SCALING OF MULTIPLICITY DISTRIBUTION IN HADRON COLLISIONS AND DIFFRACTIVE-EXCITATION LIKE MODELS

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Multiplicity distribution of secondary particles in inelastic hadron collision at high energy is studied in the semiclassical impact parameter representation. The scaling function is shown to consist of two factors: one geometrical and the other dynamical. We propose a specific choice of these factors, which describe satisfactorily the elastic scattering, the ratio of elastic to total cross-section and the simple scaling behaviour of multiplicity distribution in p-p collisions. Two versions of diffractive-excitation like models (global and local excitation) are presented as interpretation of our choice of dynamical factor.

1. *Introduction*

1a. Scaling hypothesis of multiplicity distributions

The scaling hypothesis of multiplicity distributions for hadron-hadron collisions [1] states that

$$\langle n\rangle \frac{\sigma_n}{\sigma_{\text{inel}}} \equiv \Psi(z, s) \xrightarrow[s\to\infty]{} \Psi(z), \tag{1}$$

where

$$z = \frac{n}{\langle n\rangle} \tag{2}$$

is a scaling variable and $\langle n\rangle$ is the average multiplicity which is a function of s, the c.m.s. energy squared. As usual σ_n and σ_{inel} denote the cross-section for producing n particles and total inelastic cross-section, respectively. The function $\Psi(z, s)$ must satisfy two normalization conditions

$$\int\limits_0^\infty dz\Psi(z, s) = \int\limits_0^\infty dzz\Psi(z, s) = 1. \tag{3}$$

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In writing (3) we treat n and z as continuous variables and replace the summation over n by a corresponding integration over z. In doing this we commit in general a relative error of the order $\sim 1/\langle n\rangle$. In the special case, however, where $\Psi(z, s)$ vanishes for $z = 0$, it is reduced to $1/\langle n\rangle^2$. The same is valid for the scaling function discussed below [2].

In describing the empirical data it is convenient [3] (see below) to use, instead of (2), a new scaling variable

$$w = \frac{\pi}{4} z^2 = \frac{\pi}{4}\left(\frac{n}{\langle n\rangle}\right)^2. \tag{4}$$

Then the scaling function is given by

$$\Phi(w; s) = \frac{2}{\pi z}\,\Psi(z, s) = \frac{2}{\pi}\,\frac{\langle n\rangle^2}{n}\,\frac{1}{\sigma_{\text{inel}}}\,\frac{d\sigma_{\text{inel}}}{dn} = \frac{1}{\sigma_{\text{inel}}}\,\frac{d\sigma_{\text{inel}}}{dw} \tag{5}$$

with normalization constraints corresponding to (3)

$$\int_0^\infty dw\Phi(w; s) = \frac{2}{\sqrt{\pi}}\int_0^\infty dww^{1/2}\Phi(w, s). \tag{6}$$

The last two expressions in (5) follow from our treating of n and w as continuous variables.

The original derivation of (1) concerns the asymptotic region ($\langle n\rangle \gg 1$), but it has been shown [4] that (1) is already in a good agreement with recent data of p–p collisions at 50–303 GeV/c ($5.3 \leqslant \langle n\rangle \leqslant 8.9$). On the basis of the above data we have emphasized [3] that a simple expression

$$\Phi(w) = \exp(-w), \tag{7}$$

which combines scaling and the empirical formula obtained by Bozoki *et al.* [5] gives a good approximation at least in the energy region considered. A similar formula (in terms of z) based also on Ref. [5] has been proposed by H. Weisberg [6].

This early "scaling" of multiplicity distributions can be interpreted in various ways. One cannot exclude even the picture of purely short range correlations [7], as has been carefully remarked in Ref. [4]. Many authors introduce, however, long range correlations in one way or another, in order to understand the situation. Some of them [8, 9] regard this "scaling" behaviour only apparent, while others [10–16] take it more seriously and try to find a clue for the production mechanism, even though most of them admit possible change of the scaling function at still higher energies. We feel at present that the latter point of view is more attractive, although we realize that the former may well turn out to be the reality. (Recent papers by A. Wróblewski (Warsaw preprint, June 1973) and A. J. Buras, J. Dias de Deus and R. Møller (Niels Bohr Institute preprint NBI-HE-73-14) indicate a very slow change with energy of the scaling function between 10 GeV/c and 300 GeV/c. Nevertheless, we could regard (7) as a good approximation to the available data on multiplicity distributions.)

