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QUANTUM ELECTRODYNAMICS WITH COMPENSATING CURRENT

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A formulation of quantum electrodynamics is proposed in which all the propagators and field operators are gauge invariant. It is based on an old idea of Heisenberg and Euler which consists in the introduction of the linear integrals of potentials as arguments of the exponential functions. This method is generalized by an introduction of the so-called "compensating currents", which ensure local, *i.e.* in every point of space-time, charge conservation. The linear integral method is a particular case of that proposed in this paper. As the starting point we use quantum electrodynamics with a non-zero, small photon mass (Proca theory). It is shown that, due to the presence of the compensating current, the theory is fully renormalizable in Hilbert space with positive definite scalar product. The problem of the definition of the current operator is also briefly discussed.

1. Introduction

Physical objects appearing in quantum electrodynamics can be divided into two large classes: gauge invariant and gauge dependent. The first class of objects contains all the observables (like current, S-matrix *etc.*) and some of the renormalization constants, whereas propagators or field operators can be quoted as elements of the second class.

Consider as an example the electron propagator $i\langle 0|T(\psi(x)\bar{\psi}(y))|0\rangle$. Its gauge dependence has a clear physical origin: it is the violation of the charge conservation, namely, a charge e is created in y from the vacuum and is absorbed back into the vacuum in x [4]. Therefore, the charge conservation law is satisfied globally but violated locally. Charge conservation can be made local by introducing into the propagator an object which will carry the charge back from x to y. One example of such an object is an exponential function with a linear integral of the potential as its argument [1, 2, 5, 8, 9]:

$$\exp\left(-ie\int_x^y d\xi^\mu A_\mu(\xi)\right). \tag{1.1}$$

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Then the propagator defined as:

$$i\langle 0|T[\psi(x)\exp(-ie\int_x^y d\xi^\mu A_\mu(\xi))\psi(y)]\,|0\rangle \tag{1.2}$$

is invariant under local gauge transformations:

$$\psi(x) \to \bar{\psi}(x)e^{-ie\Lambda(x)}, \qquad A_\mu(x) \to A_\mu(x)+\partial_\mu\Lambda(x). \tag{1.3}$$

The subject of the present paper is a generalization of the above method. The main idea is to replace the linear integral in the exponent by a more general object of the following type [4, 6]:

$$\int d^4z[\mathfrak{J}^\lambda(z-x)-\mathfrak{J}^\lambda(z-y)]A_\lambda(z), \tag{1.4}$$

where $\mathfrak{J}_\lambda$is a non-conserved current with point source: $\partial^\lambda\mathfrak{J}_\lambda(x) = -e\delta^{(4)}(x)$. This current carries the charge back from x to y and can be interpreted as a macroscopic current flowing in the sources and the detectors of charged particles.

In Section 2 we construct propagators with an arbitrary numbei of external lines using such a compensating current $\mathfrak{J}_\lambda$. It is shown that they are gauge invariant, where by the gauge invariance we mean invariance under gauge transformations of the free photon propagator. In Section 3 the problem of renormalizability is discussed. It is shown that the theory can be made renormalizable, even if as the starting point quantum electrodynamics with small photon mass (Proca theory) is used. In contrast to the formulation proposed by Zimmerman [10, 11], in our formulation the introduction of the indefinite metric in the state space is not needed. In the last Section we discuss briefly the problem of the definition of the current operator in the theoiy with compensating current.

2. *Gauge invariant propagators*

As the basic objects in our gauge independent, but compensating current dependent formulation of quantum electrodynamics we choose propagators defined in the following way:

$$\begin{aligned} &G_{\mu_1\ldots\mu_k}[x_1, \ldots, x_n, y_n, \ldots, y_1, z_1, \ldots, z_k|\mathscr{A}a] = \\ &= i^n\langle 0;\text{out}\,|T(\psi(x_1)\ldots\psi(x_n)\bar{\psi}(y_n)\ldots\bar{\psi}(y_1)A_{\mu_1}(z_1)\ldots A_{\mu_k}(z_k)\Phi[a])\,|0;\text{in}\rangle, \end{aligned} \tag{2.1}$$

