THE EQUILIBRIUM DEFORMATIONS AND THE QUADRUPOLE 0^+ AND 2^+ VIBRATIONAL STATES IN EVEN ISOTOPES OF W, Os AND Pt

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(Received June 16, 1973)

The theoretical quadrupole and hexadecapole ground state equilibrium deformations and deformation energies are calculated by the method of summation of the single-particle energy levels, for even tungsten, osmium and platinum isotopes. They are in a good agreement with the known experimental data.

The reduced transition probability $B(E2)$, and Rasmussen parameter for the one-phonon vibrational 0^+ and 2^+ states are calculated for the same elements, in the framework of the "quasiboson" approximation. The results show that with the use of proper quadrupole and hexadecapole equilibrium deformations of the nucleus one is able to fit the experimental data for tungsten and osmium quite satisfactorily, while for platinum isotopes this method fails.

1. Introduction

In the last years many new experimental data for the collective levels in tungsten, osmium and platinum region appeared [1, 2]. So it seems to be interesting to look for the theoretical description of those experimental results.

Theoretical investigations of the properties of the collective vibrational states in the deformed nuclei are usually based on a microscopic model. In such a model one assumes that the nucleons move in a deformed average field and interact *via* pairing and multipole-multipole forces [3, 4, 5, 6]. For the single-particle potential one uses the Nilsson or Woods-Saxon model. The short range pairing interaction is treated by means of the BCS

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method and for the multipole-mulipole interaction the Random Phase Approximation is applied.

We would like to present here such a description of the properties of the one-phonon 0^+ and 2^+ collective vibrational levels in W, Os and Pt isotopes, with the inclusion of the hexadecapole deformation parameter (ε_4) which seems to play quite an important role. We are especially interested in the problem how far one can go with this formalism into the transition region. Also, we add the calculation of the deformation energies and the ground state equilibrium deformations for these nuclei. We insert the comparison of our results with the values obtained by Kumar and Baranger in their model which does not include the hexadecapole deformation [7, 8].

In Section 2 we present the method of evaluation of the equilibrium deformations. Section 3 gives the formulae for the energies, wave functions and the decay characteristics for the quadrupole vibrational collective 0^+ and 2^+ states. In Section 4 our results are presented and discussed.

2. *Deformation energies and equilibrium deformations*

It was suggested in the Ref. [2] that, for even isotopes of W, Os and Pt the deformed shape is the most stable if the neutron number N is equal 108. It may be expected then that the deformation energies for these elements, calculated as the functions of N with constant proton number Z, should have a maximum for $N = 108$. However, the deformation energies as calculated by Kumar and Baranger [8] appear to vary monotonically with N in the vicinity of $N = 108$ (Ref. [2]). This behaviour could be a result of the neglection of the hexadecapole deformation of the nuclei.

In the present work we have calculated the deformation energies and the equilibrium deformations for even isotopes of W, Os and Pt taking into account both the quadrupole (ε) and the hexadecapole (ε_4) deformation parameters. The calculations have been done with the use of the Mottelson-Nilsson-Bés-Szymański method [9]. This method consists in the minimization of the total energy of the nucleus with respect to ε and ε_4, with the condition that the nuclear volume remains constant as the shape of the nucleus is changed. The total energy of the nucleus, $E(\varepsilon, \varepsilon_4)$, is given as a sum of the single-particle Nilsson energy levels, with pairing energy, E_{BCS}, (both for protons and neutrons) and the Coulomb energy, E_{coul}, for protons:

$$E(\varepsilon, \varepsilon_4) = E^{\mathrm{p}}_{\mathrm{BCS}} + E^{\mathrm{n}}_{\mathrm{BCS}} + E_{\mathrm{coul}} \tag{1}$$

with

$$E_{\mathrm{BCS}} = \sum_n 2v_n^2 e_n - \Delta^2/G, \tag{2}$$

where

$$v_n^2 = \frac{1}{2}\left[1 - \frac{e_n - \lambda}{\sqrt{(e_n-\lambda)^2 + \Delta^2}}\right],$$

$e_n(\varepsilon, \varepsilon_4)$ — is the single-particle energy of a proton or neutron, λ — denotes the position of the Fermi level, and Δ — the energy gap.

