OVERLAP FUNCTION FROM THE MULTIPERIPHERAL MODEL

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Monte Carlo phase space calculations of the overlap function for Chan-Łoskiewicz-Allison and Chew-Pignotti models were performed up to laboratory momentum 1500 GeV. The slope of the overlap function was found to be about four times smaller than the experimental value. Shrinkage of the elastic peak does not exceed the value found in experiment. The qualitative explanation of these results is given and the discrepancy with recent estimates obtained by Hamer and Peierls, Hwa, and Henyey is explained. Finally, it is argued that the random walk picture of the multiperipheral models is not valid for the realistic particle density in the rapidity scale.

1. Introduction

The measurements of the elastic and inelastic scattering cross-sections at NAL and ISR energies have stimulated a new interest in the problem of the Van Hove overlap function [1]. In particular, several authors have recently discussed this problem in the framework of the multiperipheral model [2–4].

In the present paper we also study the overlap function in two specific versions of the multiperipheral model, namely the Chan-Łoskiewicz-Allison and Chew-Pignotti models. Our arguments and conclusions are based on exact Monte Carlo calculations performed up to energy 1500 GeV, and taking into account energy and momentum conservation. These calculations show that the slope of the overlap function is much smaller than that of the elastic amplitude. Thus they agree with earlier calculations at lower energies but are in strong disagreement with conclusions of Refs [2–4].

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In order to introduce the reader to the problem let us write down the equation for the imaginary part of the two body (elastic) amplitude which is a direct consequence of the S-matrix unitarity:

$$\text{Im}\, A_2(p_a p_b;\, p_{a'} p_{b'}) = \sum_{n=2}^{\infty} \int d\tau A_n(p_a p_b;\, p_1, p_2, \ldots, p_n) A_n^*(p_{a'} p_{b'};\, p_1, p_2, \ldots, p_n), \tag{1}$$

where A_2 is an elastic scattering amplitude $a+b \to a'+b'$, A_n is an amplitude for production of n particles $a+b \to 1+2+\ldots+n$, and $d\tau_n$ is the Lorentz invariant phase space element.

An adequate model of particle production inserted into the RHS of Eq. (1) has to generate the correct amplitude of the elastic scattering, which is believed to be dominantly imaginary in the high energy limit. Thus Eq. (1) provides a test of models for particle production. As has been shown by Michejda et al. [5–8], this test is rather severe and many models do not satisfy it.

The problem can be formulated as follows [5]: is it possible to generate the correct forward diffraction peak, starting from a realistic description of many particle production in terms of a given phenomenological model?

Throughout this paper by realistic description we mean the production amplitude which, inserted under the phase space integral, is able to reproduce the main features of inelastic scattering data; that is to say, at least the mean values of transverse and longitudinal momenta (nucleon inelasticity).

The first attempt to solve this problem was made by Michejda et al. [5–8] for the π^+p reaction at 8 GeV. It was found that neither the Uncorrelated Jet Model nor the Multiperipheral Models (CŁA, OPE) generate an overlap function with a slope comparable to the one expected from elastic scattering data. The resulting slope of the overlap function appears to be too small by an order of magnitude. It was concluded that the unitary model of multiparticle production at the energy 8 GeV should provide either a momentum dependent phase of the matrix element or some correlations stronger than those in MPM (the spin of produced objects we regard as some sort of correlations).

The authors of papers [2–4] came to the conclusion that the situation is different at very high energies. In their opinion the Multiperipheral Model in this simplest version (CP) gives a too steep overlap function and predicts a too rapid shrinkage of the diffractive peak.

In the present paper we show that the conclusions in Refs [2–4] are based on approximations which are not justified for the experimentally observed average density of particles in the rapidity scale.

Exact Monte Carlo calculations with Chan-Łoskiewicz-Allison and Chew-Pignotti models lead us to a result very similar to that found at 8 GeV [7] — the overlap function appears to be very flat and the estimated shrinkage of the elastic peak appears to be reasonable. Furthermore, we found that the random walk picture of the multiperipheral model [12] is not valid for the realistic particle density in rapidity scale. Calculations were made for pp collisions at p_{lab} 50, 300, 1500 GeV, and multiplicities up to 30.

The paper is arranged as follows. In the following Section 2 we describe the results of our calculations with the CŁA model and we show explicitly the origin of the difference from the calculations in Refs [2–4]. In Section 3 we discuss the Chew-Pignotti model, which enables us to discuss more directly the approximations used in Refs [2–4], and the validity of the random walk picture. The effects of clustering in the rapidity space on the overlap function behaviour are qualitatively discussed in Section 4. The paper ends with conclusions.

