IMPACT PARAMETER ANALYSIS OF MULTIPLICITY DISTRIBUTION IN HIGH ENERGY pp COLLISIONS

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Dedicated to the memory of Professor Ziro Koba

Assuming the geometrical model of particle production, the average multiplicity of negative particles produced in high-energy proton-proton collisions at fixed impact parameter is determined from experimental multiplicity distributions and elastic scattering data. The effects of multiplicity fluctuations and two-component structure at fixed impact parameter are discussed. The results are compared with the predictions of some simple mechanisms of particle production.

1. *Introduction*

In this paper we discuss some properties of the multiplicity distribution in the impact-parameter representation. In particular we attempt a determination of the average multiplicity of negative particles produced in the proton-proton collisions at a given impact parameter.

The application of the impact-parameter representation to the description of the multiplicity distributions observed in particle production at high energies was discussed already by Heisenberg [1] and by Landau [2], in their models of multiparticle production. A rather complete general discussion of the problem was given by Michejda [3]. Recently, this idea was used in several papers [4–12] for explanation of the apparent scaling observed in the multiplicity distributions [13].

The multiplicity distribution is represented by

$$P(n) = \frac{1}{\sigma}\int d^2b\sigma(b)p(n, b), \tag{1}$$

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where $\sigma = \int d^2b\sigma(b)$ is the total inelastic cross-section and $\sigma(b)$ is the total inelastic cross--section at impact parameter b. $p(n, b)$ is the multiplicity distribution of particles produced in the collision at impact parameter b.

Equation (1) is quite general. To derive from it the properties of the distribution $p(n, b)$, further assumptions are needed. In this paper we follow the approach used in the so-called geometrical models of particle production [4–7], *i.e.* we assume that, at high energies, the distribution $p(n, b)$ is very narrow

$$d(b)/\bar{n}(b) \simeq 0. \tag{2}$$

Here $d(b)$ and $\bar{n}(b)$ are the dispersion and the average multiplicity respectively, at impact parameter b:

$$\bar{n}(b) = \sum_n np(n, b), \tag{3}$$

$$d^2(b) = \sum_n [n-\bar{n}(b)]^2 p(n, b). \tag{4}$$

In the high-energy limit, the condition (2) follows from the short-range correlation hypothesis applied to the collision at fixed impact parameter, and provides also a simple justification of the Koba-Nielsen-Olesen [13] scaling. Thus it is likely to be satisfied at least for non-diffractive collisions which dominate the inelastic cross-section at energies available at present.

In this paper, using the condition (2), we determine the average multiplicity of negative particles $\bar{n}(b)$ from the existing data on multiplicity distributions and on elastic scattering in proton-proton collisions. The properties of the function $\bar{n}(b)$ obtained in this way are discussed and compared to the expectations of some simple models of particle production.

Since the condition (2) is vital for the method of determination of $\bar{n}(b)$ used in this paper, it is necessary to investigate the possible corrections which arise if a non-vanishing width is given to the distribution $p(n, b)$. We considered two possible origins of this effect.

i) At finite energies we do expect the distribution $p(n, b)$ to have a non-vanishing width even for non-diffractive production. We showed however that, provided this width does not exceed significantly the one expected from weakly correlated models, it does not influence the determination of $\bar{n}(b)$.

ii) If the two-component structure of the collisions[1] is accepted at each impact parameter, the distribution $p(n, b)$ has a non-vanishing width even in the high-energy limit. However, we checked that under reasonable conditions of diffractive cross-section and multiplicity distribution, the general features of $\bar{n}(b)$ are not changed by the presence of the diffractive component.

Perhaps the most important conclusion from our analysis is that the function $\bar{n}(b)$ depends rather strongly on b. This fast variation of $\bar{n}(b)$ is at the origin of the strong positive correlations between produced particles. Since this feature does not seem to be significantly influenced by the presence of the diffractive component of the interactions, we expect strong, long-range correlations between particles produced in non-

[1] For the complete list of references on two-component models see, for example, Ref. [14].

-diffractive interactions. This is to be contrasted with predictions of the two-component models of particle production [14].

In Section 2 we describe the determination of $\bar{n}(b)$. The corrections implied by the non-vanishing width of the distribution $p(n, b)$ are discussed in Section 3. In Section 4 we discuss the properties of $\bar{n}(b)$ and compare them with the predictions of several models. Our conclusions are summarized in the last section.