The single-particle energy levels $e_n(\varepsilon, \varepsilon_4)$ are calculated in the "new Nilsson" model [10] with the inclusion of the coupling terms between the N and $N\pm2$ oscillator shells, as was suggested by Möller [11]. For the Nilsson model parameters κ and μ we take the interpolated values corresponding to $A = 187$, from Ref. [12]:

$$\kappa_p = 0.0620,\ \mu_p = 0.614 \text{ for protons and}$$

$$\kappa_n = 0.0636,\ \mu_n = 0.393 \text{ for neutrons.}$$

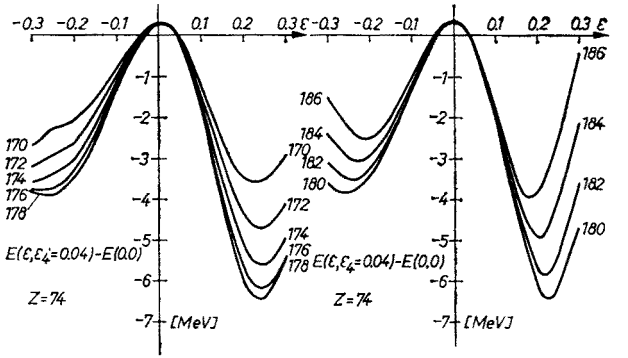

Fig. 1. Total energy in MeV, as a function of the deformation parameter ε with $\varepsilon_4 = \text{const.}$, $E(\varepsilon, \varepsilon_4 = 0.04) - E(0,0)$, plotted for isotopes of tungsten, $Z = 74$

The pairing interaction is taken into account in the BCS approximation [13]. The number of levels included in the BCS equatations

$$2/G = \sum_n \frac{1}{\sqrt{(e_n-\lambda)^2+\Delta^2}} \quad \text{and} \quad N = \sum_n \left[1 - \frac{e_n-\lambda}{\sqrt{(e_n-\lambda)^2+\Delta^2}}\right] \tag{3}$$

is equal to Z and N for protons and neutrons, respectively. The pairing force strength is $G_p = 20.8$ [MeV/A] for protons and $G_n = 15.6$ [MeV/A] for neutrons.

The Coulomb energy is calculated from the formula

$$E_{coul}(\varepsilon, \varepsilon_4) = E_{coul}(\varepsilon = 0, \varepsilon_4 = 0) \times Bc, \tag{4}$$

where

$$E_{coul}(\varepsilon = 0, \varepsilon_4 = 0) = 0.6\, Z^2 e^2/R = 0{,}71996\, Z^2/A^{1/3} \text{ MeV}$$

is the energy of a sphere, and the coefficients Bc describing the change of the Coulomb energy, as the nucleus is distorted from the spherical to the deformed shape, are taken from Ref. [14].

The calculations of the total energy surfaces as a function of the deformation parameters are performed for the positive and negative values of the quadrupole deformation ε, and for the positive values of the hexadecapole deformation ε_4. The equilibrium deforma-

tions in the minima of the surfaces are denoted as ε_0^+ and $(\varepsilon_4)_0^+$ for the positive values of ε (prolate shape of the nucleus) and as ε_0^- and $(\varepsilon_4)_0^-$ for the negative ones (oblate shape). The corresponding deformation energies are defind as $\mathscr{E}_{\text{def}}^+ = E(0,0) - E(\varepsilon_0^+, (\varepsilon_4)_0^+)$ and $\mathscr{E}_{\text{def}}^- = E(0,0) - E(\varepsilon_0^-, (\varepsilon_4)_0^-)$.

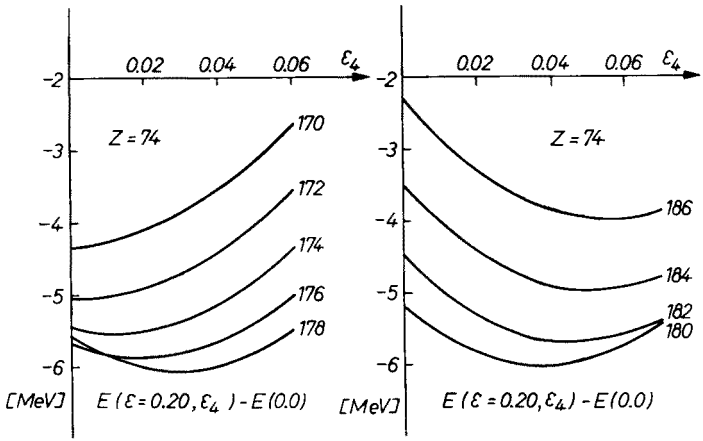

Fig. 2. Total energy in MeV, as a function of the deformation parametr ε_4 with ε = const., $E(\varepsilon = 0.20, \varepsilon_4) - E(0, 0)$, plotted for isotopes of tungsten

The minimum of the total energy surface in $(\varepsilon, \varepsilon_4)$ plane for the given nucleus is determined as a crossing point of the two curves — one of them joins the minima of the total energy E as a function of ε with various values of ε_4 = const. (Fig. 1) and the second one — the minima of the total energy as a function of ε_4 with various values of ε = const. (Fig. 2).