2. *The Chan-Łoskiewicz-Allison model*

We analysed two types of multiperipheral parametrization for the matrix element. One of them was the Chan-Łoskiewicz-Allison model [9, 10], widely studied at accelerator energies. In this model, the modulus of the matrix element for n particles production is of the form:

$$|A_n| = \prod_{i=1}^{n-1}\left(\frac{g_i s_i + ca}{s_i + c}\right)\left(\frac{s_i+a}{a}\right)^{\alpha^0_i}\left(\frac{s_i+b_i}{b_i}\right)^{t_i},$$

$$s_i = (p_i+p_{i+1})^2-(m_i+m_{i+1})^2,$$

$$t_i = (p_a - \sum_{k=1}^{K} p_k)^2. \tag{2}$$

Parameters g, c, a, b were taken from the original paper [9][1]. The kinematics is shown in Fig. 1.

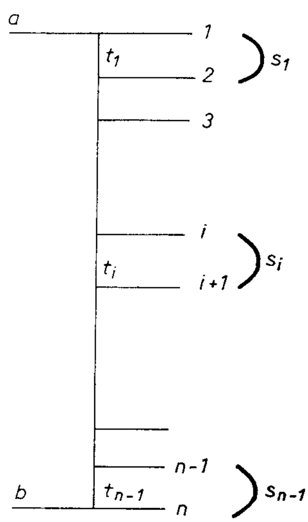

Fig. 1. Kinematics of the multiperipheral chain

[1] It is remarkable that parameters adjusted to fit the data at accelerator energies enable us to reproduce average transverse and longitudinal momenta at ISR energies.

The following formulae, taken from Ref. [11], were used to calculate partial overlap $F_n(t)$ and its slope Γ_n:

$$F_n(t) = \int d\tau_n |A_n|^2 H_n(t),$$

$$H_n(t) = \exp\left\{-4\left(1-\sqrt{1-\frac{t}{4p_a^{CM}}}\right)\sum_{i=1}^{n-1} P_{L,i}^{CM}\alpha_i(s, s_1, \ldots, s_{n-1})\right\},$$

$$\Gamma_n = \int d\tau_n |A_n|^2 S_n,$$

$$S_n = \frac{1}{2p_a^{CM}}\sum_{i=1}^{n-1} P_{L,i}^{CM}\alpha_i(s, s_1, s_2, \ldots, s_{n-1}),$$

$$P_{L,i}^{CM} = \frac{\vec{p}_a^{\,CM}}{p_a^{CM}}\sum_{k=1}^{i} \vec{p}_k^{\,CM}, \tag{3}$$

for the amplitude A_n of the general form:

$$A_n(p_a p_b; p_1, p_2, \ldots, p_n) = F(s, s_1, s_2, \ldots, s_{n-1}) \exp\left(\sum_{i=1}^{n-1} \alpha_i(s, s_1, s_2, \ldots, s_{n-1})t_i\right). \tag{4}$$

In the particular case of the CŁA model:

$$\alpha_i(s, s_1, \ldots, s_{n-1}) = \ln\left(1+\frac{s_i}{b_i}\right). \tag{5}$$

To obtain the full overlap function and its slope at $t = 0$ we must average them over the multiplicity distribution.

$$F(t) = \sum_{n=2}^{\infty} \frac{\sigma_n}{\sigma_T} F_n(t),$$

$$\Gamma = \sum_{n=2}^{\infty} \frac{\sigma_n}{\sigma_T} \Gamma_n.$$

We have performed Monte Carlo calculations of the inelastic overlap function for the whole total multiplicity spectrum at 50 and 300 GeV, and for multiplicities close to the average at 1500 GeV (estimated from logarithmic fit).

We considered only one multiperipheral graph, shown in Fig. 1, and only with meson exchanges, with intercept 0.5. We disregarded all other graphs with permutated positions of the final particles.

In our calculations we used a new method for generating multiperipheral Monte Carlo events, developed for high energy calculations. A detailed description of the method will be published in TPJU preprints.