In Ref. [3] we gave a sketch of a naive model which led to the formula (7). In this paper a substantially generalized version of our arguments will be presented, including an alternative interpretation.

1b. Outline of the model

Our picture consists of two parts. This is reflected in the general structure of the scaling function (see (22) below) which is a product of two factors, one geometrical and the other dynamical.

i) Geometrical part

We regard the elastic scattering of hadrons at very high energy as pure shadow scattering and describe it in the impact parameter representation [17]. Then a real non-

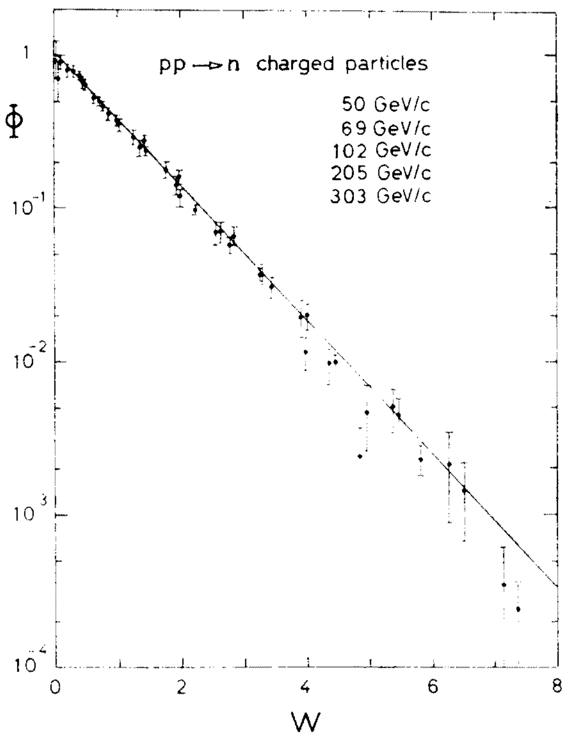

Fig. 1. Plot of Φ as given by (5) *vs* w for the reaction pp → n charged particles (50, 69, 102, 205 and 303 GeV/c). The straight line represents $\Phi(w) = \exp(-w)$. This figure is taken from Ref. [3]

-negative function $\varrho(b^2, s)$, which is the imaginary phase of the reduced S-matrix elements, determines the elastic differential cross-section $d\sigma_{el}/dt$ [18]. Through the local unitarity relation (valid at each impact parameter b), it also determines σ_{el}/σ_{tot}. The first factor in (22) is fixed by the function $\varrho(b^2, s)$, too.

Following Chou and Yang [19] one can imagine that hadrons are extended absorptive objects and interpret $\varrho(b^2, s)$ as overlap of the colliding hadrons when they go through each other. Because of this geometrical picture, we call the first factor geometrical.

ii) Dynamical part

In order to derive multiplicity distribution, one obviously needs further specifications of production mechanism, corresponding to the second factor in (22), which in the present picture may be represented by a relation between $\varrho(b^2, s)$ and the number distribution of particles produced in a collision at a given b.

In this paper we replace, for simplicity, this number distribution by its average value $n(b^2, s)$ (no fluctuation approximation) and then we find that an ansatz of the form $n \sim \varrho^{1/2}$ is required for getting agreement with empirical data.

We shall present a kind of diffractive-excitation model which leads to this form. This can, in turn, be naturally understood when we regard the hadron as composed of a number of small constituents and describe a hadron-hadron collision of high energy as an accumulation of "elementary collisions" between the constituents of two hadrons. More detailed specification of properties of such "elementary collisions" and ways of decay of the excited hadrons will be discussed in Sect. 5.