3. Microscopic description of the one-phonon vibrational states

In order to find the energy of the quadrupole collective vibrational states with $K = 0^+$ and 2^+ and their decay properties we apply the quasiboson approximation [4]. We start with the nuclear Hamiltonian given by:

$$H = H_{\text{sp}} + H_{\text{pair}} + H_{\text{coll}}, \tag{6}$$

where the single-particle part has the form

$$H_{\text{sp}} = \sum_{ks} (e_k - \lambda) a_{ks}^+ a_{ks} \quad \text{with} \quad s = \pm 1, \tag{7}$$

the pairing term is given by

$$H_{\text{pair}} = -G \sum_{kk'} a_{k+}^+ a_{k-}^+ a_{k'-} a_{k'+} \tag{8}$$

and the collective part

$$H_{\text{coll}} = -\tfrac{1}{2} Q^+ Q \tag{9}$$

with

$$Q = \sum_{\substack{ks \\ k's'}} q_{ksk's'} a^+_{ks} a_{k's'}, \tag{10}$$

where $q_{ksk's'}$ is the single-particle matrix element of $r^2 Y_{20}$ in the case of 0^+ state and of $r^2(Y_{22}+Y_{2-2})$ for 2^+ state.

The ground state of our system is the phonon vacuum defined by the equation $P_i \Psi = 0$, where P_i is the phonon operator given by

$$P_i = \tfrac{1}{2} \sum_{kk'} [\psi^i_{kk'} A_{kk'} - \varphi^i_{kk'} A^+_{kk'}]. \tag{11}$$

$A^+_{kk'}$ and $A_{kk'}$ are the quasiboson creation and annihilation operators respectively. In order to find the energy and the wave function (the amplitudes $\psi^i_{kk'}$ and $\varphi^i_{kk'}$) of the one-phonon state $P_i^+ \Psi$ we use the variational principle

$$\delta \left\{ \langle \Psi P_i | H | P_i \Psi \rangle - \langle \Psi | H | \Psi \rangle - \frac{\omega_i}{2} \left[\sum_{kk'} ((\psi^i_{kk'})^2 - (\varphi^i_{kk'})^2) - 2 \right] \right\} = 0 \tag{12}$$

which leads us to the dispersion relation[1]

$$1 - \kappa_q F(\omega_i) = 0 \tag{13}$$

with

$$F(\omega_i) = 2 \sum_{kk'} \frac{(E_k + E_{k'})\,(u_k v_{k'} + v_k u_{k'})^2 q^2_{kk'}}{(E_k + E_{k'})^2 - \omega_i^2} - \frac{4\Delta^2}{\gamma(\omega_i)} \sum_k \frac{q_{kk} \Gamma_k(\omega_i)}{E_k (4E_k^2 - \omega_i^2)}, \tag{14}$$

where E_k is the quasiparticle energy $E_k = \sqrt{(e_k - \lambda)^2 + \Delta^2}$,

$$\gamma(\omega_i) = \sum_{kk'} \frac{4\Delta^2 - \omega_i^2 + 4(e_k - \lambda)\,(e_{k'} - \lambda)}{E_k E_{k'} (4E_k^2 - \omega_i^2)\,(4E_{k'}^2 - \omega_i^2)}, \tag{15}$$

$$\Gamma_k(\omega_i) = \zeta(\omega_i) - 4(e_k - \lambda)\eta(\omega_i), \tag{16}$$

$$\zeta(\omega_i) = \sum_{kk'} \frac{q_{kk}[4\Delta^2 - \omega_i^2 - 4(e_k - \lambda)\,(e_{k'} - \lambda)]}{E_k E_{k'} (4E_k^2 - \omega_i^2)\,(4E_{k'}^2 - \omega_i^2)} \tag{17}$$

and

$$\eta(\omega_i) = \sum_{kk'} \frac{q_{kk}(e_k - e_{k'})}{E_k E_{k'} (4E_k^2 - \omega_i^2)\,(4E_{k'}^2 - \omega_i^2)}. \tag{18}$$

For the amplitudes $\psi^i_{kk'}$ and $\varphi^i_{kk'}$ we obtain the formulae

$$\psi^i_{kk'} = \sqrt{\frac{2}{Y}} \left[\frac{q_{kk'}(u_k v_{k'} + v_k u_{k'})}{E_k + E_{k'} - \omega_i} - \delta_{kk'} \frac{\Delta[2\Gamma_k(\omega_i) E_k + 4E_k^2 - \omega_i^2]}{\gamma(\omega_i) E_k (4E_k^2 - \omega_i^2)} \right], \tag{19}$$

[1] Due to the inclusion of the particle-particle interaction into the quasiparticle interaction, the spurious state is excluded from all the formulae for 0^+ state characteristics [15].