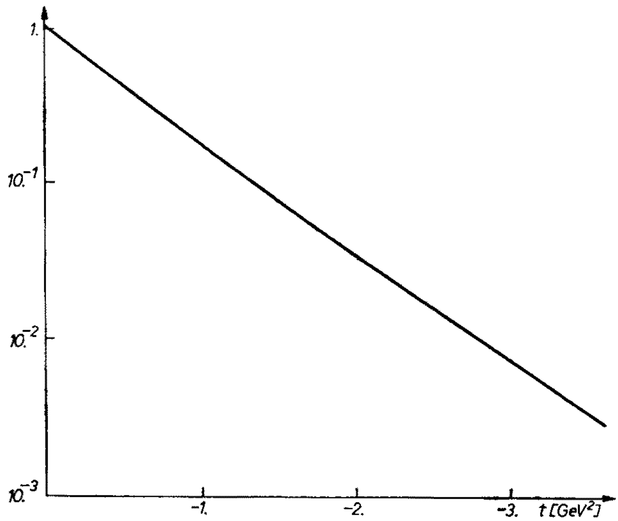

Fig. 2. The partial overlap function for the reaction pp → 10πpp at p_{lab} 300 GeV, in the CŁA model

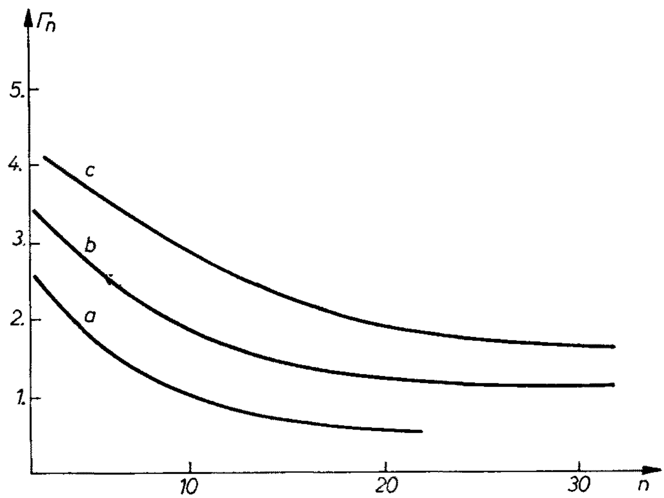

Fig. 3. Partial overlap slopes versus total multiplicity in the CŁA model at p_{lab}: *a*) 50, *b*) 300, *c*) 1500 GeV

In Fig. 2 we show the partial inelastic overlap function against t for pp → 10π pp, at 300 GeV, calculated in the CŁA model.

In Fig. 3 we have slopes of partial overlaps against the number of particles produced. As we see, in general they are small in comparison with the slope of the elastic amplitude (6 GeV^{-2}) and do not increase with multiplicity n of the intermediate state, as the random walk picture would predict.

In Fig. 4 we show slopes of partial overlap functions calculated for $n = \langle n \rangle_{exp}$ versus ln (s). It is seen that the obtained shrinkage is comparable to the experimental one.

Thus our results agree with calculations at lower energies [7]. They disagree entirely with those of Refs [2–4], in which it was calculated that (i) the slope of the overlap function in the Multiperipheral Model is greater than that experimentally determined, and (ii) the shrinkage is much stronger than that observed.

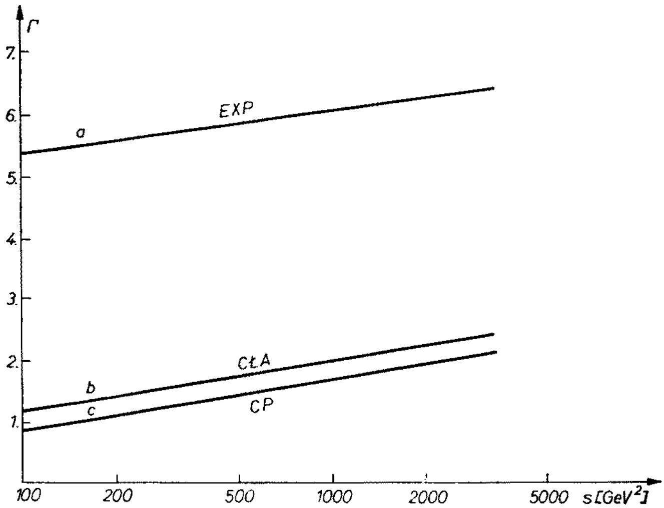

Fig. 4. Slope of the overlap function versus ln (s) at average total multiplicity a) experimental, b) from CŁA model, c) from CP model

