Vol. B5 (1974) ACTA PHYSICA POLONICA No 4

DIFFRACTIVE PRODUCTION OFF NUCLEI — SHADOW OF HADRONIC BREMSSTRAHLUNG

BY A. BIAŁAS* AND W. CZYŻ**

Max-Planck-Institute for Physics and Astrophysics, München

(Received March 9, 1974)

Diffractive production on nuclei is calculated using as an input a specific model for diffractive production on nucleons. In this model diffractive production is described as a shadow of non-diffractive multiple production of particles. The mechanism for non-diffractive production is taken to be hadronic bremsstrahlung of independently produced clusters. It is shown that such a model naturally explains the strikingly simple pattern of absorption observed in coherent production on nuclei. Possible generalizations of these results are indicated.

1. Introduction

It is widely recognized that the elastic scattering of high energy hadrons from nuclei can be successfully described in terms of the hadron-nucleon elastic scattering amplitudes [1]. It remains an interesting and as yet unsolved problem whether an analogous description is possible in the case of multiple production processes on nuclear targets [2, 3]. In particular, recently the problem of diffractive production on nuclei has been frequently discussed in this spirit (for recent references see Refs [2, 3]).

Several authors pointed out that in particle production on nuclear targets the short time behaviour of the production amplitudes on free nucleons is of primary importance [2, 4, 5, 6]. Thus to compute the production amplitudes on nuclei from "elementary" processes a model for such "elementary" production amplitudes, valid for short times after the collision, is needed.

The purpose of the present paper is to compute explicitly the diffractive production on nuclei starting from a specific model of diffractive production on nucleons proposed in Ref. [7]. In this model the diffractive production on nucleons is calculated as a shadow

* On leave of absence from the Institute of Physics, Jagellonian University, and Institute of Nuclear Physics, Kraków. Address: Instytut Fizyki, Uniwersytet Jagielloński, Reymonta 4, 30-059 Kraków, Poland.

** On leave of absence from the Institute of Nuclear Physics, Kraków. Address: Instytut Fizyki Jądrowej, Radzikowskiego 152, 31-342 Kraków, Poland.

of non-diffractive interactions given by a bremsstrahlung mechanism. We believe that the model of Ref. [7] can be interpreted as a model for the short time behaviour of production amplitudes. Since this model describes reasonably well both diffractive [7] and non--diffractive [8] particle production in nucleon-nucleon collision, we feel that it is of interest to apply it to nuclear production processes.

Our main result is that this model can explain the small nuclear absorption of the diffractively produced systems [9, 10] provided independent clusters of particles are emitted in the hadronic bremsstrahlung process [8, 11].

The paper is organized as follows. In the next section the relations between the amplitudes for diffractive production on nuclei and nucleons are reviewed. In Section 3 the model for diffractive production on nucleons is summarized. The calculations of diffractive production on nuclei is presented in Section 4. Discussion and conclusions close the paper.

2. *Relation between diffraction on nuclei and nucleons*

We use the following formula for the diffractive production on nuclei [12]:

$$\langle \tilde{f}|\mathcal{T}^{(A)}|\tilde{i}\rangle = \sum_l a^*_{fl}a_{il}\langle l|\mathcal{T}^{(A)}|l\rangle =$$

$$= \frac{ik}{2\pi}\sum_l a^*_{fl}a_{il}\int d^2b e^{i\vec{\Delta}\cdot\vec{b}}\{1-(1-\Gamma(b))^A\} \tag{2.1}$$

where

$$\Gamma(b) = \frac{2\pi}{ik}\langle l|\mathcal{T}^{(N)}|l\rangle \int_{-\infty}^{+\infty} dz\varrho(\vec{b}, z). \tag{2.2}$$