$$\varphi^i_{kk'} = \sqrt{\frac{2}{Y}}\left[\frac{q_{kk'}(u_k v_{k'} + v_k u_{k'})}{E_k + E_{k'} + \omega_i} - \delta_{kk'}\frac{\Delta[2\Gamma_k(\omega_i)E_k - 4E_k^2 + \omega_i^2]}{\gamma(\omega_i)E_k(4E_k^2 - \omega_i^2)}\right], \tag{20}$$

with

$$Y = \sum_{kk'}\frac{(E_k + E_{k'})\,(u_k v_{k'} + v_k u_{k'})^2 q^2_{kk'}\omega_i}{[(E_k + E_{k'})^2 - \omega_i^2]^2} +$$

$$+ \sum_k \frac{\Gamma_k^2(\omega_i)}{E_k(4E_k^2 - \omega_i^2)^2}\frac{2\Delta^2\omega_i}{\gamma^2(\omega_i)} - 4\Delta^2\frac{\omega_i}{\gamma(\omega_i)}\sum_k \frac{q_{kk}\Gamma_k(\omega_i)}{E_k(4E_k^2 - \omega_i^2)^2}. \tag{21}$$

In the case of the γ-vibrations (2^+ state) all the diagonal matrix elements q_{kk} are equal zero, and

$$F(\omega_i) = 2\sum_{kk'}\frac{(E_k + E_{k'})(u_k v_{k'} + v_k u_{k'})^2 q^2_{kk'}}{(E_k + E_{k'})^2 - \omega_i^2}, \tag{14a}$$

$$\psi^i_{kk'} = \sqrt{\frac{2}{Y}}\frac{q_{kk'}(u_k v_{k'} + v_k u_{k'})}{E_k + E_{k'} - \omega_i}, \tag{19a}$$

$$\varphi^i_{kk'} = \sqrt{\frac{2}{Y}}\frac{q_{kk'}(u_k v_{k'} + v_k u_{k'})}{E_k + E_{k'} + \omega_i} \tag{20a}$$

with

$$Y = \sum_{kk'}\frac{(E_k + E_{k'})\,(u_k v_{k'} + v_k u_{k'})^2 q^2_{kk'}\omega_i}{[(E_k + E_{k'})^2 - \omega_i^2]^2}. \tag{21a}$$

It is now easy to find the matrix elements for the electric multipole transition $E\lambda$

$$M(E\lambda) = \frac{e}{\sqrt{2}}\sum_{kk'}(u_k v_{k'} + v_k u_{k'})f^{(\lambda)}_{kk'}(\psi_{kk'} + \varphi_{kk'}), \tag{22}$$

where $f^{(\lambda)}_{kk'}$ is the single-particle matrix element for $E\lambda$ transition. So, for the transition between the ground 0^+ state and the γ-vibrational 2^+ state we find

$$B(E2; 0^+0 \to 2^+2) = 0.674\,A^{-2/3}\times$$

$$\times\left[(1 + e_{\text{eff}})\sum_{\substack{kk'\\ \text{protons}}}\frac{q^2_{kk'}(u_k v_{k'} + v_k u_{k'})^2(E_k + E_{k'})}{(E_k + E_{k'})^2 - \omega_i^2} + e_{\text{eff}}\sum_{\substack{kk'\\ \text{neutrons}}}\frac{q^2_{kk'}(u_k v_{k'} + v_k u_{k'})^2}{(E_k + E_{k'})^2 - \omega_i^2}\times\right.$$

$$\left.\times(E_k + E_{k'})\right]\Big/\sum_{\substack{kk'\\ \text{protons,}\\ \text{neutrons}}}\frac{q^2_{kk'}(u_k v_{k'} + v_k u_{k'})^2(E_k + E_{k'})\omega_i}{[(E_k + E_{k'})^2 - \omega_i^2]^2} \tag{23}$$

in the single-particle units. Here $q_{kk'} \sim \langle k|r^2(Y_{22} + Y_{2-2})|k'\rangle$.

For the decay of 0^+ collective vibrational state (β-vibration) one usually calculates the so-called Rasmussen parameter [16],

$$X = \frac{e^2 R_0^4 \varrho^2(E0)}{B(E2; 0^+_{\text{vibrational}} - 2^+_{\text{rotation on the ground state}})}, \tag{24}$$

where $\varrho(E0)R_0^2 = M(E0)$, $R_0 = 1.2\,A^{1/3}$ [fm] is the nuclear radius and e — the electron charge. For this parameter we have the following formula

$$X = \frac{4}{(F(\omega_i))^2}\left[(1+e_{\text{eff}})\left[\sum_{\substack{kk' \\ \text{protons}}} \frac{r_{kk'}q_{kk'}(E_k+E_{k'})\,(u_k v_{k'}+v_k u_{k'})^2}{(E_k+E_{k'})^2-\omega_i^2} - \right.\right.$$

