PROPERTIES OF SCATTERING AMPLITUDES THAT INCREASE LINEARLY WITH ENERGY*

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We investigate properties of scattering amplitudes that lead to cross sections increasing linearly with energy. A possible application is to neutrino-nucleon scattering.

The scaling hypothesis [1] as applied to weak interactions, implies that total and elastic cross-sections increase with energy. This result also is a consequence of the usual (current-current) theory of weak interactions [2]. However, the usual current-current theory corresponds to a point interaction and thus the linear increase of the cross-sections with energy must certainly fail at very high energies because of unitarity [2, 3]. Experimental results on neutrino-nucleon scattering are consistent with a linear increase of the total cross-section, with the neutrino-nucleon cross-section being approximately three times the antineutrino-nucleon cross-section [4]. With these facts in mind, it is, therefore, of interest to investigate the properties of scattering amplitudes that lead to cross-sections increasing linearly with energy, even if the linear increase is only over a finite, but, large interval of energy.

Consider the elastic scattering of neutrinos and antineutrinos off protons. We shall denote all anti-particles physical observables by placing a bar over them; S and t are the usual Mandelstam variables. The phase of the forward scattering amplitude is defined as $\varrho(S) = \mathrm{Re}\, A/\mathrm{Im}\, A$ and our amplitude is normalized so that $\sigma_T(S) = \mathrm{Im}\, A/S$.

We use the following assumptions: (a) The total and elastic cross-sections increase linearly with energy and $\sigma_T(S) > \bar{\sigma}_T(S)$. (b) The scattering amplitude has enough analyticity so that the usual crossing relations hold [5], *i.e.*, $A(S+i\varepsilon, t) = [\bar{A}(-S+i\varepsilon, t)]^*$. (c) The

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high-energy forward scattering amplitudes, consistent with (a) and (b), are

$$A(S) = c_1(-iS)^2 \log(-iS) - iC_2(-iS)^2, \tag{1a}$$

$$\bar{A}(S) = C_1(-iS)^2 \log(-iS) + iC_2(-iS)^2, \tag{1b}$$

where $A(S) \equiv A(S, 0)$ and (C_1, C_2) are positive constants with $(\pi/2)C_1 > C_2$.

We now list properties of the scattering amplitudes given in Eq. (1).

The total cross-sections are

$$\sigma_T(S) = \left[\left(\frac{\pi}{2}\right) C_1 + C_2\right] S, \tag{2a}$$

$$\bar{\sigma}_T(S) = \left[\left(\frac{\pi}{2}\right) C_1 - C_2\right] S. \tag{2b}$$

Note that we do not have a Pomeranchuk-type theorem [6] for the total cross-sections. Also, the total cross-section difference increases with energy, *i.e.*, $\sigma_T(S) - \bar{\sigma}_T(S) = 2C_2 S$.

The phase of the forward scattering amplitude is

$$\varrho(S) = -C_1 \log S \Big/ \left[\left(\frac{\pi}{2}\right) C_1 + C_2\right], \tag{3a}$$

$$\bar{\varrho}(S) = -C_1 \log S \Big/ \left[\left(\frac{\pi}{2}\right) C_1 - C_2\right]. \tag{3b}$$

The phase is negative for both neutrino and antineutrino scattering. If we measure S in units of the proton mass-squared and set [7] $\sigma_T(S) \simeq 3\bar{\sigma}_T(S)$, we find that the phase can be quite large even for moderate energies[1].

We may also obtain an estimate of the effective number of partial waves contributing to the scattering amplitude. (The amplitude in Eq. (1), unlike the amplitude obtained from the usual current-current weak interactions theory [8], is a full scattering amplitude and thus contains all partial waves.) We make use of the following relation [9]

$$|\varrho(S)| \leqslant L/[S\sigma_T(S)]^{1/2}, \tag{4}$$

where L is the effective number of partial waves contributing to the scattering amplitude. Putting the results of Eqs (2) and (3) into Eq. (4) and taking the equality sign in Eq. (4), we obtain

$$L = C_3 S \log S. \tag{5}$$

It is interesting that this value for L corresponds to the Greenberg-Low bound [10].

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[1] A more realistic unit to measure S is the inverse of the weak coupling constant, G. Note that G has dimensions $(\text{GeV})^{-2}$, thus S will always occur in the combination GS. We expect that for energies in the interval, $M_\varrho^2 \ll S \ll G^{-1}$, the analysis of this paper would apply to neutrino-nucleon scattering. In this case, we obtain from Eq. (3) the result that the phases are small and positive.

The forward differential cross-section, asymptotically, is

$$\frac{d\sigma(S,0)}{dt} \sim C_1^2 S^2 (\log S)^2/16\pi. \tag{6}$$

If we define the diffraction width as [11]

$$\Delta(S) = \sigma_{\mathrm{EL}}(S) \Big/ \frac{d\sigma(S,0)}{dt}, \tag{7}$$

then, we find, using the fact that $\sigma_{\mathrm{EL}}(S) = C_4 S$,

$$\Delta(S) = C_5/S\,(\log S)^2. \tag{8}$$

The fact that the diffraction width decreases so rapidly with energy is a consequence of unitarity, *i.e.*, $\sigma_{\mathrm{EL}}(S) < \sigma_T(S)$.

Using the estimate, given in Eq. (5), for the effective number of partial waves contribution, we may also obtain a lower bound for the elastic cross-section. We make use of the following relation [12]

$$\sigma_{\mathrm{EL}}(S) \geqslant S\sigma_{\mathrm{T}}^2(S)/16\pi L^2, \tag{9}$$

obtaining

$$\sigma_{\mathrm{EL}}(S) \geqslant C_6 S/(\log S)^2. \tag{10}$$

Finally, as stated before, the results derived above follow from our three assumptions and are expected to hold for any scattering process where the total cross-sections increase linearly with energy and particle, antiparticle cross-sections are not equal. Even with the conventional current-current theory, which is expected to break down at about 300 GeV, there should exist a range of energies over which our results, Eqs (3), (6), (8) and (10) will apply. Such results are important, in view of the fact, that at present, no complete theory of the weak interactions exists.

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