2. *Determination of the average multiplicity*

Let us now describe how it is possible to determine $\bar{n}(b)$ from the data, using the condition (2). It was shown by Buras and Koba [4] and by Moreno [6] that Eqs. (1) and (2) imply

$$\bar{N}P(n) = \frac{2\pi b\sigma(b)}{\sigma} \frac{\bar{N}}{\left|\frac{d\bar{n}(b)}{db}\right|_{b=b_n}}, \tag{5}$$

where b_n is the solution of the equation $n(b_n) = n$, and $\bar{N}$ is the average multiplicity of the collision

$$\bar{N} = \sum_n nP(n) = \frac{1}{\sigma}\int d^2b\sigma(b)\bar{n}(b). \tag{6}$$

We observe that the only unknown quantity in Eq. (5) is $|d\bar{n}(b)/db|_{b=b_n}$. Indeed,

i) the function $\bar{N}P(n) = \psi(z, \bar{N})$, where $z = n/\bar{N}$, can be determined from the experimental multiplicity distribution at each energy;

ii) the partial inelastic cross-section $\sigma(b)$ is simply the impact parameter representation of the Van Hove overlap function [15]

$$F(\Delta) = \int d^2be^{i\vec{\Delta}\cdot\vec{b}}\sigma(b) \tag{7}$$

and thus can be determined from the measured elastic scattering.

Consequently, Eq. (5) can be considered as a differential equation for $\bar{n}(b)$:

$$\frac{dw}{db} = \pm \frac{2\pi b\sigma(b)}{\sigma} \frac{1}{\psi(w)} \tag{8}$$

where, for convenience, we denoted $w(b) = \bar{n}(b)/\bar{N}$.

The sign ambiguity in Eq. (8) implies that there are two solutions for $\bar{n}(b)$. The negative sign on the right-hand side of Eq. (8) gives $\bar{n}(b)$ decreasing with increasing b. This is the so-called "intuitive" solution. It corresponds to the geometrical picture of the collision in which "central" collisions at small impact parameters lead to the production of many particles, whereas in "peripheral" collisions at large impact parameters only a few particles are produced.

The positive sign on the right-hand side of Eq. (8) gives $\bar{n}(b)$, which increases with increasing b. Such a situation arises in simple field-theory models, for example in a multiperipheral model.

The solution $w = w(b)$ of Eq. (8) is obtained by integration:

$$\int_0^{w(b)} \psi(w)dw = \frac{1}{\sigma}\int_b^{\infty} d^2b\sigma(b), \tag{9a}$$

$$\int_0^{w(b)} \psi(w)dw = \frac{1}{\sigma}\int_0^{b} d^2b\sigma(b), \tag{9b}$$

where Eq. (9a) provides the decreasing solution and Eq. (9b) the increasing one.

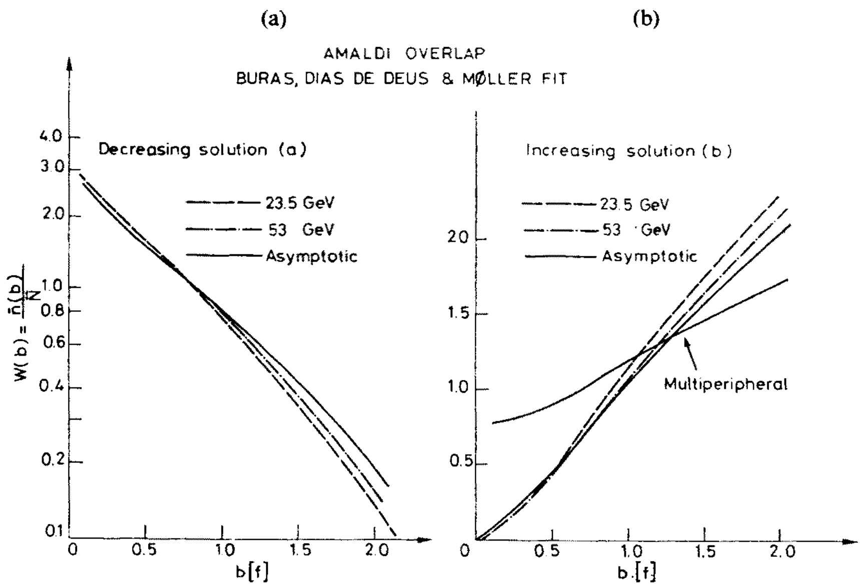

Fig. 1. Average multiplicity $\bar{n}(b)$ of negative particles produced in high-energy proton-proton collisions as a function of the impact parameter: (a) decreasing solution, (b) increasing solution. The results were obtained using the overlap function calculated from data shown by Amaldi [17] and the multiplicity distribution according to fit done by Buras, Dias de Deus and Møller [16]

We have performed numerical integration of Eqs. (9a) and (9b) in the region $0 \lesssim b \lesssim 2f$ and thus obtained explicitly the two solutions for $\bar{n}(b)$. They are plotted in Figs 1a and 1b. The function $\psi(z, \bar{N})$ was taken from the fit to multiplicity distributions presented recently by Buras, Dias de Deus and Møller [16]. The partial cross-sections were taken from the analysis of Amaldi [17].