$$\left. - \frac{2\Delta^2}{\gamma(\omega_i)} \sum_{\substack{k \\ \text{protons}}} \frac{r_{kk}\Gamma_k(\omega_i)}{E_k(4E_k^2-\omega_i^2)}\right] + e_{\text{eff}}\left[\sum_{\substack{kk' \\ \text{neutrons}}} \frac{r_{kk'}q_{kk'}(E_k+E_{k'})\,(u_k v_{k'}+v_k u_{k'})^2}{(E_k+E_{k'})^2-\omega_i^2} - \right.$$

$$\left.\left. - \frac{2\Delta^2}{\gamma(\omega_i)} \sum_{\substack{k \\ \text{neutrons}}} \frac{r_{kk}\Gamma_k(\omega_i)}{E_k(4E_k^2-\omega_i^2)}\right]\right]^2, \tag{25}$$

where $q_{kk'} \sim \langle k|r^2\,Y_{20}|k'\rangle$, $r_{kk'} \sim \langle k|r^2 Y_{00}|k'\rangle$ and $F(\omega_i)$ is given by the formula (14a).

4. Results and discussion

Table I presents the values of the quadrupole and hexadecapole ground state equilibrium deformations, for prolate $(\varepsilon_0^+, (\varepsilon_4)_0^+)$ and oblate $(\varepsilon_0^-, (\varepsilon_4)_0^-)$ shapes of nuclei, calculated by the method described in Section 2. It contains also the corresponding deformation energies $\mathscr{E}^{\pm}_{\text{def}} = E(0,0) - E(\varepsilon_0^{\pm}, (\varepsilon_4)_0^{\pm})$, the energy differences between the oblate and the prolate minimum, $\Delta\mathscr{E}_{\text{def}} = E(\varepsilon_0^-, (\varepsilon_4)_0^-) - E(\varepsilon_0^+, (\varepsilon_4)_0^-) = \mathscr{E}^+_{\text{def}} - \mathscr{E}^-_{\text{def}}$, and the values of β_2 and β_4. The relation between (β_2, β_4) and $(\varepsilon, \varepsilon_4)$ parameters has been taken from work [12] where it is assumed that $R(\Theta) = R_0(\beta_2\beta_4\beta_6 \ldots)(1+\beta_2 Y_{20}+\beta_4 Y_{40}+\beta_6 Y_{60}+\ldots)$. The calculations have been performed for about ten isotopes od W, Os, and Pt including light nuclei lying far from the β-stability line. For comparison, some experimental values of β_2 taken from Ref. [2] are also presented. The experimental values of β_4 for these isotopes are unknown. One can see that the theoretical values calculated by this simple method are in quite good agreement with the experimental ones. They are also close to the results obtained by means of the Strutinski renormalization method, with Nilson potential in the single particle Hamiltonian [12, 17].

The quadrupole (β_2) and hexadecapole (β_4) ground state deformations for some isotopes of W, Os, and Pt have been calculated in the paper of Gareev *et al.* [18] and recently in the paper of Götz *et al.* [19], by means of the Strutinski method with Woods-Saxon single-particle energy levels. Our β_2 values for W and Os nuclei and for light isotopes of Pt are close to the values given in [19], the differences are less than 10%. The β_2 value for Pt ^{192} and β_2 from [19] have different signs. The values of β_4 given

TABLE I

The ground state equilibrium deformations and the deformation energies of W, Os and Pt isotopes