There k is the incident laboratory momentum, $\vec{\Delta}$ the momentum transfer, $\varrho(\vec{b}, z)$ the nuclear single particle density at the impact parameter $\vec{b}$. The incident particle direction is along the z-axis. The states $|l\rangle$ are the eigenstates of the scattering matrix which gives diffractive production and elastic scattering on one nucleon [13]. Hence $\langle l|\mathcal{T}^{(N)}|l\rangle$ are the eigenvalues of this matrix. The physical states $|\tilde{l}\rangle$ are related to the eigenstates $|l\rangle$ through the expansion coefficients a_{kl}:

$$|k\rangle = \sum_l a^*_{lk}|\tilde{l}\rangle. \tag{2.3}$$

The most important physical assumption included in the formulas (2.1), (2.2) is that all non-diffractive intermediate states are neglected. This is presumably a good approximation for purely coherent processes. We also accepted that each nucleon in the target interacts only once. Furthermore to simplify the formalism we approximated the ground state wave function of the target nucleus by a product of identical single particle wave functions (whose squares give the density ϱ). In Eq. (2.2) the impact parameter distribution of $\langle l|\mathcal{T}^{(N)}|l\rangle$ is assumed to be much narrower (in $\vec{b}$) than $\varrho(\vec{b}, z)$.

It follows from Eqs (2.1) and (2.2) that the diffractive amplitudes on nuclei can be computed provided one knows the diffractive amplitudes on nucleons and if one can perform the diagonalization of the matrix defined by these nucleon amplitudes in the space of diffractive states.

Thus to proceed further one needs a model for diffractive processes on nucleons. In this paper we are going to use the model developed in Ref. [7] which is summarized in the next section.

3. *A model of diffractive processes on nucleons*

In this model any diffractive process (elastic or inelastic) is described as a shadow of the non-diffractive multiple particle production which, in turn, was assumed to be a hadronic bremsstrahlung process [14, 15].

The amplitudes for diffractive processes are obtained by sandwiching the following overlap operator [7]

$$F = T_N^\dagger T_N = \frac{1}{(2\pi)^8}\int d^4x d^4x' e^{-i(P\cdot x - P'\cdot x')}\tilde{T}_n^\dagger(x')\tilde{T}_n(x)\tilde{S}_\pi^\dagger(x')\tilde{S}_\pi(x) \tag{3.1}$$

between the initial and final states. Here P, P' are the total initial and final four momenta and

$$\tilde{T}_n(x) = \exp\left(i\int\frac{d^3k}{E_k}\,k\cdot x b^\dagger(k)b(k)\right)T_n, \tag{3.2}$$

$$T_n = \int\frac{d^3k_A}{E_A}\frac{d^3k_B}{E_B}\frac{d^3k_C}{E_C}\frac{d^3k_D}{E_D}\,\psi(k_A, k_B, k_C, k_D)b^\dagger(k_C)b^\dagger(k_D)b(k_A)b(k_B), \tag{3.3}$$

$$\tilde{S}_\pi(x) = \exp\left(i\int\frac{d^3k}{E_k}\,k\cdot x a^\dagger(k)a(k)\right)S_\pi, \tag{3.4}$$

$$S_\pi = \exp\left(i\int\frac{d^3q}{E_q}\,\varrho^*(q)a(q)+i\int\frac{d^3q}{E_q}\,\varrho(q)a^\dagger(q)\right). \tag{3.5}$$

In Eqs (3.2)–(3.5) $b(k)$, $a(k)$ are the free field nucleon and meson annihilation operators,

$$\{b(k), b^\dagger(k')\} = [a(k), a^\dagger(k)] = E_k\delta^{(3)}(k-k'), \tag{3.6}$$

$\varrho(q)$ is the probability amplitude for non-diffractive emission of one meson. ψ (k_A, k_B, k_C, k_D) is the nucleon transition amplitude in the non-diffractive processes which are generated by the operator

$$T_N = \delta^{(4)}(P-\hat{P})T_n S_\pi, \tag{3.7}$$

where $\hat{P}$ is the operator of the total four-momentum.

From (3.1) one can see that indeed the diffractive amplitudes are the shadows of the non-diffractive processes given by Eq. (3.7). Note that the shadow of diffractive processes is neglected in this approximation. One may argue [7] that such an approximation is reasonable for a description of the gross features of diffractive processes.

The non-diffractive matrix elements (given by Eq. (3.7)) give, as seen from Eq. (3.5), the independent emission of pions [15] (apart from energy and momentum conservation), whereas the nucleons are treated classically. These two characteristics of the non-diffractive processes are typical of the hadronic bremsstrahlung models [14].

In the next section which contains our main results we diagonalize the operator (3.1) and, after inserting it into Eq. (2.1), we discuss the properties of the nuclear diffractive amplitudes obtained in this way.