In the region considered the results are not sensitive to small changes in input data. To check this, we calculated $\bar{n}(b)$ taking as input the Czyżewski-Rybicki [18] and Wróblewski [19] fits to multiplicity distributions. This does not change the results significantly. Also, replacing the Amaldi fit to $\sigma(b)$ by that of Henyey *et al.* [20] does not affect the curves shown in Fig. 1.[2]

3. *Corrections for non-vanishing width of $p(n, b)$*

The next important problem is to estimate the possible influence of the non-vanishing width of the distribution $p(n, b)$. To illustrate the kind of effects which may be expected, let us write the formula for the dispersion of the multiplicity distribution $P(n)$

$$D^2 \equiv \sum_n (n-\bar{N})^2 P(n) = \frac{1}{\sigma}\int d^2b\sigma(b)d^2(b) + \frac{1}{\sigma}\int d^2b\sigma(b)\,(n(b)-\bar{N})^2. \tag{10}$$

We see that the width of the multiplicity distribution is a sum of two positive terms. The first of these terms is neglected in the zero-width approximation. Consequently, if a non--vanishing width is given to the distribution $p(n, b)$, its effects should be compensated by a decrease of the second term in Eq. (10). This would lead to a less steep dependence of $\bar{n}(b)$ on b, at least in the region where $\sigma(b)$ is large.

The accuracy of the zero-width approximation (2) depends on the ratio of the contributions from the two terms in Eq. (10). Assuming a Poisson-like relation $d^2(b) \simeq \simeq \alpha\bar{n}(b)+\beta$, we obtain

$$D^2 = \alpha\bar{N}+\beta+\bar{N}^2\frac{1}{\sigma}\int d^2b\sigma(b)\,(w(b)-1)^2. \tag{11}$$

From the Wróblewski fit to the dispersion [21] we have $D = 0.585\ (\bar{N}+0.5)$ and thus conclude that

$$\alpha = \frac{1}{\sigma}\int d^2b\sigma(b)\,[w(b)-1]^2 \simeq 0.335 \quad \text{and} \quad \beta \simeq 0.085.$$

The error in the determination of $1/\sigma \int d^2b\sigma(b)\ (w-1)^2$ is thus $(\alpha\bar{N}+\beta)/\bar{N}^2$, which is below 10% at ISR energies, and should decrease with increasing incident energy.

The argument given above indicates also that it should be possible to describe all observed energy variation of the multiplicity distribution by the "geometrical" Ansatz $\bar{n}(b, s) = \bar{N}(s)\bar{w}(b)$ and Poisson-like distribution at given b.

To obtain a rough idea of how this affects the determination of $w(b)$ we have calculated $w(b)$ from fits to multiplicity distributions extrapolated to infinite energy, where the

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[2] The decreasing solution at large b is sensitive to the behaviour of the Koba–Nielsen–Olesen function $\psi(z)$ at small z. In this region data are not good enough to determine $\psi(z)$ with good accuracy and thus the same is true for $\bar{n}(b)$. At extremely small impact parameters the decreasing solution is sensitive to the behaviour of $\psi(z)$ at large z, $z \gtrsim 3$, where the statistics reached in present experiments are small and, consequently, the errors are large.

The increasing solution is much better determined. At large b the accuracy is limited mostly by inaccuracies in determination of $\sigma(b)$.

assumption (2) is expected to be exact. The corresponding curves are shown in Fig. 1. It is seen that they are indeed a little flatter than those obtained for fits at finite energies. The difference does not exceed a few per cent however. Thus we conclude that the corrections to width are not likely to change our results significantly.