As we shall show below, this difference arises from the fact that the authors of Refs [2–4] neglected the longitudinal part of the four momentum transfer. First, let us discuss the situation in the CŁA model. The parameter

$$\alpha_i = \ln\left(1+\frac{s_i}{b_i}\right)$$

which governs the momentum transfer distribution in the CŁA model (see Eq. (2)), depends on the invariant mass of neighbouring particles in the multiperipheral chain. From Monte Carlo calculations we found for central multiplicities (close to average) average s_i of order 0.5 GeV^2, giving $\langle\alpha_i\rangle = 0.4$. This agrees with the value obtained from analysis at 8 GeV [9, 10]. For large multiplicities $\langle\alpha_i\rangle$ is even smaller, so the bounds on the *four momentum transfers* are very weak. Nevertheless *transverse momenta* are strongly bounded, and we get their average values consistent with the data. To see how it happens, let us write a formula for momentum transfer along the i-th link in the multiperipheral chain (see Fig. 1)

$$t_i = t_i^{\mathrm{L}}+t_i^{\mathrm{T}},$$

$$t_i^{\mathrm{L}} = (E_a-\sum_{k=1}^{i} E_k)^2-(p_a-\sum_{k=1}^{i} p_k^{\mathrm{L}})^2,$$

$$t_i^{\mathrm{T}} = -(\sum_{k=1}^{i} \vec{p}_k)^2. \tag{6}$$

We shall show that the bounds on transverse momenta are provided by the longitudinal part t_i^{L} of the momentum transfer rather than its transverse part t_i^{T}. For later purpose it will be very useful to rewrite the longitudinal transfer t_i^{L} as a function of rapidities. *Taking into account momentum and energy conservation* we can derive (see Appendix):

$$t_i^{\mathrm{L}} = -(m_a e^{-y_a} - \sum_{k=1}^{i} m_k^{\mathrm{T}} e^{-y_k})(m_b e^{y_b} - \sum_{k=i+1}^{n} m_k^{\mathrm{T}} e^{y_k}), \tag{7}$$

where $m_k^{\mathrm{T}} = \sqrt{m_k^2 + p^{\mathrm{T}2}}$ is the transverse mass of the k-th particle. To estimate t_i^{L} we shall calculate it assuming that the particles are equally spaced in the rapidity scale with distance d, and all transverse masses are equal to average. For links far from the ends of the multiperipheral chain we obtain (see Appendix):

$$t_i^{\mathrm{L}} \cong -\langle m_i^{\mathrm{T}} \rangle^2 \frac{e^{-d}}{(1-e^{-d})^2}. \tag{8}$$

The approximate transverse momentum dependence of the amplitude through t_i^{L} is $\exp(-\alpha_{\mathrm{eff}} p_i^{\mathrm{T}2})$ where $\alpha_{\mathrm{eff}} = \langle \alpha_i \rangle e^{-d}/(1-e^{-d})^2$. Taking $d = \ln(s)/\langle n \rangle_{\mathrm{exp}} = 0.3$, from the logarithmic fit of the average multiplicity we obtain $\alpha_{\mathrm{eff}} = 4.5$, i. e. a reasonable description of the transverse momentum distribution. We obtain also $|\langle t_i^{\mathrm{L}} \rangle| = 1.7$ GeV^2 which is in good agreement with our results from Monte Carlo calculations for multiplicities close to average.

We would like to emphasize that these estimates of d, α_{eff}, and t_i^{L} at average multiplicity are approximately energy independent.

The authors of Refs [2–4] write the multiperipheral matrix element in the general form:

$$|A_n| = \prod_{i=1}^{n-1} f(s_i) \exp(\alpha_i' t_i^{\mathrm{T}}) \tag{9}$$

which is equivalent to Eq. (3), provided that the longitudinal parts of the four momentum transfer are neglected. In order to fit the transverse momentum distribution using formula (9), it is necessary to take:

$$2\alpha_i' \cong \frac{1}{|\langle t \rangle|} \cong \frac{2}{\langle (p^{\mathrm{T}})^2 \rangle} \cong 11.0. \tag{10}$$

This value is more than one order of magnitude larger than that used in the CLA model. Such a large value of α_i' is the main reason for the large value of the overlap slope and strong shrinkage found in Refs [2–4] (see Eq. (2)). The approximation $t_i^{\mathrm{L}} \cong 0$ used in Refs [2–4] can be justified for events of low multiplicity, where the density of the particles in the rapidity space is very small. Then, the average distance between particles becomes very large, and as is seen from Eq. (8), the longitudinal part of the momentum transfer is close to zero. However, as shown above, for multiplicities close to average this approximation does not hold. On the contrary, the transverse momentum distribution is controlled by the longitudinal part of the momentum transfer.