where:

$$\Phi[a] = \exp\left[-ie\int d^4z'\sum_{i=1}^{n}(a^\lambda(z'-x_i)-a^\lambda(z'-y_i))\,(A_\lambda(z')+\mathscr{A}_\lambda(z'))\right] \tag{2.2}$$

and $\mathscr{A}_\lambda(z')$ is an external c-number electromagnetic potential. Since $ea^\lambda(z)$ plays the role of the compensating current, it must satisfy the divergence condition with a delta-type right-hand side:

$$\partial^\lambda a_\lambda(z) = \delta^{(4)}(z). \tag{2.3}$$

Therefore, the propagator (2.1) is invariant under simultaneous gauge transformations of the fields $\psi(x)$ and $A_\mu(z)$ or under the same transformations applied to ψ and to the external field $\mathscr{A}_\mu$. To show the invariance of $G_{\mu_1\ldots\mu_k}$ under gauge transformations of the free photon propagator we will use the following brief notation for the perturbation series in the theory without a compensating current [16]

$$G_{\mu_1\ldots\mu_k}[x_1, \ldots, x_n, y_n, \ldots, y_1, z_1, \ldots z_k|\mathscr{A}] =$$
$$= i^k \frac{\delta^k}{\delta J^{\mu_1}(z_1) \ldots \delta J^{\mu_k}(z_k)} \Big\{ (V[\mathscr{A}])^{-1} \exp\left(\frac{i}{2}\int J\Delta_F J\right) \exp\left(-\int J\Delta_F \frac{\delta}{\delta\mathscr{A}}\right) \times$$
$$\times \exp\left(-\frac{i}{2}\int \frac{\delta}{\delta\mathscr{A}} \Delta_F \frac{\delta}{\delta\mathscr{A}}\right) C[\mathscr{A}] K_F[x_1, \ldots, x_n, y_n, \ldots, y_1|\mathscr{A}] \Big\}\Big|_{J=0}. \tag{2.4}$$

In this equation K_F denotes the propagator in the theory of non-interacting electrons moving in an external electromagnetic field and $C[\mathscr{A}]$ is the vacuum to vacuum amplitude in this theory:

$$K_F[x_1, \ldots, x_n, y_n, \ldots, y_1|\mathscr{A}] =$$
$$= i^n \langle 0; \text{out}| T(\psi_0(x_1) \ldots \psi_0(x_n)\bar{\psi}_0(y_n) \ldots \bar{\psi}_0(y_1)) |0; \text{in}\rangle, \tag{2.5a}$$

$$\left(-i\gamma^\nu \frac{\partial}{\partial x^\nu} + m + e\gamma^\nu \mathscr{A}_\nu(x)\right) \psi_0(x) = 0, \tag{2.5b}$$

$$C[\mathscr{A}] = \langle 0; \text{out}|0; \text{in}\rangle = \langle 0; \text{in}| T \exp\left(-ie \int d^4z : \bar{\psi}_0^{\text{in}} \gamma^\lambda \psi_0^{\text{in}} : \mathscr{A}_\lambda\right) |0; \text{in}\rangle, \tag{2.5c}$$

$$V[\mathscr{A}] = \exp\left(-\frac{i}{2}\int \frac{\delta}{\delta\mathscr{A}} \Delta_F \frac{\delta}{\delta\mathscr{A}}\right) C[\mathscr{A}]. \tag{2.5d}$$

Symbols of the type $\int J\Delta_F J$ *etc.* stand for the following integrals:

$$\int J^\mu(x) \Delta_{F\mu\nu}(x-y) J^\nu(y) d^4x d^4y,$$

where $\Delta_{F\mu\nu}$ is the free propagator of the vector field with mass μ:

$$\Delta_{F\mu\nu}(x) = \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \frac{-g_{\mu\nu} + \mu^{-2} k_\mu k_\nu}{\mu^2 - k^2 - i\varepsilon}. \tag{2.6}$$