2. *General formulation*

2a. Basic formulae of optical approximation

We shall work in the impact parameter representation [17–19] and assume that the unitarity relation is valid locally for each value of the impact parameter b. (This representation has already been used for discussion of multiplicity scaling and elastic scattering by Nielsen and Olesen [10] and by S. Barshay [11]. See also H. Moreno, Phys. Rev. **D8**, 268 (1973) and M. Le Bellac *et al.*, Nucl. Phys. **B62**, 350 (1973), which appeared at the same time as the preprint version of the present paper (Niels Bohr Institute preprint NBI-HE-73-6).) Then we can define $\sigma_{\text{el}}(b^2, s)$ and $\sigma_{\text{inel}}(b^2, s)$ as the part of elastic and inelastic cross-section, respectively, which are due to the collision at impact parameter larger than b, and write

$$\sigma_{\text{el}}(b^2, s) = \pi \int_{b^2}^{\infty} db'^2 |1 - S(b'^2, s)|^2, \tag{8}$$

$$\sigma_{\text{inel}} = \pi \int_{b^2}^{\infty} db'^2 \{1 - |S(b'^2, s)|^2\}, \tag{9}$$

or equivalently

$$-\frac{d\sigma_{\text{el}}}{db^2} = \pi |1 - S(b^2, s)|^2, \tag{10}$$

$$-\frac{d\sigma_{\text{inel}}}{db^2} = \pi \{1 - |S(b^2, s)|^2\}. \tag{11}$$

Here $S(b^2, s)$ is the transmission coefficient at impact parameter b, or the reduced S-matrix element of elastic scattering. We put it

$$S(b^2, s) = \exp(-\varrho(b^2, s)), \tag{12}$$

and assume ϱ to be real, non-negative

$$0 \leqslant \varrho < \infty \tag{13}$$

i.e. we regard that the elastic scattering is completely due to shadow of inelastic process. In a semi-classical picture ϱ stands for the absorption of the incident wave and in Chou--Yang's picture of extended hadrons [19], it is regarded as overlap of incident hadrons during the collision. We assume that $\varrho(b^2, s)$ is a monotonically decreasing function of b^2,

$$\frac{d\varrho}{db^2} \leqslant 0 \quad (s \text{ fixed}). \tag{14}$$

(This assumption may break down in the vicinity of $b^2 = 0$, where the whole scheme of this formalism becomes dubious.)

2b. Multiplicity distributions

We introduce a probability distribution function for producing n particles by a collision at impact parameter b and c.m.s. energy $\sqrt{s}$, which is normalized as

$$\int_0^\infty dn\bar{Q}(n; b; s) = 1. \tag{15}$$

Then we get

$$\frac{1}{\sigma_{\text{inel}}}\frac{d\sigma_{\text{inel}}}{dn} = \frac{1}{\sigma_{\text{inel}}}\int_0^\infty db^2 \left|\frac{d\sigma_{\text{inel}}}{db^2}\right| \bar{Q}(n; b; s). \tag{16}$$

This can be expressed, changing the variables n and b^2 to w and ϱ, as

$$\Phi(w; s) = \frac{1}{\sigma_{\text{inel}}}\frac{d\sigma_{\text{inel}}}{dw} = \frac{1}{\sigma_{\text{inel}}}\int_0^\infty d\varrho \frac{d\sigma_{\text{inel}}}{d\varrho} Q(w; \varrho; s), \tag{17}$$

where the new probability distribution $Q(w; \varrho; s)$ is obtained by

$$Q(w; \varrho; s) = \frac{\langle n\rangle}{\sqrt{\pi w}} \bar{Q}(2\langle n\rangle \sqrt{w/\pi}; b^2(\varrho); s) \tag{18}$$

with normalization

$$\int_0^\infty dwQ(w; \varrho; s) = 1. \tag{19}$$

2c. No local scaling in general

A sufficient condition for multiplicity scaling (1) is obviously to put

$$Q(w; \varrho; s) = Q(w; \varrho). \tag{20}$$

This amounts to assuming that the multiplicity scales for each value of b, as has been the case in the naive model of Ref. [3]. In this paper we are going to present an ansatz which in general violates the local multiplicity scaling (20) and still yields the multiplicity scaling (1).

2d. No fluctuation approximation

For simplicity we shall neglect fluctuations of multiplicity distributions for a given value of b (*i.e.* for a given value of ϱ) around its mean value. Namely, we put

$$Q(w;\varrho;s) = \delta(w-f(\varrho;s)), \tag{21}$$

f being a monotonically increasing function of ϱ. (Our naive picture is that, in a central collision with b small and ϱ large, a larger number of particles is produced, whereas in a peripheral collision (b large and ϱ small) a smaller number emerges. This is in contrast to the usual impact parameter picture of the multiperipheral model.)