We have also directly fitted the distribution (1) to the experimental data using $p(n, b)$ in the form of the truncated Gaussian

$$p(n, b) = Z(b) \exp\left\{-\frac{[n-\gamma(b)]^2}{2\delta\gamma(b)}\right\} \quad \text{for} \quad n \geqslant 0,$$
$$= 0 \quad \text{for} \quad n < 0, \tag{12}$$

where $Z(b)$ is determined by the normalization condition and δ is the parameter responsible for the width of the distribution (12). For $\delta = 0$ we recover the condition (2). For $\delta = 1$, $p(n, b)$ is close to the Poisson distribution. The function $\gamma(b)$ was fitted to reproduce the data with δ being kept fixed. The results of the fitting for the case of the decreasing solution are shown in Fig. 2, where the function $w(b) = \bar{n}(b)/\bar{N}$ is plotted for different values of δ and $\bar{N} = 4.3$. It is seen that for $\delta \lesssim 0.5$ the results do not differ considerably from the ones obtained with condition (2). We feel that $\delta \simeq 0.5$ is perhaps a reasonable guess if one assumes that, for given b, there are no important dynamical correlations between pions [22]. Consequently we believe that our results are rather insensitive to the details of the distribution $p(n, b)$.

It should be emphasized, however, that it is certainly possible to construct models in which the behaviour of $w(b)$ is rather different from the one shown in Figs 1 and 2, and which can still describe well the experimental data[3]. Our estimate of error was based on the assumption that the departures from the Koba-Nielsen-Olesen scaling law, observed at low energies, are mostly caused by a non-vanishing width of the distribution $p(n, b)$. This seems a natural explanation in geometrical models[4]; however other effects are also possible.

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[3] A simple example is to take $p(n, b) \equiv P(n)$.

[4] It is interesting to note that the generalized Koba–Nielsen–Olesen scaling law advocated recently by Buras, Dias de Deus and Møller [15] and by Wróblewski [19]

$$\langle (n-\bar{N})^k \rangle = C_k(\bar{N}+\lambda)^k$$

is satisfied exactly if the moments of the multiplicity distribution $p(n, b)$ fulfil the following conditions

$$\bar{n}(b) = w(b)\bar{N},$$
$$d^2(b) = w_2(b)\bar{N}+\lambda^2[w(b)-1]^2,$$
$$\mu_k(b) \equiv \sum [n-\bar{n}(b)]^k p(n, b) = \lambda^k[w(b)-1]^k.$$

Here $w(b)$ is given by Eq. (8) and

$$w_2(b) = \frac{2\lambda}{\psi[w(b)]} \frac{1}{\sigma} \int_0^b d^2b\sigma(b)[w(b)-1].$$

If these conditions are satisfied, all corrections to the Koba–Nielsen–Olesen scaling are indeed provided by non-vanishing of the central moments of the distribution $p(n, b)$.

A particularly important effect may arise if the multiplicity distribution has the two--component structure at each impact parameter, since in this case the condition (2) is violated even in the high-energy limit. To estimate possible corrections, some information on diffractive cross-section and multiplicity distribution of diffractively produced particles is necessary. We assumed that the partial diffractive cross-section is strongly peripheral and concentrated at impact parameters around 1, as suggested by the analysis of Sakai and White [23] and by Henyey [24]. The diffractive multiplicity distribution was taken from

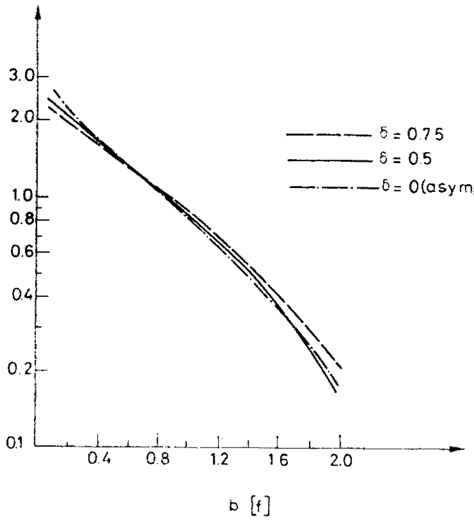

Fig. 2. $\bar{n}(b)$ in the model with non-vanishing width of the multiplicity distribution at fixed impact parameter

experiment at 303 GeV/c [25]. With these assumptions, and using the condition (2) for a non-diffractive component, it was possible to determine the average multiplicity $\bar{n}_{\mathrm{ND}}(b)$ of negative particles produced in non-diffractive collisions[5]. In Fig. 3 $\bar{n}_{\mathrm{ND}}(b)$ is compared with the decreasing solution at $\sqrt{s} = 23.5$ GeV obtained in Section 2. We also constructed the average multiplicity for all collisions, assuming that $\bar{n}_{\mathrm{diffractive}}(b)$ does not depend on b. This last assumption is justified because the diffractive component gives a significant contribution only in a rather narrow region around 1 [23, 24]. The $\bar{n}(b)$ determined in this way is also plotted in Fig. 3.