Expression (2.4) can be made gauge invariant by inserting into it a suitable phase factor in the front of K_F. This phase factor must be composed of a^λ and $\mathscr{A}^\lambda$ in the following way:

$$\exp\left[-ie \int d^4z' \sum_{i=1}^{n} (a^\lambda(z'-x_i) - a^\lambda(z'-y_i)) \mathscr{A}_\lambda(z')\right] \equiv \exp\left(-ie \int a\mathscr{A}\right). \tag{2.7}$$

The perturbation expansion for the gauge independent propagator $G[a]$ can now be written as:

$$G_{\mu_1\ldots\mu_k}[x_1, \ldots, x_n, y_n, \ldots, y_1, z_1, \ldots, z_k|\mathscr{A}a] =$$
$$= i^k \frac{\delta^k}{\delta J^{\mu_1}(z_1) \ldots \delta J^{\mu_k}(z_k)} \Big\{ (V[\mathscr{A}])^{-1} \exp\left(\frac{i}{2}\int J\Delta_F J\right) \exp\left(-\int J\Delta_F \frac{\delta}{\delta\mathscr{A}}\right) \times$$
$$\times \exp\left(-\frac{i}{2}\int \frac{\delta}{\delta\mathscr{A}} \Delta_F \frac{\delta}{\delta\mathscr{A}}\right) \exp\left(-ie\int a\mathscr{A}\right) K_F[x_1, \ldots, x_n, y_n, y_{n-1}, \ldots, y_1|\mathscr{A}] \Big\}\Big|_{J=0}. \tag{2.8}$$

It is easy to show that

$$\exp\left(-\frac{i}{2}\int\frac{\delta}{\delta\mathscr{A}}\Delta_F\frac{\delta}{\delta\mathscr{A}}\right)\exp\left(-ie\int a\mathscr{A}\right)=$$

$$=\exp\left(-ie\int a\mathscr{A}\right)\exp\left(-\frac{i}{2}\int\left(iea-\frac{\delta}{\delta\mathscr{A}}\right)\Delta_F\left(iea-\frac{\delta}{\delta\mathscr{A}}\right)\right) \tag{2.9a}$$

and

$$\exp\left(-\int J\Delta_F\frac{\delta}{\delta\mathscr{A}}\right)\exp\left(-ie\int a\mathscr{A}\right)=$$

$$=\exp\left(-ie\int a\mathscr{A}\right)\exp\left(-\int J\Delta_F\left(\frac{\delta}{\delta\mathscr{A}}-iea\right)\right). \tag{2.9b}$$

Using (2.9) we can insert $\exp(-ie\int a\mathscr{A})$ in (2.8) before the large bracket[1]

$$G_{\mu_1\ldots\mu_k}[x_1,\ldots,x_n,y_n,\ldots,y_1,z_1,\ldots,z_k|\mathscr{A}a]=$$

$$=\exp\left(-ie\int a\mathscr{A}\right)i^k\frac{\delta^k}{\delta J^{\mu_1}(z_1)\ldots\delta J^{\mu_k}(z_k)}\Big\{(V[\mathscr{A}])^{-1}\exp\left(\frac{i}{2}\int J\Delta_F J\right)\times$$

$$\times\exp\left(-\int J\Delta_F\left(\frac{\delta}{\delta\mathscr{A}}-iea\right)\right)\exp\left(-\frac{i}{2}\int\left(\frac{\delta}{\delta\mathscr{A}}-iea\right)\Delta_F\left(\frac{\delta}{\delta\mathscr{A}}-iea\right)\right)\times$$

$$\times C[\mathscr{A}]K_F[x_1,\ldots,x_n,y_n,\ldots,y_1|\mathscr{A}]\Big\}\Big|_{J=0}. \tag{2.10}$$

