i*) of each incident hadron fly through with small momentum transfer and give rise to the corresponding leading particle or cluster, shaking off or not some of the accompanying glue *iii*) in the course of the collision. The central clusters are droplets of this glue (gluons or gluon pairs?) and are therefore mainly neutral. Since glue is shaken off by one or the other leading hadron, the two-jet grouping of central clusters discussed at the end of Section 5 appears naturally. Because of the small momentum they carry the quark-antiquark pairs *ii*) should only play a small part in the collisions here discussed.

In our picture of hadron-hadron collisions, the role attributed by Feynamn to "wee partons" is taken over by the glue, hadronic stuff carrying energy-momentum and having strong interactions but deprived of direct coupling to photons and leptons. Of course, the glue must couple to quarks and to ordinary hadrons, because the pieces [*i*), *ii*), *iii*)] of a hadron somehow stick together and because a glue droplet or central cluster decays in pions, $K\bar{K}$, $N\bar{N}$, *etc.* But this coupling cannot be too strong at least inside the hadron, otherwise *iii*) above would conflict with *ii*). The strongest interaction of the glue may well be with itself. The general picture has then a certain analogy with recent ideas on hadron structure proposed by T. D. Lee [56], who suggests that among the constituents of hadrons the ones interacting most strongly are spinless fields with super-renormalizable interaction, the Fermion constituents interacting more weakly with these spinless gluon fields. Most of what we said above on cluster production would hold *mutatis mutandis* in such a picture of hadron structure.

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NOTE ADDED IN PROOF

The results of Hamer and Peierls [49] and of Henyey [50, 51] on the slope of the overlap function in the multi-Regge model are found to be incorrect for energies up to 1500 GeV in a more recent study of S. Jadach and J. Turnau (Overlap function from the multiperipheral model, Cracow Preprint TPJU-3/74, January 1974; to be published in *Acta Phys. Pol.*, **B5**, No 5 (1974)). The discrepancy results from neglect of the longitudinal parts of the four-momentum transfers in Refs. [49–51], which is unjustified for realistic values of the parameters. Jadach and Turnau find that the overlap function slope is 4 times smaller than the experimental value and that shrinkage is small. In case of cluster production, the slope becomes larger, i.e., closer to the experimental value.

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