Thus we conclude that, although the calculations of Refs [2–4] correctly estimate the slope of the overlap function for amplitude given by Eq. (9), they cannot be considered as representative for the multiperipheral model, which is formulated in terms of the four momentum transfers, and not in terms of the transverse momenta.

In concluding this Section, we wish to point out that the dominance of the longitudinal momentum transfer in the description of the transverse momentum distribution has still another important consequence: it questions the validity of the random walk picture of the multiperipheral model [2–4, 12].

We discuss this problem in some detail in the next Section, using a more simplified version of the multiperipheral model, namely the Chew-Pignotti model [14].

3. The Chew-Pignotti model

We have performed the same calculations for the simple Chew-Pignotti model (CP)

$$|A_n| = \prod_{i=1}^{n-1} e^{at_i}, \tag{11}$$

at the same energies and multiplicities. This was done so as to make clearer some of the kinematical aspects discussed previously. This parametrization was used in papers [2–4] hence our Monte Carlo calculations enable us to perform a direct check of the approximations made there. In order to reproduce the experimental average transverse momenta (which are around 350 MeV) we had to take for constant a a value about 0.5–0.4 GeV^{-2} in rough agreement with the CLA value $\langle\alpha_i\rangle = 0.4$ and in strong disagreement with the approximation $a = 1/\langle p^{T2}\rangle = 5.5$ GeV^{-2} used in Refs [2–4].

To show explicitly that the bounds on the transverse momenta are imposed mainly by the bounds on the longitudinal momentum transfers t_i^L, we have put in formula (11)

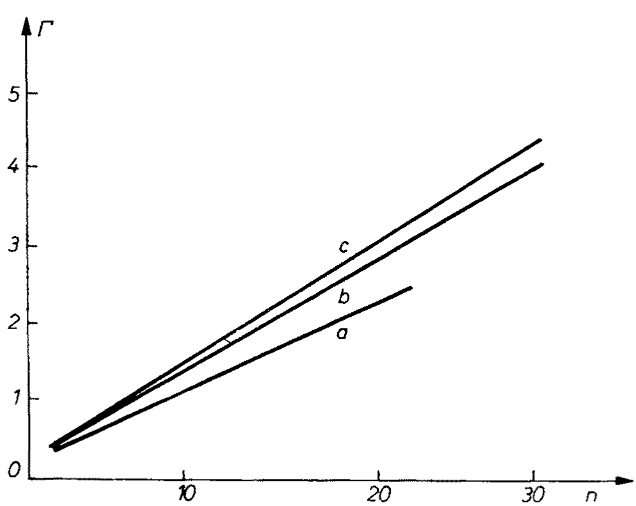

Fig. 5. Partial overlap slopes versus multiplicity in the Chew-Pignotti model at p_{lab}: *a*) 50, *b*) 300, *c*) 1500 GeV

t_i^L instead of t_i. The resulting average transverse momenta and other average values did not change more than 10% with respect to the results of calculations with amplitude given by Eq. (11). Thus it appears that t_i^T rather than t_i^L can be neglected.

From the fact that $t_i \cong t_i^L$ we conclude once more that amplitude does not depend on transverse momenta directly by t_i^T but mainly by transverse masses in t_i^L. Thus the random walk picture is, for the multiplicities close to and larger than average, completely false. The situation appears to be similar to that in the Uncorrelated Jet Model, though transverse momentum is cut more strongly when multiplicity increases. This last property implies that, when multiplicity increases, a) the average transverse momentum decreases, and b) the overlap slope increases (see Fig. 5). In the CŁA model the dependence of average p^T and slope on multiplicity is different (see Fig. 3) because $\langle \alpha_i \rangle$ decreases with multiplicity.

When comparing the results of the Monte Carlo calculations with CŁA and CP models at average total multiplicities for different energies (see Fig. 4), we can see that there is no substantial difference between them as far as the overlap function and its shrinkage are concerned. We conclude that flatness of the overlap function cannot be attributed to the particular CŁA parametrization but (in the framework of the models with single emission from each vertex) is a general feature of the multiperipheral kinematics.