We see that (2.10) can be obtained from (2.4) simply by insertion of $\delta/\delta\mathscr{A}-iea$ instead of $\delta/\delta\mathscr{A}$ and multiplication by an overall phase factor, equal to one in the absence of the external electromagnetic field. Similarly as in [4], we replace $\delta/\delta\mathscr{A}-iea$ by:

$$\frac{\delta}{\delta\mathscr{A}}-iea\to\frac{\delta}{\delta\mathscr{A}_\mu}-\int d^4w\frac{\partial}{\partial z^\lambda}a^\mu(z-w)\frac{\delta}{\delta\mathscr{A}_\lambda(w)}=$$

$$=\int d^4w\left[g^{\mu\lambda}\delta^{(4)}(z-w)-\frac{\partial}{\partial z_\lambda}a^\mu(z-w)\right]\frac{\delta}{\delta\mathscr{A}^\lambda(w)}. \tag{2.11}$$

Formula (2.11) follows from the equations describing the dependence of C and K_F on the gauge [16]:

$$\partial_\mu\frac{\delta}{\delta\mathscr{A}_\mu(z)}C[\mathscr{A}]=0, \tag{2.12a}$$

$$\partial_\mu\frac{\delta}{\delta\mathscr{A}_\mu(z)}K_F[x_1,\ldots,x_n,y_n,\ldots,y_1|\mathscr{A}]=$$

$$=ie\sum_{i=1}^n[\delta^{(4)}(z-x_i)-\delta^{(4)}(z-y_i)]K_F[x_1,\ldots,x_n,y_n,\ldots,y_1|\mathscr{A}]. \tag{2.12b}$$

[1] A procedure of this kind has been applied by Białynicki-Birula to the propagator G with $k=0$ in Ref. [4].

By substitution of (2.11) into (2.10) we obtain:

$$G_{\mu_1 \ldots \mu_k}[x_1, \ldots, x_n, y_n, \ldots, y_1, z_1, \ldots, z_k | \mathscr{A} a] =$$

$$= \exp\left(-ie \int a\mathscr{A}\right) i^k \frac{\delta^k}{\delta J^{\mu_1}(z_1) \ldots \delta J^{\mu_k}(z_k)} \Big\{ (V[\mathscr{A}])^{-1} \exp\left(\frac{i}{2} \int J\Delta_F J\right) \times$$

$$\times \exp\left(-\int J D_F^{(a)} \frac{\delta}{\delta\mathscr{A}}\right) \exp\left(-\frac{i}{2} \int \frac{\delta}{\delta\mathscr{A}} \mathscr{D}^F \frac{\delta}{\delta\mathscr{A}}\right) \times$$

$$\times C[\mathscr{A}] K_F[x_1, \ldots, x_n, y_n, \ldots, y_1 | \mathscr{A}] \Big\} \Big|_{J=0}, \tag{2.13}$$

where the photon propagators $D_F^{(a)}$ and $\mathscr{D}^F$ are given by the following formulae:

$$D_{F\mu\nu}^{(a)}(z-w) = \int d^4 z' \left[g_\lambda^\nu \delta^{(4)}(z'-w) - \frac{\partial}{\partial z'^\lambda} a^\nu(z'-w) \right] \Delta_{F\mu\lambda}(z-z'), \tag{2.14a}$$

$$\mathscr{D}_{\lambda\varrho}^F(w-w') = \int d^4 z d^4 z' \left[g_{\lambda\mu} \delta^{(4)}(z-w) - \frac{\partial}{\partial z^\lambda} a_\mu(z-w) \right] \Delta_F^{\mu\nu}(z-z') \times$$

$$\times \left[g_{\varrho\nu} \delta^{(4)}(z'-w') - \frac{\partial}{\partial z'^\varrho} a_\nu(z'-w') \right]. \tag{2.14b}$$

It follows from (2.3) that the singular part of the free vector field propagator $\mu^{-2}\partial_\mu\partial_\nu\Delta_F$ does not appear in (2.14). Therefore, the propagator $\Delta_{F\mu\nu}$ in these formulae can be replaced by the usual Feynman-gauge propagator $D_{F\mu\nu}$. In order to avoid infrared divergences we maintain non-zero mass in the denominator of its Fourier transform.