Z	N	A	ε_0^-	$(\varepsilon_4)_0^-$	ε_0^+	$(\varepsilon_4)_0^+$	$\mathscr{E}_{\text{def}}^-$ MeV	$\Delta\mathscr{E}_{\text{def}}$ MeV	$\mathscr{E}_{\text{def}}^+$ MeV	β_2^{exp}	β_2	β_4
74	96	170	—	—	.225	.000	3.0	1.5	4.5		.224	.022
	98	172	−.265	.000	.240	.000	3.5	1.9	5.4		.261	.025
	100	174	−.255	.000	.240	.015	4.0	2.0	6.0		.262	.007
	102	176	−.250	.000	.245	.024	4.3	2.0	6.3		.268	−.003
	104	178	−.240	.005	.240	.034	4.4	2.1	6.5		.263	−.016
	106	180	−.240	.009	.230	.043	4.2	2.3	6.5	.251	.253	−.029
	108	182	−.240	.011	.225	.053	3.8	2.2	6.0	.247	.248	−.042
	110	184	−.225	.017	.215	.054	3.3	1.8	5.1	.233	.236	−.045
	112	186	−.205	.020	.190	.050	2.7	1.3	4.0	.224	.208	−.045
76	98	174	—	—	.225	.004	2.7	1.4	4.1		.245	.017
	100	176	−.260	.000	.230	.011	3.1	1.5	4.6		.251	.009
	102	178	−.250	.002	.230	.020	3.3	1.7	5.0		.251	−.002
	104	180	−.250	.003	.225	.032	3.4	1.7	5.1		.246	−.017
	106	182	−.240	.006	.225	.040	3.3	1.8	5.1	.228	.247	−.027
	108	184	−.225	.011	.215	.050	2.9	1.8	4.7	.213	.236	−.041
	110	186	−.200	.014	.195	.051	2.4	1.5	3.9	.205	.214	−.045
	112	188	−.180	.019	.180	.048	1.9	1.2	3.1	.194	.197	−.044
	114	190	−.165	.024	.165	.047	1.5	0.7	2.2	.180	.180	−.045
	116	192	−.150	.023	.145	.044	1.1	0.5	1.6	.167	.158	−.044
78	100	178	—	—	.230	.008	2.2	1.1	3.3		.250	.013
	102	180	−.230	.002	.230	.015	2.4	1.1	3.5		.251	.005
	104	182	−.230	.004	.230	.027	2.5	1.1	3.6		.251	−.010
	106	184	−.220	.007	.220	.036	2.3	1.1	3.4	.226	.241	−.023
	108	186	−.200	.014	.200	.040	2.0	0.9	2.9	.197	.219	−.031
	110	188	−.170	.017	.175	.038	1.7	0.5	2.2	.183	.191	−.033
	112	190	−.155	.020	.150	.036	1.4	0.2	1.6	.165	.163	−.034
	114	192	−.140	.022	.130	.032	1.0	0.1	1.1	.169	.141	−.031
	116	194	−.125	.020	.115	.031	0.7	−0.1	0.6	.149	−.130	−.017
	118	196	−.110	.018	.100	.028	0.5	−0.3	0.2	.135	−.115	−.016

The first three columns identify the nucleus. The next seven columns list the results obtained in this paper: the quadrupole (ε) and hexadecapole (ε_4) ground state deformations for prolate and oblate shapes of nuclei, the deformation energies for oblate shape ($\mathscr{E}_{\text{def}}^-$), the energy differences between the oblate and the prolate minimum ($\Delta\mathscr{E}_{\text{def}}$) and the deformation energies for prolate shape ($\mathscr{E}_{\text{def}}^+$). All energies are given in MeV. In column 11 some experimental values of the quadrupole deformation β_2^{exp} are given. The last two columns present the quadrupole (β_2) and hexadecapole (β_4) deformations evaluated from (ε, ε_4) parameters.

in [19] differ much more from our results. Their absolute values are always greater than ours. This could be partly connected with the fact that in evaluating the (β_2, β_4) parameters the authors of work [19] assumed that $R(\Theta) = R_0(\beta_2\beta_4)\ (1+\beta_2 Y_{20}+\beta_4 Y_{40})$ neglecting β_6 and the others components.

The equilibrium deformations are also presented in Fig. 3 in the $(\varepsilon, \varepsilon_4)$ plane. It can be seen that for isotopes lighter than the isotope with $N = 108$, the hexadecapole deformation increases strongly with A, while for the isotopes with $N > 108$ it changes very slowly. On the contrary, the quadrupole deformation changes very slowly with A for isotopes lighter than $N = 108$ and it strongly decreases for $N > 108$.

In Figure 4 the deformation energies are plotted as the function of the neutron number N for constant proton number Z. It appears that these curves really have a maxi-

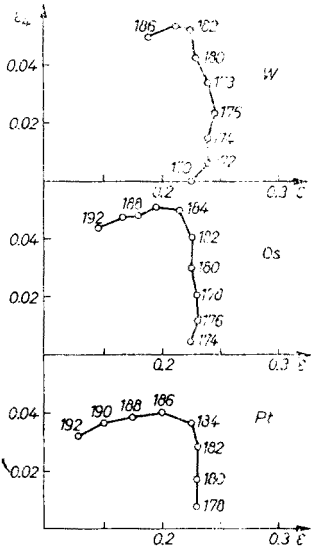

Fig. 3. Ground state equilibrium deformations in $(\varepsilon, \varepsilon_4)$ plane, for W, Os and Pt nuclei

mum, as was suggested in Ref. [2]. The position of this maximum is in the vicinity of $N = 104, 106$. The curves presenting the variation of $\Delta\mathscr{E}_{\text{def}}$ with N (Fig. 5) also exhibit a maximum. The greatest energy differences appear for $N = 106, 108$.