4. *Cluster formation and the overlap function*

Let us summarize what we found when investigating Multiperipheral Models with single emission from each vertex.

We found that for multiplicities close to average: (*i*) invariant masses of particles neighbouring in the multiperipheral chain are small with respect to momentum transfers $\langle s_i \rangle \cong 0.5 < |\langle t_i \rangle|$; (*ii*) momentum transfers are much greater than transverse momentum square $\langle p_i^{T2} \rangle \ll |\langle t_i \rangle| \cong 1.5 \text{ GeV}^2$.

Both these results were already known from earlier analysis at lower energies [9, 10]. We would like to emphasize, however, that since these results come from high density of particles in the rapidity scale, they are expected to be energy independent for multiplicities close to average.

The slope of the overlap function is approximately proportional to $\langle n \rangle / |\langle t_i \rangle|$. Since $|\langle t_i \rangle| = 1.5 \text{ GeV}^2$, the resulting values of the slope are rather small, much smaller than required by comparison with elastic scattering data.

On the other hand, we can see from formula (3) that at 300 GeV even two peripheral exchanges with negligible t_i^L and α_i of order $1/(p^T)^2 = 5.5$ would give a slope of the right order of magnitude. Therefore it is possible that in the model in which particles form a few clusters in the rapidity space, we would get better behaviour of the overlap function. In such a model with many particle emission from one multiperipheral vertex, we would have a few really peripheral exchanges, instead of having many pseudo-reggeon exchanges with peripherality killed by the longitudinal transfer. In fact, cluster formation is the only way of obtaining really peripheral reggeons.

Finally in this Section let us notice that the CŁA model is not a clustering one as was suggested by Henyey [4]. He expects a large amount of clustering between particles, since the matrix element in the CŁA model has a minimum when all subenergies are small and equal. Such a situation is not confirmed by detailed investigations. A statistical analysis of event to event fluctuations of the longitudinal momenta applied to 2-, 3-, 4-, and 6-pion production at 28 GeV for proton-proton collision has shown [13] that the CŁA model is very close to the non-clustering reference (Uncorrelated Jet Model), especially for higher multiplicities.

This behaviour can be understood as follows. In the dominating factor exp $(\alpha_i t_i^{\mathrm{L}})$, α_i decreases with decreasing difference of rapidity of neighbouring particles. This can be interpreted as the existence of cluster forming attractive forces. On the other hand, the longitudinal transfer has opposite behaviour:

$$t_i^{\mathrm{L}} \cong -\langle m_i^{\mathrm{T}} \rangle^2 \frac{e^{y_{i+1}-y_i}}{(1-e^{-d})^2}, \tag{12}$$

(see Appendix), which can be interpreted as a source of repulsive forces in the rapidity scale. The net result is very close to the Uncorrelated Jet Model.

To sum up this Section let us assemble its main points.

i) The Multiperipheral Model with a single emission from each vertex leads to a much too flat overlap function because of the high density of particles in the rapidity scale. In this point we disagree with the conclusion reached in Refs [2–4].

ii) Since the introduction of clusters reduces the density of produced objects in the rapidity scale, it is natural to expect that it will improve the behaviour of overlap function. This conclusion is similar to that reached in Refs [2–4]. However, our motivation is entirely different, as can be seen from point (*i*) above. Furthermore, we feel that, for the time being, this idea is only a theoretical guess and should be checked by detailed calculations.

iii) Contrary to the opinion expressed in Ref. [4] the small value of the slope in the CŁA model is not caused by clustering effects but is typical for any multiperipheral amplitude with a single emission from each vertex.

5. *Conclusions*

It was shown by means of the Monte Carlo phase space calculations for pp collisions at energies 50, 300, and 1500 GeV that in the Chan-Łoskiewicz-Allison and Chew-Pignotti models without momentum dependent phases, the slope of the overlap function is four times smaller than was found experimentally. Shrinkage was found close to the experimental value.

These results are in disagreement with calculations based on the approximation in which longitudinal momentum transfers are neglected. We proved that this approximation (which is, incidentally, the basis of the random walk picture) cannot be justified for multiplicities close to and larger than average. Non-negligible longitudinal momentum transfers result from the high particle density in the rapidity scale and energy and momentum conservation.

Thus we conclude that in Multiperipheral Models with realistic particle density in the rapidity scale:

i) The slope of the overlap function is much smaller than that in the elastic scattering data,

ii) The random walk picture in impact parameter space is not valid.