4. *Diffractive production on nuclei*

The first step is to diagonalize the oprator F of diffractive processes on one nucleon given by Eq. (3.1). To this end let us observe that the states

$$|p_1 \dots p_n\rangle = S_\pi^\dagger \widetilde{|p_1 \dots p_n\rangle} \tag{4.1}$$

are the eigenstates[1] of the pionic part $\tilde{S}_\pi^\dagger(x')\tilde{S}_\pi(x)$ of the diffractive operator F. Here $S_\pi^\dagger$ given by Eq. (3.5) and

$$\widetilde{|p_1 \dots p_n\rangle} = \frac{1}{\sqrt{n!}} a^\dagger(p_1) \dots a^\dagger(p_n)\,|0\rangle, \tag{4.2}$$

where $|0\rangle$ is the pionic vacuum state. Since $S_\pi^\dagger$ is a unitary operator, the states $|p_1 \dots p_n\rangle$ form an orthonormal set.

In the basis (4.1) we have

$$\begin{aligned} &\langle p_1' \dots p_n'|\tilde{S}_\pi^\dagger(x')\tilde{S}_\pi(x)\,|p_1 \dots p_n\rangle = \\ &= \langle p_1' \dots p_n'|\exp\left(i\int \frac{d^3k}{E_k}\, k\cdot(x-x')a^\dagger(k)a(k)\right)\widetilde{|p_1 \dots p_n\rangle} = \\ &= \widetilde{\langle p_1' \dots p_n'}\widetilde{|p_1 \dots p_n\rangle}\, e^{iQ_\pi\cdot(x-x')}, \end{aligned} \tag{4.3}$$

where $Q_\pi = \sum_i p_i = \sum_i p_i'$. Thus the operator $\tilde{S}_\pi^\dagger(x')S_\pi(x)$ becomes indeed diagonal and $\exp[iQ_\pi\cdot(x-x')]$ are its eigenvalues. Inserting Eq. (4.3) into Eq. (3.1) we obtain

$$\begin{aligned} &\langle p_1' \dots p_n'; Q_C Q_D|F|q_A q_B; p_1 \dots p_n\rangle = \delta^{(4)}(P-P')\,\widetilde{\langle p_1' \dots p_n'}\widetilde{|p_1 \dots p_n\rangle}\times \\ &\times \int \frac{d^3k_1}{E_1}\frac{d^3k_2}{E_2}\,\delta^{(4)}(P-Q_\pi-k_1-k_2)\psi^*(Q_C, k_1, Q_D, k_2)\psi(q_A, k_1, q_B, k_2), \end{aligned} \tag{4.4}$$

where q_A, q_B (Q_C, Q_D) are the initial (final) nucleon momenta.

Eq. (4.4) is still rather complicated. In order to see the most important results implied by the model of Ref. [7] we introduce the following drastic simplifications which, we believe, have little influence on our further discussion and conclusions:

(a) We neglect the transverse motion in the initial, final and all intermediate states. This amounts to suppressing all transverse momenta in Eq. (4.4).

[1] Note that (4.1) is a specific realization of the transformation (2.3).

(b) We make the high energy approximation which means that we neglect masses of all individual particles.

Furthermore we find it convenient to write[2]

$$\psi(q_1, q_2, q_3, q_4) = \sqrt{E_1 E_2 E_3 E_4}\, \Phi(q_1, q_2, q_3, q_4). \tag{4.5}$$

Using these simplifications and inserting Eq. (4.5) into (4.4) we obtain in the high energy limit (after integrating over the longitudinal components):

$$\langle p'_1 \dots p'_n; Q_C Q_D | F | p_1 \dots p_n; q_A q_B \rangle = \delta^{(4)}(P-P') \langle \overbrace{p'_1 \dots p'_n} | \overbrace{p_1 \dots p_n} \rangle \times$$
$$\times \sqrt{E_C E_D E_A E_B}\, \Phi^*(Q_C, k_+, Q_D, k_-) \Phi(q_A, k_+, q_B, k_-), \tag{4.6}$$

where $k_\pm = \pm\frac{1}{2}(E - E_\pi \mp Q_\pi)$, E being the total c.m. energy. We remind the reader that in Eq. (4.6) only the longitudinal momenta are present.

Eq. (4.6) gives a specific form of the scattering amplitude of the eigenmode l which was symbolically denoted by $\langle l | \mathscr{T}^{(N)} | l \rangle$ in Eq. (2.2).

