ISOSPIN-INDUCED CORRELATIONS IN A LARGE CLASS OF MODELS FOR HIGH MULTIPLICITY EVENTS

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Assuming that in high-energy collisions only pions are produced and that the multi-pion wave function is completely symmetric in all momentum variables, we derive bonds for the different multiplicity-correlations between charged and neutral pions. It is found that in pp and in $\pi^{\pm}$p inelastic scattering the bound for f_{c0}, i. e., the correlation between charged and neutral pions, is violated by the experimental data. The conclusion is that the wave function cannot be fully symmetric. It is shown that with ϱ-production, i. e., with an antisymmetric wave function, the observed charged-neutral correlation can be explained.

In two previous papers [1, 2] it was shown how for many particle processes the exact conservation of isospin could be taken into account. A good fit was obtained for the topological cross sections in inelastic p–p collisions and a calculation of the dispersion of the charged multiplicity distribution gave the asymptotic formula of Wróblewski $D \sim \beta \overline{N}_c$, with $\beta = 0.58$. With a single component model [1, 3], however, it was impossible to get a positive value for the correlation between charged and neutral pions, i. e., for $f_{c0} = \overline{n_c n_0} - \overline{n}_c \overline{n}_0$. This was due to the fact that in these models the two-particle correlation $f_2 = \overline{n(n-1)} - \overline{n}^2$ is not large enough. Since $f_{2c} = \overline{n_c(n_c-1)} - \overline{n}_c^2$ and $f_{20} = \overline{n_0(n_0-1)} - \overline{n}_0^2$ are quite large and positive, one must therefore have a strong cancellation in the right hand side of the relation $f_2 = f_{2c} + f_{20} + 2f_{c0}$, causing f_{c0} to be negative. In the model of Ref. [2] three components were introduced, one for each value of the total isospin of the pions. In this way we could obtain a very broad distribution of the total number of pions and therefore a large value of f_2. This made it possible, at least in principle, to make f_{c0} positive. In this note, however, we will show, that for a large class of final states, this possibility is not realized, so that f_{c0} remains negative.

We will restrict ourselves to many particle final states, which are completely symmetric for the interchange of any two pion momenta. For instance, the production of pions, three at a time, with total isospin zero, is therefore excluded, since such a triplet has a

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completely antisymmetric wave function. For the same reason production of pion-pairs with $I = 1$ (ϱ-mesons) does not qualify. With this restriction of symmetry the most general pion state with $I = l$ and $I_3 = m$, which can be produced when two nucleons with momenta $\vec{p}_1$ and $\vec{p}_2$ collide to give two other nucleons with momenta $\vec{q}_1$ and $\vec{q}_2$, is

$$|lm\rangle = \sum_{n=0}^{\infty} \frac{1}{n!} \int d\vec{\tau} Y_{lm}(\vec{\tau}) \int d_4k_1 \dots d_4k_n \Psi_l(p_1 p_2 q_1 q_2 k_1 \dots k_n) \times$$

$$\times \delta_4(p_1 + p_2 - q_1 - q_2 - k_1 \dots - k_n) \prod_{i=1}^{n} [\sqrt{2k_{0i}}\, \delta(k_i^2 - m^2)\, \theta(k_{i0})\, (\vec{\tau} \cdot \vec{a}^*(k_i))]\, |0\rangle. \quad (1)$$

Here $\vec{\tau}$ is a vector of unit length. The dependence of $|lm\rangle$ on $\vec{p}_1$, $\vec{p}_2$, $\vec{q}_1$ and $\vec{q}_2$ has not been explicitly indicated. We further write (pp) for a state of two protons with momenta $\vec{q}_1$ and $\vec{q}_2$ and similarly $(1/\sqrt{2})\,(pn+np)$ for an $I = 1$, $I_3 = 0$ state of a proton and a neutron with momenta $\vec{q}_1$ and $\vec{q}_2$. For a proton-proton collision we can now construct the most general final state, which is completely symmetric in the pions and which has the same isospin as the initial state. Neglecting baryon pair production this state is

$$|\text{final}\rangle = \int d\vec{q}_1\, d\vec{q}_2 \Bigg[A(pp)|00\rangle + B\left\{\frac{1}{\sqrt{2}}\,(pp)\,|10\rangle - \frac{1}{2}\,(pn+np)\,|11\rangle\right\} +$$

$$+ C\left\{\frac{1}{\sqrt{10}}\,(pp)\,|20\rangle - \sqrt{\frac{3}{20}}\,(pn+np)\,|21\rangle + \sqrt{\frac{3}{5}}\,(nn)\,|22\rangle\right\} + \frac{1}{\sqrt{2}}\,D(pn-np)\,|11\rangle \Bigg]. \quad (2)$$