2e. The basic formula

From (17) and (21) we obtain

$$\Phi(w;s) = \left[\frac{1}{\sigma_{\text{inel}}}\frac{d\sigma_{\text{inel}}}{d\varrho}\frac{d\varrho}{dw}\right]_{\varrho=f^{-1}(w,s)},$$
$$w = f(\varrho;s). \tag{22}$$

It is seen from this expression that the distribution function $\Phi(w;s)$ consists of two factors. One of them, $1/\sigma_{\text{inel}}\, d\sigma_{\text{inel}}/d\varrho$, is determined by specifying the geometrical distribution of the transmission coefficient or opaqueness, or by the matter distribution inside the hadron; this we shall call geometrical factor. The other, $d\varrho/dw$, which is determined by the production mechanism (21), will be called dynamical factor.

It turns out to be more convenient to specify the geometrical factor in terms of a function $\xi(b^2;s)$ which is defined by

$$1-\exp(-2\varrho(b^2;s)) = \exp(-\xi(b^2;s)). \tag{23}$$

After simple calculation we obtain

$$\Phi(w;s) = \left[\frac{2\pi}{\sigma_{\text{inel}}}\left[\frac{d\xi}{db^2}\right]^{-1}\exp(-2\varrho)\frac{d\varrho}{dw}\right]_{\varrho=f^{-1}(w,s)}. \tag{24}$$

3. *Geometrical and dynamical ansatz leading to multiplicity scaling*

We propose to make the following ansatz for the geometrical factor

$$\frac{d\xi(b^2;s)}{db^2} = B(s)\exp(\lambda(s)\varrho(b^2;s)) \tag{25}$$

which determines, together with (23), the function ϱ or ξ. Here $B(s)$ and $\lambda(s)$ are functions of s only. (The simple case of Ref. [5] corresponds to $\lambda(s) = 0$.)

Since (25) gives, taking into account (11), (12) and (23),

$$\sigma_{\text{inel}} = \pi\int_0^\infty d\xi\,\frac{db^2}{d\xi}\exp(-\xi) = \frac{\pi}{B(s)}\,\frac{2}{\lambda(s)+2}, \tag{26}$$

we get from (24) and (25)

$$\Phi(w; s) = (\lambda(s)+2)\exp(-(\lambda(s)+2)\varrho(w; s))\,\frac{d\varrho(w; s)}{dw}. \tag{27}$$

It is now evident that a dynamical ansatz

$$\varrho = \frac{w}{\lambda(s)+2} \tag{28}$$

yields the empirically suggested form of the scaling function

$$\Phi(w; s) = \Phi(w) = \exp(-w).$$

Notice that both the geometrical and the dynamical ansatz, (25) and (28), depend explicitly on the energy, unless λ and B are constant. Thus the local scaling of multiplicity assumed in Ref. [3] does not hold in general. Nevertheless, energy-dependent factors are cancelled in the distribution function so that the latter scales. In the non-asymptotic energy region, it is rather likely that the slope of the elastic scattering and the ratio σ_{el}/σ_{tot} change slowly with energy. Thus a compensation mechanism, such as illustrated here, may give an explanation of early scaling of multiplicity.

4. *Geometrical factor and elastic scattering data*

The ansatz (25) together with (10), (11), (12), (23) and (26) leads to the following expressions for σ_{el}/σ_{tot} and $A(s, 0)$, the slope of $d\sigma_{el}/d|t|$ at $t = 0$

$$\frac{\sigma_{el}}{\sigma_{tot}} = 1 - \frac{1}{2T(\lambda)\,(\lambda+2)}, \tag{29}$$

$$\frac{A(s, 0)}{\sigma_{tot}} = \frac{1}{4\pi T^2(\lambda)} \sum_{\nu=0}^{\infty} \frac{T(2\lambda+4+2\nu)}{2\nu+2+\lambda}, \tag{30}$$

where we introduced

$$T(\lambda) \equiv \sum_{\nu=0}^{\infty} \frac{(-1)^\nu}{\lambda+2+\nu}. \tag{31}$$

The values of σ_{el}/σ_{tot} for various λ are given in Table I. σ_{el}/σ_{tot} and $A(s, 0)/\sigma_{tot}$ are monotonously decreasing and increasing functions of λ, respectively.