It is seen from Fig. 3 that the presence of the diffractive component does not influence $\bar{n}(b)$ for b below 1f. For larger impact parameters, the curve flattens significantly but the general picture remains unchanged. The details of the curves shown in Fig. 3 do not

[5] The method was the same as described in Section 2. The non-diffractive partial cross-section was calculated as the difference between the Amaldi fit to the overlap function and the Sakai-White diffractive partial cross-section. Also a non-diffractive multiplicity distribution was obtained by subtracting the experimental diffractive multiplicity distribution at 303 GeV (smoothed out, to avoid discontinuities) from the fit of Ref.]16].

have much significance, because the assumptions on the diffractive component were rather crude. However, it seems safe to conclude that the presence of the peripheral diffractive component does not change drastically the impact parameter dependence of the average multiplicity in the high-energy collisions.

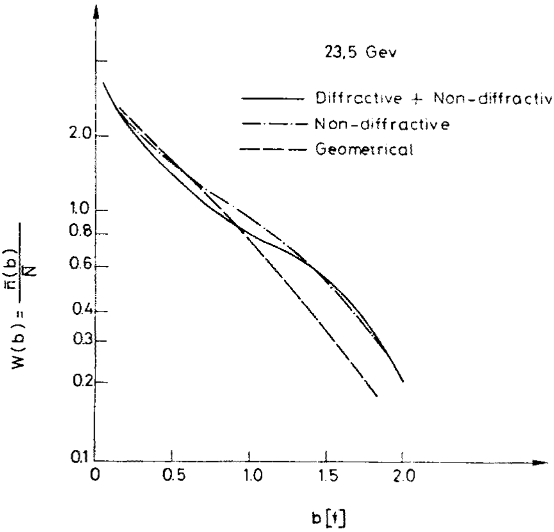

Fig. 3. $\bar{n}(b)$ in two-component model

4. *Discussion*

Let us now discuss the behaviour of $w(b)$ as presented in Fig. 1. First, we notice that there is very little energy dependence. As pointed out above, this energy dependence can be entirely explained by the corrections due to the non-vanishing width of $p(n, b)$. Thus we conclude that within the accuracy of our analysis the data are compatible with energy--independent $w(b)$ given by the "asymptotic" curves in Fig. 1.[6]

Several models make statements about the behaviour of the function $w(b)$. The geometrical models choose the decreasing solution. In these models, $w(b)$ is usually related to the "opaqueness" or eikonal $\Omega(b)$ defined as

$$2\sigma(b) = 1 - e^{-2\Omega(b)}. \tag{13}$$

To see the relation between $w(b)$ and $\Omega(b)$ we plotted in Fig. 4 w *versus* Ω. The first observation is that the "optical" relation [9–11] $\bar{n}(b) = \lambda\Omega(b)$ is not satisfied. Instead, the dependence of w on Ω is characterized by a steep rise at small Ω followed by a linear increase over a rather large interval. At very large Ω (*i. e.* small b) there are deviations from this linear law.

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[6] The observed energy dependence of $\sigma(b)$ can be approximately taken into account by assuming that $\sigma(b, s) = \sigma[b/R(s)]$[26]. All our conclusions remain valid after the substitution $b \to b/R(s)$.

The Buras and Koba relation [4] $w(b) \sim \sqrt{\Omega(b)}$ is also plotted in Fig. 3. It is in good agreement with our curve up to $\Omega \simeq 1$. A discrepancy is seen, however, at large Ω.

Rechenberg and Robertson [12] have noted that the Heisenberg model [1] predicts, in general, $w(b) \sim [\Omega(b)]^{\alpha}$, where $0 < \alpha < 1$ is a measure of the strength of the interaction. The Buras and Koba relation discussed above is a special case of this formula. Thus we conclude that the Heisenberg model describes reasonably the data, provided the impact paiameter is not too small. This was noted already by Rechenberg and Robertson. We feel that the possible rescattering effects may be at the origin of the discrepancy.