The results for the deformation energies in W, Os, and Pt nuclei obtained by Götz *et al.* are plotted in Fig. 6. Unfortunately, in Os and Pt nuclei the authors did not present the results for isotopes lighter than isotopes with $N = 106$. So, the comparison with our work for these nuclei is difficult. The deformation energies taken from Ref. [19] are greater than ours at about 1 MeV for Pt nuclei to 3 MeV for W nuclei. In all nuclei the deformation energies decrease with increasing N for $N > 108$. For isotopes with $N < 108$ the values of $\mathscr{E}_{\text{def}}$ are equal to the value of $\mathscr{E}_{\text{def}}$ $(N = 108)$ or smaller than this value

With the use of the formulae (23) and (25) we have calculated the values of the reduced transition probability $B(E2; 0^+_{\text{ground}} - 2^+_{\text{vibr}})$ for the transition from the 0^+ ground state to the second 2^+ state (the γ-vibrational level) and the Rasmussen parameter X which characterizes the decay properties of the one-phonon vibrational 0^+ state (the β-vibrational level). The calculation was made for seven isotopes of Pt, six Os, and four W nuclei. The quadrupole equilibrium deformations were obtained from the experimental values of $B(E2)$ between the ground state and the first rotational 2^+ state [2]. For the hexadecapole

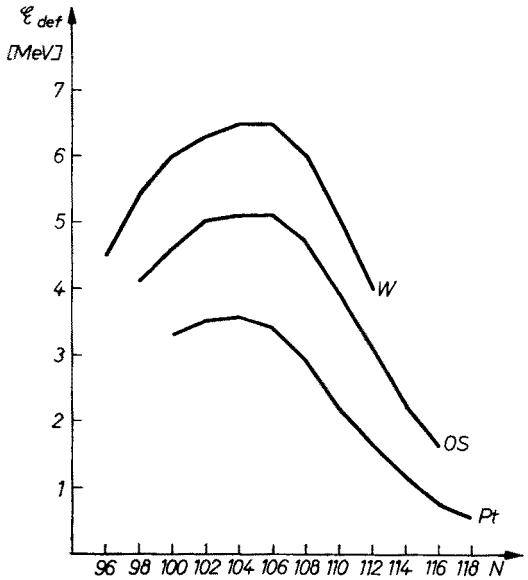

Fig. 4. Deformation energies in MeV, as a function of neutron number N, for even isotopes of W, Os and Pt

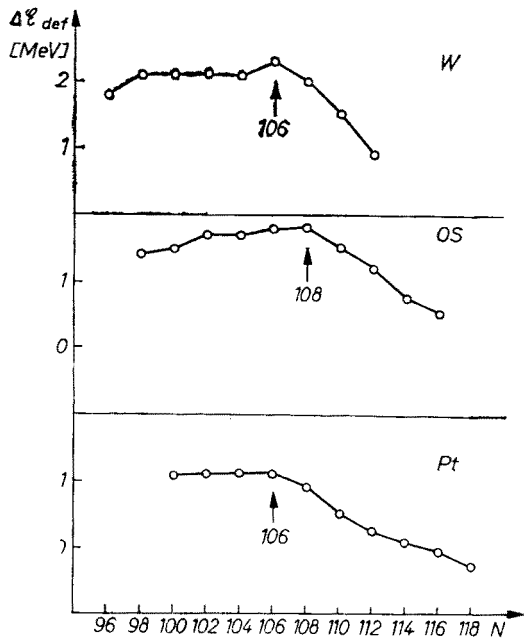

Fig. 5. Energy differences in MeV between the oblate and the prolate minimum, as a function of neutron number N, for even isotopes of W, Os and Pt

deformations we took the theoretical values given in Ref. [20] for tungsten and osmium. For platinum we used the extrapolated values. Both the ε and ε_4 deformations agree fairly well with the values evaluated in this paper. For the rest of the Nilsson model parameters and for the pairing force strength we use the numbers given in Section 2. All the summations in (23) and (25) goes over all single-particle states in the energy range $\lambda - 2\hbar\omega_0 < e_k < \lambda + \hbar\omega_0$ where λ is the energy of the Fermi level. It corresponds to about 50 levels for protons and about 60 for neutrons.

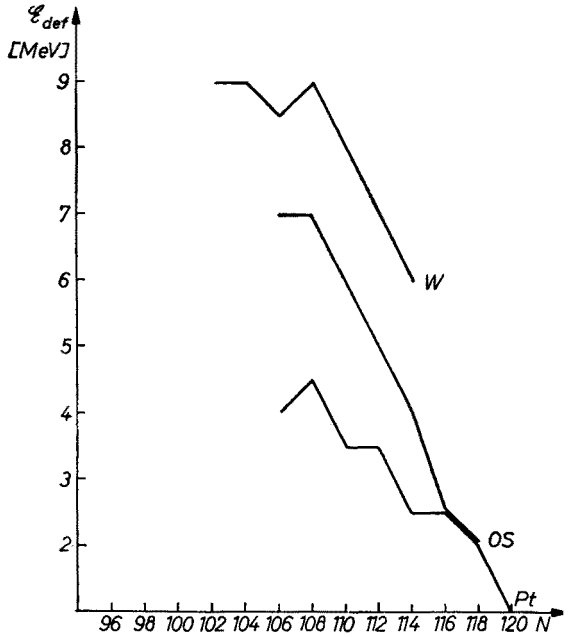

Fig. 6. Deformation energies in MeV, as a function of neutron number N, for W, Os and Pt nuclei, taken from Ref. [19]

All the calculations are performed for the experimental values of the energy of the one-phonon state. The only free parameter of this calculation is the effective charge for neutrons which was chosen to be equal to 0.15.