In order to establish the energy dependence in our model we have to estimate empirically the dependence of λ on s. To this end we first assume σ_{tot} to be constant (this is a good approximation for say, 50 GeV/$c \leqslant p_{lab} \leqslant$ 1000 GeV/c for pp scattering) and take $\sigma_{tot} = 39$ mb, the value measured at 303 GeV/c [21]. (Being interested only in a rough estimation of the dependence of λ on s we neglect the recently observed slow rise of σ_{tot} with energy

TABLE I

The values of $\frac{\sigma_{el}}{\sigma_{tot}}$ and $\frac{A(s, 0)}{\sigma_{tot}}$ for various λ

λ	$\frac{\sigma_{el}}{\sigma_{tot}}$	$\frac{A(s,0)}{\sigma_{tot}}$
−0.6	0.233	0.086
−0.4	0.215	0.096
−0.2	0.199	0.105
0.0	0.185	0.115
0.4	0.163	0.135
0.8	0.145	0.156
1.2	0.131	0.177
1.6	0.119	0.198
2.0	0.109	0.219

at ISR energies.) Next we use (30) and the empirical formula [20] for the energy dependence of the slope $A(s, t)$ for pp scattering at very small $|t|$ $(0.05 \leqslant |t| \leqslant 0.09)$ to obtain

$$\lambda = 0.152 \ln(s) - 0.864. \tag{32}$$

By means of (32) and (29) we can now estimate the energy dependence of σ_{el}/σ_{tot} in this model. As can be seen from Table II this dependence is consistent with the empircal values of Ref. [22].

TABLE II

The values of $\frac{\sigma_{el}}{\sigma_{tot}}$ and the slope of $\frac{d\sigma_{el}}{d|t|}$ at $t = 0$ for different values of p_{Lab}. $\left(\frac{\sigma_{el}}{\sigma_{tot}}\right)_{exp}$ has been evaluated by means of the empirical fit from Ref. [22]

$p_{Lab}\left[\frac{\text{GeV}}{c}\right]$	λ	$A(0)$ [GeV^{-2}]	$\frac{\sigma_{el}}{\sigma_{tot}}$	$\left(\frac{\sigma_{el}}{\sigma_{tot}}\right)_{exp}$
25	−0.28	9.87	0.205	0.209 ±0.03
50	−0.175	10.36	0.197	0.191 ±0.02
100	−0.07	10.86	0.190	0.180 ±0.02
200	0.036	11.37	0.183	0.172 ±0.015
300	0.098	11.67	0.1795	0.169 ±0.014
400	0.14	11.88	0.177	0.167 ±0.013
500	0.175	12.05	0.175	0.1655±0.012
1000	0.280	12.57	0.169	0.1626±0.011

To estimate the t-dependence of $A(s, t)$ for fixed s in this model it is sufficient to consider the case $\lambda = 0$. Then (25) reduces to

$$1 - \exp(-2\varrho) = C \exp(-Bb^2) \tag{33}$$

the parametrization originally proposed by Van Hove [23] and used in Ref. [3].

In our model as one can see from (4), (28) and (33) C is related to the maximal value $(b^2 = 0)$ of $n/\langle n \rangle$

$$z_{\max} = \left(-\frac{4}{\pi} \ln (1-C) \right)^{1/2}, \tag{34}$$

and thus C must be at least 0.9995 if we want to take into account all events observed at present energies $(0.2 \leqslant z \leqslant 3.2)$. Thus we put $C = 1$. As has been shown by Heckman and Henzi [24] (33) with $C = 1$ reproduces recently observed [20] t-dependence of the slope of $d\sigma_{el}/d|t|$ for pp-collisions. This is an improvement compared to Chou-Yang's simplest parametrization [19]

$$1 - \exp(-\varrho) = C \exp(-Bb^2),$$

where the slope of $d\sigma_{el}/d|t|$ is t-independent.

We should, however, emphasize that (33) with $C = 1$ as required by (34) gives a constant ratio $\sigma_{el}/\sigma_{tot} = 0.185$. We therefore have to introduce the general parametrization (25) in order to describe the slow decrease of σ_{el}/σ_{tot} with energy.