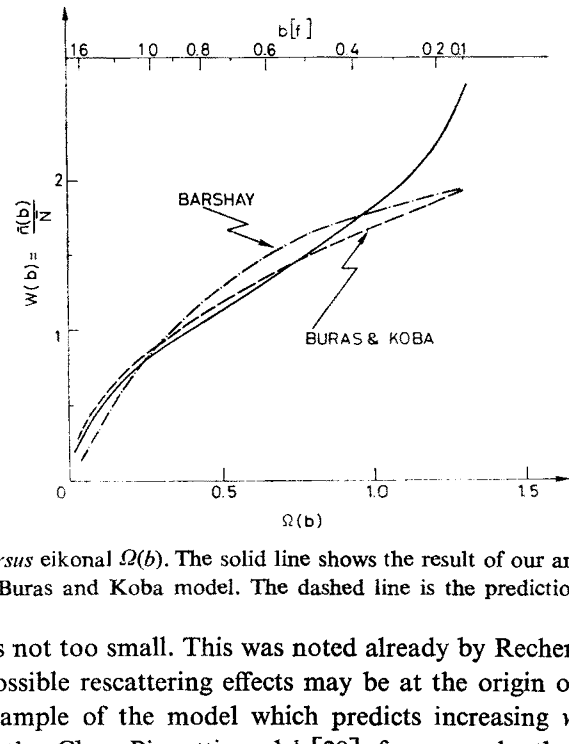

Fig. 4. Plot of $w(b)$ *versus* eikonal $\Omega(b)$. The solid line shows the result of our analysis. The dotted line is the prediction of the Buras and Koba model. The dashed line is the prediction of the Barshay model

The simple example of the model which predicts increasing $w(b)$ is the multiperipheral model[7]. In the Chew-Pignotti model [28], for example, the multiplicity distribution at fixed b is given by [9]

$$\sigma(b)p(n, b) = \frac{\overline{N}^n}{n!}\,\frac{1}{na}\,e^{-b^2/na}. \tag{14}$$

In Fig. 1b the function $w(b)$ following from Eq. (14) is shown for $\overline{N} = 3$ and $a = 0.25$.[8] It is seen that $w(b)$ from the Chew-Pignotti model is much flatter than the one obtained in our analysis. This is so because (*i*) the distribution $p(n\ b)$ following from Eq. (14) is rather broad and (*ii*) the Chew-Pignotti model predicts too narrow a multiplicity distribution (Poisson distribution), which does not fit correctly the high-energy data.

Finally, we note that the linear parametrization of the incieasing solution, proposed by Moreno [6], reproduces appioximately our results.

[7] Sakai discussed recently a large class of multiperipheral models which provide increasing $w(b)$ [27].
[8] For this value of a, $\sigma(b)$ following from Eq. (14) reproduces approximately the experimental data.

5. *Conclusions*

Our conclusions can be summarized as follows:

i) In a geometrical model of high-energy collisions, it is possible to determine the average multiplicity $\bar{n}(b)$ of particles produced at fixed impact parameter, using the experimental multiplicity distributions and elastic scattering data.

ii) There are two possible solutions for $\bar{n}(b)$: the "multiperipheral" one with $\bar{n}(b)$ increasing with increasing b and the "intuitive" one with $\bar{n}(b)$ decreasing with increasing b.

iii) The existing data allow the determination of the average multiplicity of the negative particles produced in proton-proton collisions at fixed impact parameter. The errors do not exceed 10% in the region of b below $2f$.

iv) The corrections for non-vanishing width of the multiplicity distribution $p(n, b)$ at fixed impact parameter do not influence significantly the determination of $\bar{n}(b)$, provided $p(n, b)$ is narrower than the Poisson distribution.

v) $\bar{n}(b)$ is not proportional to the eikonal $\Omega(b)$.

vi) The Buras-Koba and Heisenberg models describe correctly the behaviour of $\bar{n}(b)$ for $b \gtrsim 0.4$ f. At smaller impact parameters the average multiplicity is higher than expected from these models.

vii) The data are compatible with an energy-independent ratio $w(b) = \bar{n}(b)/\bar{N}$. All energy dependence of the multiplicity distribution can be explained by an energy-dependent (increasing) $\bar{N}$ and the corresponding decrease of the ratio $d(b)/\bar{n}(b)$ with increasing energy.

viii) The presence of a peripheral diffractive component, does not change the general behaviour of $\bar{n}(b)$. Thus our analysis suggests the existence of strong long-range correlations[9] between particles produced in non-diffractive interactions.

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[9] Here we use the term "long-range correlations" to describe the situation in which a correlation parameter f_2 is a quadratic function of $\bar{N}$ rather than a linear one, which would be characteristic for "short-range correlations". Since we analyse only multiplicity distributions we, of course, cannot make any statements about correlation length in rapidity.

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