Formula (2.13) shows that in the perturbation series of the general propagator $G_{\mu_1 \ldots \mu_k}$ appear three types of free photon propagators: $D_{F\mu\nu}^{(a)}$, $\mathscr{D}_{\mu\nu}^F$ and $\Delta_{F\mu\nu}$. The first two can be obtained from the Feynman-gauge propagator by a transformation of the following type:

$$D_{F\mu\nu} \to D_{F\mu\nu} + \partial_\mu f_\nu + \partial_\nu f_\mu + \partial_\mu \partial_\nu g. \tag{2.15}$$

On the other hand, one can easily verify the invariance of the propagators (2.14) under gauge transformations of the same type. This implies invariance of $G_{\mu_1 \ldots \mu_k}$ under (2.15) (if the Proca propagator $\Delta_{F\mu\nu}$ in (2.13) is not changed). We have therefore shown that our new propagators (2.1) do not change both under transformations (1.3) and (2.15).

3. *Renormalizability of the theory*

Our further discussion of the compensating current dependent quantum electrodynamics will be based on the formula (2.13). One can see from it that the Feynman diagrams occurring in the perturbation series of the propagator $G_{\mu_1 \ldots \mu_k}$ contain three types of photon lines with different analytic expressions corresponding to them. Internal photon lines correspond to the propagator $\mathscr{D}_{\mu\nu}^F$ defined by (2.14b). External photon lines with one end free and one end attached to the rest of the diagram must be replaced by $D^{(a)}_{F\mu\nu}$, whereas external lines with two ends free correspond to the usual Proca propagator $\Delta_{F\mu\nu}$. We see therefore that the expressions corresponding to the photon lines with at least one end attached to the diagram are free of the singular $\mu^{-2}k_\mu k_\nu$ terms in momentum space. This implies that the asymptotic behaviour of the momentum space integrands in the perturbation expansion of $G_{\mu_1\ldots\mu_k}$ is no worse than that in Feynman-gauge quantum electrodynamics. This statement is true independently of the presence of the compensating current in $\mathscr{D}^F$ and $D^{(a)}_F$, since from (2.3) it follows, that in momentum space we have:

$$k^\lambda \tilde{a}_\lambda(k) = i. \tag{3.1}$$

This means that the asymptotic, large k behaviour cannot be made worse by the occurrence of the compensating current.

As a result of the introduction of the compensating current to the theory of massive photons we have obtained, apart from the gauge invariance of the propagators, a full renormalizability of that theory. Note that the vector meson field theory is not renormalizable in the usual formulation, *i. e.* finite field operators cannot be constructed as operator valued distributions acting in the state space with positive definite metric [10, 11, 12]. On the other hand, the gauge independent formulation, proposed in the present paper, results in fully renormalizable propagators in the usual Hilbert space with positive definite scalar product, at least for a certain class of compensating currents.

To make the role of the current a^λ in our theory more clear, let us consider the photon propagator $\mathscr{D}^F$ in greater detail. It follows from (2.14b) that it is equal to the photon propagator evaluated in a certain, particular gauge, which is in a sense defined by a^λ[4]. The analytic expressions corresponding to the remaining types of photon lines have the same form as that for the internal ones after performing the following gauge transformation:

$$A_\mu(z) \to B_\mu(z) = A_\mu(z) - \int d^4z' \frac{\partial}{\partial z'^\mu} a^\nu(z'-z)A_\nu(z'). \tag{3.2}$$

The free propagator of B_μ is equal to $\mathscr{D}^F_{\mu\nu}$. Thus, using instead of $G_{\mu_1\ldots\mu_k}$ the propagator $\mathscr{G}_{\mu_1\ldots\mu_k}$ defined as:

$$\begin{aligned}\mathscr{G}_{\mu_1\ldots\mu_k}[x_1, \ldots, x_n, y_n, \ldots, y_1, z_1, \ldots, z_k|\mathscr{A}a] &= \\ = i^n\langle 0; \mathrm{out}|T(\psi(x_1)\ldots\psi(x_n)\overline{\psi}(y_n)\ldots\overline{\psi}(y_1)B_{\mu_1}(z_1)\ldots B_{\mu_k}(z_k)\Phi[a])|0; \mathrm{in}\rangle\end{aligned} \tag{3.3}$$

we obtain a fully gauge-invariant object. However, it follows from (2.14a) that it is equal to the usual propagator in the theory without a compensating current, calculated in a certain gauge, namely the gauge defined by a^λ. We can therefore say that quantum electrodynamics with a compensating current, expressing explicitly charge conservation (gauge invariance), is equivalent to the theory formulated in a certain, particular gauge (*cf.* equivalence theorem in [4]).