5. *Dynamical factor and its physical interpretation*

The most important ingredient in the foregoing arguments is the dynamical ansatz (28) which states that the number of produced particles in a collision at impact parameter b is proportional to the square root of the overlap ϱ.

Without such a square root dependence we do not get the simple scaling function (7). In this section we present two versions of arguments to justify this ansatz.

5a. Global excitation model

One possible interpretation is the following picture.

When two hadrons collide at a given impact parameter b, they exchange a certain amount of longitudinal momentum ΔP and energy ΔE, both of which are assumed to be proportional to the overlap $\varrho(b^2, s)$ of these hadrons at this impact parameter. The proportionality factors are in general functions of energy. Thus

$$\Delta P = h(s)\varrho(b^2, s), \tag{35}$$

$$|\Delta E| = j(s)\varrho(b^2, s). \tag{36}$$

After the collision the mass of the two excited hadrons M_+ and M_- will be given by

$$M_{\pm} = \{(E \pm |\Delta E|)^2 - (P - \Delta P)^2\}^{1/2}. \tag{37}$$

Except for cases of very high and very low excitation, we can regard

$$M \ll |\Delta E| \ll E, \; M \ll \Delta P \ll P. \tag{38}$$

Then the increase $\Delta M_{\pm}$ of mass of the hadrons amounts to

$$\Delta M_{\pm} = M_{\pm} - M \approx s^{1/4}\{\Delta P \pm \Delta E|\}^{1/2} \tag{39}$$

or

$$\Delta M_{\pm} \approx s^{1/4}\{h(s) \pm j(s)\}^{1/2} \varrho^{1/2}. \tag{40}$$

Taking the simplest assumption that the number of pions produced by decay of the excited hadrons is proportional to $\Delta M = \Delta M_{+} + \Delta M_{-}$ *i.e.*

$$n = c\{\Delta M_{+} + \Delta M_{-}\} = g(s)\varrho^{1/2} \tag{41}$$

with

$$g(s) = cs^{1/4}[\{h(s)+j(s)\}^{1/2} + \{h(s)-j(s)\}^{1/2}] \quad (c \text{ being constant})$$

we obtain (28) if we notice the relation

$$\lambda(s)+2 = \frac{\pi}{4}\left[\int_0^\infty (1-e^{-2\varrho})db^2 \Big/ \int_0^\infty \varrho^{1/2}(1-e^{-2\varrho})db^2\right]^2. \tag{42}$$

It is possible [25] in a parton-model of the Kraemmer-Nielsen-Susskind type [26] where the partons are assumed to scatter only through s-waves, to justify the assumptions (35) and (36). Such a model, however, suffers from the general Nova-model disease: the exponential decreasing behaviour of the multiplicity distribution for $z > 1$ is incompatible with a flat plateau in the single particle inclusive distribution [27].

5b. Local excitation model [3]

In this model, the hadrons are regarded as consisting of a large number of constituents. When the two hadrons A and B collide at a given impact parameter b, there will be a number $N(b^2, s)$ of constituents from one hadron, any of which can make collision with any of $N(b^2, s)$ from the other hadron.

Let each of such elementary collisions take place with the probability ε, and suppose that if any of them takes place an inelastic events results. Then, by definition, the probability that nothing happens is given by

$$|S(b^2, s)|^2 = e^{-2\varrho} = (1-\varepsilon)^{N^2(b^2,s)} \tag{43}$$

or

$$2\varrho = -N^2 \log(1-\varepsilon). \tag{44}$$

Now assume that during the collision only those parts of hadrons which overlapped, *i.e.* passed through each other, can be excited. They decay into pions immediately after the collision takes place, so that the excitation cannot be shared by parts which did not overlap. This is, so to speak, a fragmentation of hadrons after the local excitation. In the constituent language this means that only $N(b^2, s)$ constituents of each hadron are excited and decay into final particles so that

$$n(b^2, s) \sim 2N(b^2, s). \tag{45}$$

Assuming next that ε is independent of b one obtains from (44) and (45)

$$n(b^2, s) = \hat{g}(s)\varrho^{1/2}(b^2, s).$$

This has the same form as (41), although $\hat{g}(s)$ and $\hat{g}(s)$ have completely different physical contents.

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