As it was discussed in Ref. [12] the dependence of the diagonal matrix elements (eigenvalues) given by Eq. (4.6) on the parameters of the eigenstates is essential for determining the absorption of the diffractively produced systems. In particular, it was argued that an approximate equality of these eigenvalues gives similar absorption in the entrance and exit channels (in agreement with experiment).

It is seen from Eq. (4.6) that the eigenvalues depend on only one relevant parameter, namely Q_π. We observe that as long as the dependence of Φ on the longitudinal momenta can be neglected, the eigenvalues do not depend on Q_π and E_π. Thus, the condition

$$\Phi(q_1, q_2, q_3, q_4) \cong \text{const}, \tag{4.7}$$

makes all the eigenvalues approximately equal and consequently, as argued in Ref. [12], will lead to approximately equal absorptions in the entrance and exit channels. This is the main result of our analysis. It sh owsthat the model can indeed explain the small absorption observed in the nuclear coherent production experiments. The significance of the condition (4.7) and the general conclusions are discussed in the next section.

5. *Discussion and conclusions*

In order to see the physical meaning of the condition (4.7) let us write the cross--section for non-diffractive production of n pions [7]:

$$d\sigma = \frac{2\pi^2}{n!} S^{-\lambda} \delta^{(4)}\Big(q_A + q_B - Q_C - Q_D - \sum_i p_i\Big) \times$$
$$\times |\Phi(q_A, q_B, Q_C, Q_D)|^2 d^3Q_C d^3Q_D \sum_{j=1}^{n} \frac{d^3 p_j}{E_j} |\varrho(p_j)|^2, \tag{5.1}$$

[2] When the Feynman scaling [16] is satisfied the function Φ depends only on the ratios of momenta, not on the momenta themselves.

where

$$\lambda = \int d^2p_\perp |\varrho(p_\perp, p_{||} = 0)|^2, \qquad S = (q_A+q_B)^2.$$

Neglecting again the transverse momenta and performing integration over the longitudinal momenta of the nucleons we obtain in the high energy limit

$$d\sigma = \frac{2\pi^2}{n!} S^{-\lambda} |\Phi(q_A, q_B, Q_+, Q_-)|^2 \prod_{j=1}^{n} \frac{d^3p_j}{E_j} |\varrho(p_j)|^2, \tag{5.2}$$

where $Q_\pm = \pm\frac{1}{2}(E - \sum_j E_j \mp \sum_j p_j)$, E being the total c.m. energy. When the condition (4.7) is satisfied, Eq. (5.2) gives an independent emission of pions. Thus we see that the description of non-diffractive production given in Ref. [7] with the additional condition (4.7) is equivalent to the one considered by Stodolsky [11]. However, from the recent measurements [17] it is known that there exist positive short range correlations between produced pions. Consequently the independent production model is unacceptable. This problem was discussed recently in Ref. [8] where it was concluded that the data are fully compatible with the independent emission of clusters of pions.

Thus we can conclude that the condition (4.7) which is necessary to explain small absorption observed in nuclear diffractive production is compatible with the data, provided the hadronic bremsstrahlung of the model of Ref. [7] refers to production of clusters, not of individual pions.

It should be stressed however that the hadronic bremsstrahlung model considered above is not the only one which is capable of explaining the approximate equality of the attenuations in the entrance and exit channels in coherent diffractive production on nuclei. As easily seen from the algebra presented in Section 4, any model whose non-diffractive transition operator can be represented in the form (Eq. (3.7))

$$T_N = \delta^{(4)}(P-\hat{P})T_n S_\pi,$$

where S_π is a unitary operator acting in pionic space, leads to the same results provided the nucleon transition amplitude Φ defined by Eq. (4.5) satisfies the condition (4.7).

The simplified way of seeing why all these models give absorption of the multiparticle states the same as that of one particle states is to observe that after omitting $\delta^{(4)}(P-\hat{P})$ in (3.7) we obtain for the diffractive amplitudes

$$F \cong T_n^\dagger T_n S_\pi^\dagger S_\pi = T_n^\dagger T_n. \tag{5.3}$$

Thus F is diagonal in the representation of the physical pion states. Furthermore when the condition (4.7) is satisfied all the diagonal elements are equal to each other. It means that the elastic scattering of all the physical states with any number of pions is the same. It immediately leads to the same absorption for all these states. The role of the δ-function is to spoil the diagonal character of F and thus to provide some diffractive production. Since however such production is weak in any specific channel the effect of the conservation laws should be small.