In Table II we present our results, the experimental data and the resulst obtained by Kumar and Baranger [7, 8]. In the case of the 2^+ states one can notice that the agreement of our results with the experimental data for tungsten and osmium isotopes is quite good. In many cases it is better than in the theory of Kumar and Baranger, while for the platinum isotopes our results are very far from the experimental data. The same trouble seems to occur with the value of the Rasmussen parameter for those nuclei. Unfortunately there are almost no experimental data for X for the isotopes of Tungsten and Osmium.

With the use of Eq. (13) one can calculate the values of the quadrupole coupling constants κ_{20} and κ_{22} which give the experimental energy of the one-phonon 0^+ and 2^+ vibrational states $1/F(\omega_i) = \kappa = \kappa^0 A^{-4/3}$. It seems to be interesting to notice here that the quadrupole coupling constant κ_{20}^0 for the 0^+ β-vibrational state decreases with the

quadrupole deformation of the nucleus while the non-axial quadrupole coupling constant κ^0_{22}, which corresponds to the 2^+ γ-vibrational state, increases as the quadrupole nuclear deformation increases. This result is in accordance with the predictions of Ref. [20].

The general aim of this work was to test how far one can go into the transition region with the formalism described above. It seems to us that using the proper values of the

TABLE II

Vibrational 2^+ and 0^+ states in W, Os, and Pt isotopes

Nucleus	ε	ε_4	$E_{2^+}^{exp}$ MeV	$B(E2)$ exp	$B(E2)$ K–B	$B(E2)$ $e_{eff}=0.15$	$E_{0^+}^{exp}$ MeV	10^2X exp	10^2X theory	10^2X K–B
^{180}W	0.24	0.056	0.828			4.70	0.904	20	45	
^{182}W	0.24	0.056	1.221	4.01	0.87	2.02	1.138		42	
^{184}W	0.22	0.056	0.903	3.92	2.71	4.81	1.004		42	
^{186}W	0.22	0.056	0.737	4.90	4.84	7.15	0.883		42	
^{182}Os	0.21	0.05	0.891			5.13				
^{184}Os	0.20	0.05	0.944			4.45				
^{186}Os	0.19	0.05	0.767	7.28	5.97	6.67				
^{188}Os	0.18	0.05	0.633	7.86	5.69	8.53	1.085		26	
^{190}Os	0.17	0.04	0.558	7.02	4.36	9.41	0.912		29	
^{192}Os	0.15	0.04	0.489	6.38	1.05	10.10				
^{184}Pt	0.21	0.05	0.649			8.24	0.492	0.8 ± 0.3	31	
^{186}Pt	0.18	0.05	0.607			8.12	0.471	0.65 ± 0.2	24	0.61
^{188}Pt	0.17	0.05	0.606			8.70	0.799	0.90 ± 0.15	22	
^{190}Pt	0.15	0.04	0.598			8.62	0.921	0.62 ± 0.12	16	
^{192}Pt	0.16	0.04	0.612	0.60	0.15	8.58	1.195	2.2 ± 0.3	18	1.9
^{194}Pt	0.14	0.04	0.622	0.36	0.15	7.71	1.267	0.8 ± 0.2	10	1.4
^{196}Pt	0.13	0.03	0.689			6.02	1.135		8	

The first column identifies the nucleus. The next two columns give the quadrupole and the hexadecapole deformations of the nuclei, used in the calculation. The next four columns list the experimental energies of 2^+ vibrational states, in MeV, the experimental values of $B(E2)$, the values of $B(E2)$ given by Kumar and Baranger and values of $B(E2)$ calculated in this paper, with $e_{eff} = 0.15$. All $B(E2)$ values are given in single particle units. In the last four columns we present the experimental energies of 0^+-vibrational state in MeV, the experimental values of Rasmussen parameter multiplied by a factor of 100, the values of this parameter calculated in this paper and the results of Kumar and Baranger.

quadrupole and hexadecapole equilibrium deformations ε and ε_4, one is able to fit the experimental data for Tungsten and Osmium isotopes quite satisfactorily while for Platinum nuclei this formalism is no longer valid.

Most of the numerical calculations were performed in the Joint Institute for Nuclear Research in Dubna, and we are indebted to the Nuclear Theory Department for the hospitality extended to us.

We would like to express our gratitude to Drs. N. I. Pyatov, A. Sobiczewski and J. Jastrzębski, and to Professor Z. Szymański for their helpful comments.

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