For fixed nucleon momenta $\vec{q}_1$ and $\vec{q}_2$ this state has exactly the same form as the corresponding state in Ref. [1] and [2]. There, however, the state $|lm\rangle$ did not contain the meson momenta and was defined as

$$|lm\rangle = N_l^{-1/2} \sum_{n=0}^{\infty} \frac{C_n(l)}{n!} \int d\vec{\tau} Y_{lm}(\vec{\tau})\, (\vec{\tau} \cdot \vec{a}^*)^n\, |0\rangle. \quad (3)$$

For the calculation of momentum distributions we need of course the states of Eq. (1). If, however, we are interested only in multiplicity distributions, the states (1) and (3) will give identical results, provided we take

$$N_l^{-1}\, |C_n(l)|^2 = \int d\vec{q}_1\, d\vec{q}_2\, d_4k_1 \dots d_4k_n\, |\Psi_l(p_1 p_2 q_1 q_2 k_1 \dots k_n)|^2 \times$$

$$\times \delta_4(p_1 + p_2 - q_1 - q_2 - k_1 \dots - k_n) \prod_{i=1}^{n} \delta(k_i^2 - m^2)\, \theta(k_{i0}). \quad (4)$$

In Ref. [2] it was shown that all multiplicity distributions and correlations for charged and for neutral particles could be expressed in terms of the coefficients A, B, C, and D and the three distributions $P_l(n)$ ($l = 0,1$ and 2) for the total multiplicity. Since these distributions are proportional to $N_l^{-1}|C_n(l)|^2$, they can now be calculated as the phase space integral occurring in the right hand side of Eq. (4). In particular this can now explain (see e. g. Ref. [4]) why the average multiplicity for each value of the isospin $I = 0,1$ and 2 rises logarithmically with $s = (p_1 + p_2)^2$, while in [1] and [2] this was external input.

As a consequence the formulas for the multiplicity-averages and -correlations, as derived in [1] and [2] are now also valid for the more general final state of Eq. (2). For later reference we quote some of the results obtained in [1] and [2]. If the numbers v_I and $w_I (I = 0, 1$ and $2)$ are defined by $\langle N_I \rangle = \sum n P_I(n) = v_I \langle n \rangle$ and $\langle N_I^2 \rangle = \sum n^2 P_I(n) = w_I \langle n^2 \rangle$, where n is the total number of pions and the averages are taken over the state (2), then we can show that in addition to the normalization condition

$$|A|^2 + |B|^2 + |C|^2 + |D|^2 = 1 \tag{5}$$

there exist also the following relations between the v_I and w_I

$$|A|^2 v_0 + (|B|^2 + |D|^2) v_1 + |C|^2 v_2 = 1 \tag{6}$$

and

$$|A|^2 w_0 + (|B|^2 + |D|^2) w_1 + |C|^2 w_2 = 1. \tag{7}$$

We can now express several averages and correlations in terms of A, B, C, D, v_I and w_I. It actually turns out that they depend only on the following combinations of these parameters $\alpha_0 = |A|^2 + 2|B|^2 - |D|^2$, $\alpha_1 = |A|^2 v_0 + 2|B|^2 v_1 - |D|^2 v_1$ and $\alpha_2 = |A|^2 w_0 + 2|B|^2 w_1 - |D|^2 w_1$. We find for example

$$\langle n_0 \rangle = \tfrac{1}{15} (4 + \alpha_1) \langle n \rangle - \tfrac{1}{10} (1 - \alpha_0) \tag{8}$$

and

$$\langle n_c \rangle = \tfrac{1}{15} (n - \alpha_1) \langle 11 \rangle + \tfrac{1}{10} (1 - \alpha_0), \tag{9}$$

for the average number of neutral and charged pions. Similar expressions can be obtained for f_{2c} and f_{c0}. For the latter, in particular, we find

$$f_{c0} = \tfrac{1}{105} (13 + \alpha_2) \langle n^2 \rangle - \tfrac{1}{225} (4 + \alpha_1)(11 - \alpha_1) \langle n \rangle^2 + O(\langle n \rangle). \tag{10}$$