The simple result (5.3) is a consequence of the fact that in the model of Ref. [7] there is a fundamental distinction in the treatment of the leading and produced particles. The leading particles are treated as sources of the field, whereas produced particles are free field quanta. Consequently, the produced particles do not interact except while "taking part" in another nucleon-nucleon collision. Thus one has to accept that the model of Ref. [7] has a chance to be valid inside of nuclei as long as the produced particles and their sources stay together. That means its possible applicability region is limited to the times during which the particles do not spread out when passing the nucleus[3]. It remains an open problem how the transition between the short and long time behaviours is realized.

From this interpretation it follows also that such a bremsstrahlung model should lead to comparatively low multiplicities in non-diffractive hadron-nucleus collisions. This problem is being under investigation.

To summarize: We have shown that the strikingly simple pattern of absorption observed in coherent diffractive production on nuclei is naturally explained in the class of models in which the diffractive processes are generated by non-diffractive production given by the transition operator (3.7) provided the condition (4.7) is satisfied. The model of hadronic bremsstrahlung of Ref. [7] is a specific example of such models. We feel that our result, in turn, gives support for the models of this class.

The authors wish to thank Professors L. Van Hove and L. Stodolsky for helpful discussions. The hospitality of the Max-Planck-Institute is also gratefully acknowledged.

REFERENCES

[1] See *e. g.* R. J. Glauber's and other articles in Columbia Univ. Conf. Proc.: *High Energy Physics and Nuclear Structure*, ed. S. Devons, Plenum Press, 1970.
[2] K. Gottfried, *Coherent and Incoherent Multiple Production on Nuclei*, CERN preprint TH 1735, to be published in Proceedings of the Fifth International Conference on High-Energy Physics and Nuclear Structure, Uppsala, Sweden, 18–22 June 1973; K. Gottfried, *Phys. Rev. Lett.* **32**, 957 (1974).
[3] L. Van Hove, *Coherent Particle Production and the Propagation of Hadronic Systems Through nuclear Matter*, CERN Preprint TH. 1685, 1973, to appear in the Proceedings of the 5th International Conference on High Energy Physics and Nuclear Structure, Uppsala, Sweden, 18–22 June, 1973.
[4] V. N. Gribov, *Yad. Fiz.*, **9**, 640 (1969).
[5] L. Van Hove, *Nucl. Phys.*, **B46**, 75 (1972).
[6] A. S. Goldhaber, *Phys. Rev.*, **D7**, 765 (1973).
[7] A. Białas, A. Kotański, *Acta Phys. Pol.*, **B4**, 659 (1973).
[8] S. Pokorski, L. Van Hove, *Acta Phys. Pol.* **B5**, 229 (1974).
[9] W. Beusch, *Acta Phys. Pol.*, **B3**, 679 (1972).
[10] P. Mühlemann *et al.*, ETH-MILANO-CERN Imperial College Collaboration, *Nucl. Phys.*, **B59**, 106 (1973).
[11] L. Stodolsky, *Proceedings of the Seventh Rencontre de Moriond*, 1972, edited by Tran Than Van, Faculté des Sciences, Univ. de Paris, 1972.
[12] W. Czyż, *Phys. Rev.*, **D8**, 3219 (1973).

---

[3] We thank Professor L. Van Hove for a discussion of this point.

[13] M. L. Good, W. D. Walker, *Phys. Rev.*, **120**, 1857 (1960).
[14] H. W. Lewis, J. R. Oppenheimer, S. A. Wouthuysen, *Phys. Rev.*, **73**, 127 (1948).
[15] L. Van Hove, *Rev. Mod. Phys.*, **36**, 525 (1964).
[16] R. P. Feynman, *High Energy Collisions*, Stony Brook Conference 1969, Gordon and Breach, New York, London, Paris, p. 237.
[17] H. Dibon *et al.*, CERN–HAMBURG–VIENNA Collaboration, *Phys. Lett.*, **44B**, 313 (1973); S. R. Amendolia *et al.*, PISA–STONY BROOK Collaboration, *Measurement of Inclusive Two-Particle Rapidity Correlations at the ISR*, CERN preprint, August 1973.