A large class of compensating currents is given by the following formula [17]:

$$\tilde{a}^\lambda(k; \varrho) = i\frac{n^\lambda(nk) - \varrho k^\lambda}{(nk - i\varepsilon)^2 - \varrho k^2}, \qquad n^\mu n_\mu = 1, \tag{3.4}$$

where the parameter ϱ can change from $-\infty$ to $+\infty$. For $\varrho = 0$ (3.4) gives the Fourier transform of the current created by a point particle moving along a time-like straight line, whereas for $\varrho = 1$ we obtain the Coulomb-gauge current. This means that substituting $a(k;1)$ to the Fourier transform of $\mathscr{D}^F_{\mu\nu}$ we obtain the Coulomb-gauge photon propagator:

$$\mathscr{D}^F_{\mu\nu}(x) = \left[-g_{\mu\nu} + \frac{\partial_\mu\partial_\nu - n_\mu(n\partial)\partial_\nu - n_\nu(n\partial)\partial_\mu}{\Box - (n\partial)^2}\right] D_F(x). \tag{3.5}$$

For $\varrho \to \infty$ the formula (3.4) gives the Fourier transform of the complex current vector:

$$\tilde{a}^\lambda(k; \infty) = i\,\frac{k^\lambda}{k^2 + i\varepsilon}, \tag{3.6}$$

corresponding to the Landau-gauge. This is the only relativistic gauge which can be described by a compensating current (by a relativistic gauge we mean a gauge in which no direction n^μ in space-time is distinguished). However, since $a^\lambda(x; \infty)$ is not real, it has no clear physical interpretation as a macroscopic current connected with the process of creation and detection of a charged particle. On the other hand, it is well known that Landau-gauge formulation of quantum electrodynamics requires "ghost" states with negative norm squared, and therefore our earlier statement concerning renormalizability is not true for the case $\varrho \to \infty$ in (3.4). Thus, as far as the class (3.4) of compensating currents is concerned, quantum electrodynamics with a compensating current can be renormalized in good Hilbert space with positive definite metric for finite ϱ. This result is consistent with the well known fact that Coulomb-gauge quantum electrodynamics is renormalizable in the state space with positive definite scalar product.

4. *Final remarks*

We have shown that the compensating current dependent quantum electrodynamics, formulated in Section 2, is a fully gauge invariant and fully renormalizable theory. This means that all objects (even those, which in the usual formulation are gauge dependent) become now gauge invariant, and that the field operators can be constructed in the state space with positive definite norm.

The next problem to consider should be the problem of field equations or the equations fulfilled by the propagators. Their right-hand sides contain various kinds of current operators, such as the Dirac current or the electromagnetic current, given by the formal products of at least two field operators in the same space-time point. It is well known that such products can be defined properly with the use of a limiting procedure of the following type [1, 5, 9, 13]:

$$\lim_{\xi \to 0} \left[\overline{\psi}(x+\xi)\gamma^\mu e^{-ie\int_{x-\xi}^{x+\xi} d\eta^\lambda A_\lambda(\eta)}\,\psi(x-\xi)\right].$$

The exponential function has been inserted into this formula to ensure gauge invariance at all steps of calculations. In our formulation this exponent must be replaced by an expression involving the compensating current:

$$e^{-ie\int d^4z[a^\lambda(z-x-\xi)-a^\lambda(z-x+\xi)]A_\lambda(z)}.$$

Since the current operator, as an observable, is a gauge-invariant object, it should not depend on a^λ. Problems connected with a proper definition of the current and its dependence on a^λ will be studied in a further publication.

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