In the same way also the parameter β in Wróblewski's relation $D_c \simeq \beta \langle N_c \rangle$ can be expressed in terms of α_1, α_2 and $\xi = \langle n^2 \rangle / \langle n \rangle^2$. This expression for β can be used to eliminate $\langle n^2 \rangle$ from Eq. (10) and in this way we obtain

$$f_{c0} \simeq \frac{(11 - \alpha_1) \langle n \rangle^2}{1800 (8 - \alpha_2)} [(13 + \alpha_2)(11 - \alpha_1)(\beta^2 + 1) - 8(4 + \alpha_1)(8 - \alpha_2)]. \tag{11}$$

For any one-component model one has $\alpha_0 = \alpha_1 = \alpha_2 = \alpha$ and it is easy, using Eq. (11), to find the region in the $\alpha - \beta^2$ plane where f_{c0} is positive. This region is indicated in Fig. 1. Since the experimental value of β^2 is $\beta^2 = 0.36$ and α lies between -1 and $+2$ (as follows from the relations (5), (6) and (7)), it is clear from this figure that, with the assumed symmetry of the wave function, a one-component theory can never give a positive f_{c0}. For the more general case α_1 and α_2 lie between -1 and $+2$, but not all points of this square are allowed. It can be shown that, due to Eqs (5), (6) and (7), only the area inside the dashed lines of Fig. 2 belongs to the parameter space in the $\alpha_1-\alpha_2$ plane. In the same figure and for a number of β-values we have drawn the (dashed) curves for which $f_{c0} = 0$. To the right of such a curve f_{c0} is positive, to the left it is negative. For the experimental value $\beta = 0.58$ only the shaded area gives a positive correlation. The experiments,

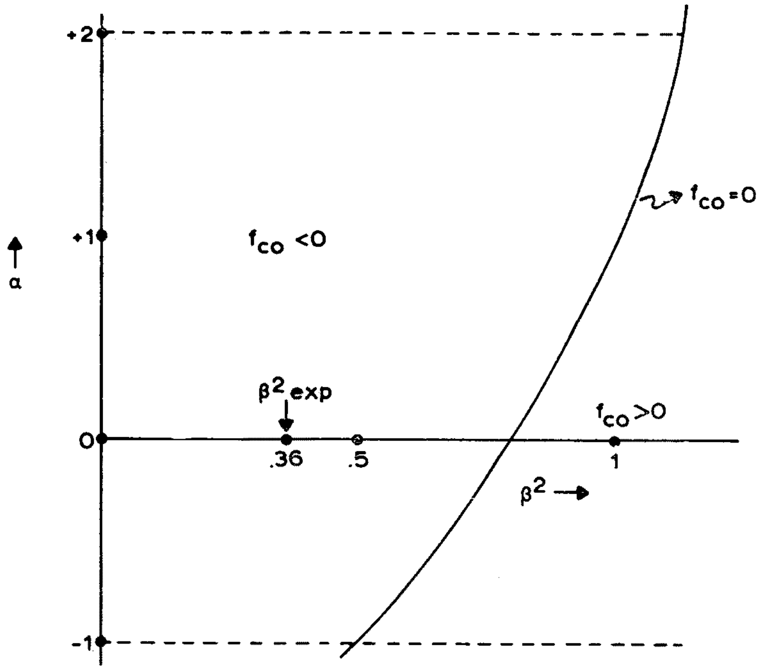

Fig. 1. Partition of the $\alpha-\beta^2$ plane into regions where $f_{c0} < 0$ and $f_{c0} > 0$ for the pp-case

however, also indicate that $\langle n_0 \rangle$ is greater than 0.5 $\langle n_c \rangle$. From Eqs (8) and (9) it follows immediately that in the high energy limit this implies that $\alpha_1 > 1$. From Fig. 2 we then see that for the experimental value $\beta = 0.58$ there can be no positive correlation f_{c0}, if we still assume that all many pion wave functions are completely symmetric, like e. g. in all independent emission models. The authors of Ref. [5] get a positive f_{c0} for an independent emission model because they use the Cerulus [6] weight factors to express the isospin independence. This is not to say that also for states with another symmetry the Cerulus weight factors will give erroneous results.