We find it quite possible that the introduction of clusters would improve the behaviour of the overlap function.

The authors would like to thank Professor A. Białas for his help and encouragement. They also thank Dr K. Fiałkowski and Professor K. Zalewski for reading the manuscript and for critical remarks.

APPENDIX

We shall derive the formulae (7), (8), (10). Let us start from the definition of the longitudinal momentum transfer:

$$t_i^{\mathrm{L}} = \left(E_a - \sum_{k=1}^{i} E_k\right)^2 - \left(p_a^{\mathrm{L}} - \sum_{k=1}^{i} p_k^{\mathrm{L}}\right)^2. \tag{A1}$$

It can be rewritten in the form:

$$t_i^{\mathrm{L}} = \left(E_a - \sum_{k=1}^{i} E_k - p_a^{\mathrm{L}} + \sum_{k=1}^{i} p_k^{\mathrm{L}}\right)\left(E_a - \sum_{k=1}^{i} E_k + p_a^{\mathrm{L}} - \sum_{k=1}^{i} p_k^{\mathrm{L}}\right). \tag{A2}$$

Inserting momentum and energy conservation rules

$$E_a - \sum_{k=1}^{i} E_k = -E_b + \sum_{k=i+1}^{n} E_k,$$

$$p_a^{\mathrm{L}} - \sum_{k=1}^{i} p_k^{\mathrm{L}} = -p_b^{\mathrm{L}} + \sum_{k=i+1}^{n} p_k^{\mathrm{L}}, \tag{A3}$$

we obtain:

$$t_i^{\mathrm{L}} = -\left[E_a - p_a^{\mathrm{L}} - \sum_{k=1}^{i} (E_k - p_k^{\mathrm{L}})\right]\left[E_b + p_b^{\mathrm{L}} - \sum_{k=i+1}^{n} (E_k + p_k^{\mathrm{L}})\right]. \tag{A4}$$

Noting that

$$E_k + p_k^{\mathrm{L}} = m_k^{\mathrm{T}} e^{y_k},$$

$$E_k - p_k^{\mathrm{L}} = m_k^{\mathrm{T}} e^{-y_k},$$

where $m_k^{\mathrm{T}} = \sqrt{m_k^2 + (\vec{p}_k^{\mathrm{T}})^2}$ and y_k is rapidity, we can write the longitudinal transfer in the form:

$$t_i^{\mathrm{L}} = -\left(m_a e^{-y_a} - \sum_{k=1}^{i} m_k^{\mathrm{T}} e^{-y_k}\right)\left(m_b e^{y_b} - \sum_{k=i+1}^{n} m_k^{\mathrm{T}} e^{y_k}\right). \tag{A5}$$

We shall evaluate t_i^L in a specific case, under the following assumptions:

i) particles 1, 2, 3, ..., i are equally spaced in the rapidity scale by the distance d,

ii) the same holds for particles $i+1$, $i+2$, ..., n,

iii) all transverse masses are equal to the average value $m_k^T = \langle m^T \rangle$.

Under these assumptions the sums in (A5) can be explicitly evaluated:

$$\sum_{k=1}^{i} m_k^T e^{-y_k} = \langle m^T \rangle \frac{e^{-y_i} - e^{-y_1}}{1-e^{-d}},$$

$$\sum_{k=i+1}^{n} m_k^T e^{y_k} = \langle m^T \rangle \frac{e^{y_{i+1}} - e^{y_n}}{1-e^{-d}}. \tag{A6}$$

When the i-th particle is far from the ends of the multiperipheral chain, the following terms in (A5) and (A6) can be neglected:

$$e^{-y_a} \approx e^{-y_1} \ll e^{-y_i},$$

$$e^{y_b} \approx e^{y_n} \ll e^{y_{i+1}},$$

and we come to the approximation:

$$t_i^L \cong -\langle m^T \rangle \frac{e^{y_{i+1}-y_i}}{(1-e^{-d})^2}. \tag{A7}$$

Formula (8) is the particular case of (A7) when $y_i - y_{i+1} = d$.

Note added in proof:

After completion of this work the paper of Yasuo Matsumoto and Fujio Takagi *Momentum Transfer in Multiparticle Production and Validity of Multiperipheral Models* (*Phys. Rev.* **D9**, 3127 (1974)) was called to our attention. Their paper contains some of our results, concerning approximate evaluation of longitudinal momentum transfer.

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