Using the states as defined in Ref. [1] we can perform a similar analysis for $\pi^\pm$p-scattering and for $p\bar{p}$-annihilation. For π^-p-scattering (Eq. (8) in Ref. [1]) there are two parameters, $\alpha_1 = |A'|^2 v_0 + |B'|^2 v_1$ and $\alpha_2 = |A'|^2 w_0 + |B'|^2 w_1$, in terms of which we find for the average high energy multiplicities

$$\langle n_0 \rangle = \tfrac{1}{15}(7-2\alpha_1)\langle n \rangle \tag{12}$$

and

$$\langle n_c \rangle = \tfrac{1}{15}(8+2\alpha_1)\langle n \rangle. \tag{13}$$

The parameter α_1 (and also α_2) is restricted to the interval (0,1). In order to reproduce the experimental result $\langle n_0 \rangle \approx \frac{1}{2} \langle n_c \rangle$, we see from Eqs (12) and (13) that α_1 must be close to its maximally allowed value of one, which can be achieved only when the $I = \frac{3}{2}$ states are suppressed, relative to the $I = \frac{1}{2}$ states. The experimental value of the correlation param-

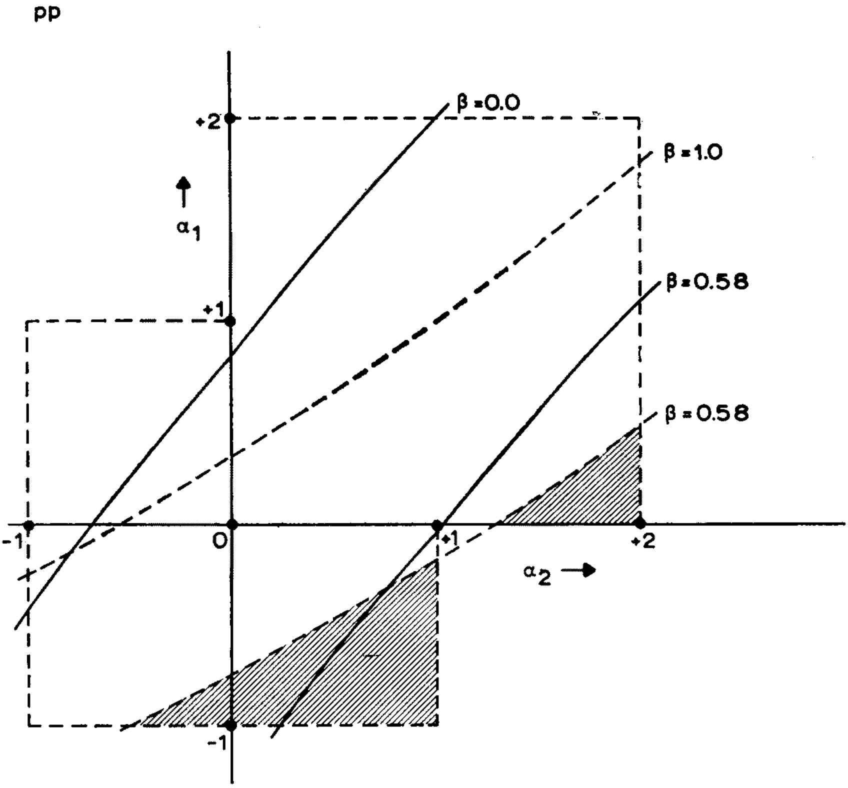

Fig. 2. For the symmetric states f_{c0} is positive to the right of the dashed lines for each β. For the "ϱ-model" f_{c0} is positive to the left of the solid lines for each β

eter f_{c0} is positive. Theoretically, if we allow only symmetric pion wave functions, it is given by

$$f_{c0} \simeq \frac{4+\alpha_1}{225\,(5+2\alpha_2)} \left[(\beta^2+1)\,(4+\alpha_1)\,(8-\alpha_2) - 2(7-2\alpha_1)\,(5+2\alpha_2)\right] \langle n \rangle^2, \qquad (14)$$

where β has the same meaning as before and is again about equal [5, 7] to 0.6. In the $\alpha_1 - \alpha_2$ plane of Fig. 3 we have plotted the contours $f_{c0} = 0$ for a number of values of β. Since f_{c0} is positive to the left of these contours, we see that also for π^-p-collisions a positive f_{c0} can be obtained only for rather unrealistic values of α_1 and α_2.

For π^+p-collisions (Eq. (10) of Ref. [1]) we find $\langle n_0 \rangle = \frac{1}{4} \langle n_c \rangle$ and $f_{c0} = \frac{4}{75}(2\beta^2 - 1)\langle n \rangle^2$. Here we obtain not only a negative f_{c0} for $\beta \approx 0.6$ (which disagrees with experiment), but already the average number of neutral pions is much too small [8].

At last we consider $p\bar{p}$-annihilation into a final state (Eq. (12) of Ref. [1])

$$|p\bar{p}\rangle = X|00\rangle + Y|10\rangle. \tag{15}$$

With $\alpha_1 = |X|^2 v_0$ and $\alpha_2 = |X|^2 w_0$, we get

$$\langle n_0 \rangle \simeq \tfrac{1}{15}(9-4\alpha_1)\langle n \rangle, \tag{16}$$

$$\langle n_c \rangle \simeq \tfrac{1}{15}(6+4\alpha_1)\langle n \rangle \tag{17}$$

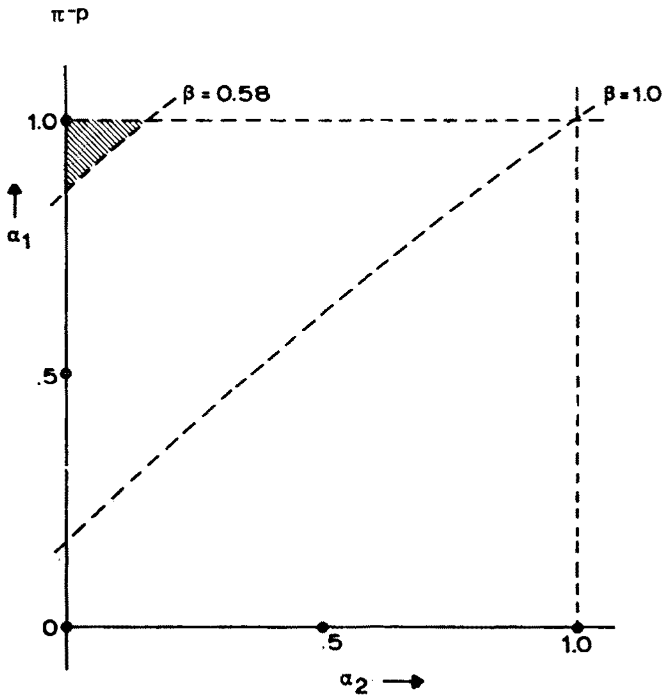

Fig. 3. f_{c0} is negative to the right of each line. For β equal to the experimental value f_{c0} is positive only in the shaded area

and

$$f_{c0} \simeq \frac{1}{225}\,\frac{3+2\alpha_1}{3+4\alpha_2}\,[(\beta^2+1)(9-2\alpha_2)(3+2\alpha_1)-2(9-4\alpha_1)(3+4\alpha_2)]\langle n \rangle^2. \tag{18}$$

The parameter α_1 should again be close to unity. A recent compilation [9] gives $\beta \approx 0.42$ for $p\bar{p}$-annihilation. A look at Fig. 4 shows that with this β the value of f_{c0} is negative in the greater part of the $\alpha_1 - \alpha_2$ plane $(0 \leqslant \alpha_i \leqslant 1)$. If therefore a negative f_{c0} is measured, this will not be inconsistent with independent pion emission in $p\bar{p}$-annihilation. At present no conclusive data are available to us.

Leaving the $p\bar{p}$-case aside, we can conclude from the above analysis that the presently available data are sufficient to rule out a symmetric pion wave function. In particular independent pion emission is in disagreement with experiment.

These conclusions will be changed completely when in the states of Eq. (1) we consider $\vec{a}^* (\vec{k}_i)$ as the operator which creates not a pion, but a composite object with momentum $\vec{k}_i$ and isospin one. For instance, if we take these objects to be ϱ-mesons with the decay modes

$$\varrho_+ \rightarrow \pi_+\pi_0,\ \varrho_- \rightarrow \pi_-\pi_0,\ \varrho_0 \rightarrow \pi_+\pi_-$$

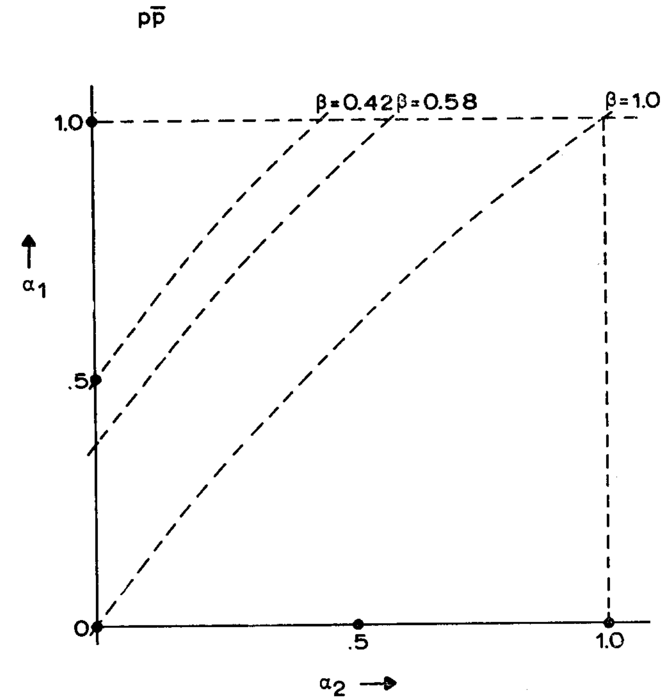

Fig. 4. f_{c0} is negative to the right of each line

then for each event we have the identities

$$n = 2N,\ n_c = 2N_0 + N_c,\ n_0 = N_c, \tag{19}$$

where the capital N's refer to ϱ-mesons and the small n's to pions. For the averages and correlations of the ϱ-mesons we then have exactly the same formulas as before for the pions. For the latter we get for pp-scattering

$$\langle n_c \rangle = 2\langle N_0 \rangle + \langle N_c \rangle = \tfrac{1}{30}(19+\alpha_1)\langle n \rangle - \tfrac{1}{10}(1-\alpha_0), \tag{20}$$

$$\langle n_0 \rangle = \tfrac{1}{30}(11-\alpha_1)\langle n \rangle + \tfrac{1}{10}(1-\alpha_0). \tag{21}$$

We can apply the same procedure to get for the correlation parameter f_{c0}

$$f_{c0} = \frac{19+\alpha_1}{900(44+5\alpha_2)}[\tfrac{3}{2}(\beta^2+1)(15-\alpha_2)(19+\alpha_1) - $$

$$-(11-\alpha_1)(44+5\alpha_2)]\langle n \rangle^2 + O(\langle n \rangle), \tag{22}$$

where β again is the coefficient in the Wróblewski relation for the dispersion of the charged pions. For some β-values we have drawn (Fig. 2) the curves for which $f_{c0} = 0$ (in the figure these are the solid ones). Now f_{c0} is positive to the left of such a curve and we see that for the experimental value of $\beta = 0.58$ f_{c0} is positive for the greater part of the $\alpha_1 - \alpha_2$ plane, in particular for $\alpha_1 \simeq 1$ which is again the value for which $\langle n_0 \rangle \simeq 0.5 \langle n_c \rangle$. The same conclusion also holds for $\pi^{\pm}$p and $\bar{\text{p}}$p-scattering. This follows directly from the identity

$$f_{c0} = 2F_{c0} + F_{2c} \tag{23}$$

between the correlation parameters for pions and ϱ-mesons, which can be derived from (19). In (23) we have for the correlation parameters for ϱ-mesons F_{2c} and F_{c0} the same formulas as obtained for the pions in the symmetric model. Because F_{2c} is quite large and positive, while F_{c0} is small and negative, f_{c0} will be positive in a large part of the $\alpha_1 - \alpha_2$ plane. Summarizing we can conclude that any model in which the wave function of the pions is symmetric with respect to the interchange of two pion momenta can never explain the observed positive charged-neutral correlation. This means that one has to introduce cluster production (e. g. ϱ-meson production as was done in the last part of this paper) to explain this positive correlation.

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