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THEORY CHALLENGES AT FUTURE LEPTON COLLIDERS* **

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High energy, high luminosity, future lepton colliders, circular or linear, may possibly give us a hint about fundamental laws of Nature governing at very short distances and very short time intervals, the same which have brought our Universe to live. Currently considered projects are on the one hand, linear electron–positron colliders, which offer higher energy and lower beam intensities and on the other hand, circular electron–positron colliders, limited in energy but offering tremendous interaction rates. On the far future horizon, muon circular colliders are the only viable projects which can explore > 10 TeV teritory of the lepton colliders. Experiments in all these future colliders will require theoretical calculations, mainly of the Standard Model processes (including QED), at the precision level one or even two orders better than available today. After brief characterization of theory puzzles in the fundamental interactions, we shall overview main challenges in the precision calculations of the Standard Model effects, which have to be removed from data in order to reveal traces of new unexpected phenomena.

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1. Introduction

High-energy colliders considered for the future construction and exploitation would collide hadron (proton) beams or lepton (electron, muon) beams. What are presently the main proposals for the future lepton colliders worldwide? The leading candidates are: the circular e^+e^- collider FCC-ee [1, 2] in CERN delivering huge crop of events from 150 inverse attobarns (150 ab^{-1}) at 91 GeV to 1.5 ab^{-1} at 365 GeV [3], the compact linear e^+e^- collider CLIC in CERN hopefully providing 1 ab^{-1} at 380 GeV and up to 2.5 ab^{-1} at

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1.5 TeV, international linear collider ILC in Japan which may deliver 2 ab^{-1} at 250 GeV and 4 ab^{-1} at 500 GeV, and finally, another circular e^+e^- collider CEPC in China which would get 16 ab^{-1} at 91 GeV and 6 ab^{-1} at 240 GeV [4]. Muon circular colliders with $\mu^+\mu^-$ beams will remain the only option for another lepton collider at energies $\sim$ 15 TeV, thanks to much weaker energy loss due to bremsstrahlung. The most precise measurement of cross sections, asymmetries, extremely rare decays would come from the circular collider FCC-ee. It will be able to provide 5×10^{12} Z-boson decays, 10^8 WW events, 10^6 HZ events, and 10^6 top-quark pairs [3]. This is several orders of magnitude more than in previous similar experiments (as compared to numbers of Z and WW at LEP), reducing experimental errors of cross sections, asymmetries, masses, decay rates by the factor of 10–100. As we shall see in the following, it will be an enormous challenge to perform theoretical calculations for these observables, within the Standard Model (SM) and beyond, in order to match the above anticipated experimental precision. In this short note, we shall be able to overview only the main problems of the above theory challenges.

2. Puzzles of the fundamental physics

Successful experimental verification of the SM of the electroweak and strong interactions, the absence of direct signs of new physics at multi-TeV experiments at the proton–proton collider LHC, discovery of striking new properties of neutrinos, and a wealth of new observations in astrophysics, result in a number of burning questions on the nature of the fundamental laws governing our Universe [5]. Theorists are deeply worried that the Nature has different opinion about the "naturalness" then we do: Higgs dynamics at the scale of the electroweak symmetry breaking requires to be protected by the very "un-natural" fine tuning of the dynamics at higher energies (shorter distance). This is also called a "hierarchy problem". Moreover, we have no clue why do we have three families quark and leptons and there is no systematic theoretical explanation of their masses and mixings. Recent discovery of the neutrino masses and mixings add to the confusion. Meeting point of gravity and quantum mechanics is still not understood. According to accumulated knowledge, what we see as today's Universe was shaped to a large extent at the end of the "inflation era" in the early stage of the Big Bang, but we do not know the origin and the details of the inflation, except that it has to be closely related to Higgs dynamics [6]. The mechanism of producing striking matter–antimatter asymmetry in the present Universe still begs for explanation. Better knowledge of the Higgs potential parameters would be valuable for the inflation modelling. In particular, measurement of the triple Higgs coupling with precision $\sim 1\%$ in the collider experiment would be of great interest for astrophysics. The existence of the abundant dark matter everywhere in the Universe, interacting gravitationally with the ordinary matter, is another great puzzle. Last but not least, the hypothetic dark energy speeding up the expansion of the Universe remains completely unexplained.

There is presently no satisfactory theory candidate, which could explain the above puzzling phenomena and new hints from experiments are badly needed. Experiments in high-energy colliders are the most promising source of such a hint. At high-energy colliders, one may possibly see new particles and/or discover new interactions of the known particles — in particular, decays of known unstable particles into forbidden final states could provide a valuable hint. Very precise measurements of the properties of the known particles may depart from the SM predictions, signalling new types of forces, or existence of unknown much heavier particles.

3. Precision measurements of the electroweak observables

The role of the SM theoretical predictions for the future lepton collider measurements will be different from the past role. In the past, testing and verifying SM was the main aim. In the future experiments, SM will be assumed to be correct, while searching for the deviation of the SM predictions from data as a sign of new physics will be the main objective. SM will be the tool and not the aim. Perturbative calculations within the SM are commonly organised in such a way that internal (Lagrangian level) SM parameters[1] are determined using limited number of the *SM input parameters*. Typically, they are observables known most precisely, for instance Z mass, the electromagnetic coupling $\alpha_{\rm QED}(M_Z)$, Fermi constant G_μ and the strong coupling constant $\alpha_{\rm S}$. Mass of the t quark plays an important role as an input parameter. All other observables, cross sections, asymmetries, masses of W boson and Higgs boson H, width and decay rates of Z, W and H can be calculated perturbatively, in principle, with an arbitrary precision and will be confronted with the experiment [7].

The SM input parameters are known with a certain experimental error, which propagates to all SM predictions, and are called *parametric errors*. The additional uncertainty of the SM predictions due to technical uncertainties of the perturbative calculations (uncalculated higher orders, numerical problems) are commonly referred to as *intrinsic errors*. The map of errors in the SM calculations is shown schematically in Fig. 1.

Alternatively, all observables can be treated in the same way in the global fit of all observables to the SM, without any of them playing a privileged role of the SM input parameters.

For more discussion, the reader may consult Refs. [7–10].

[1] Most important are three gauge coupling constants, EW symmetry breaking scale and mass of the heaviest t quark. Fermion masses and mixings also have to be added.

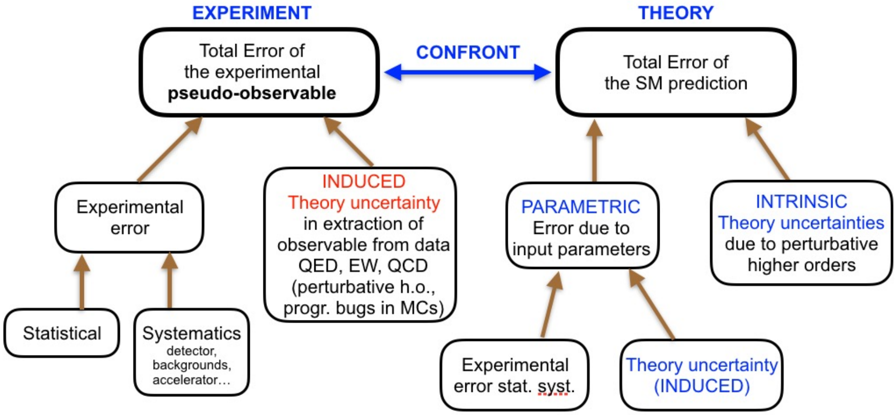

Fig. 1. Scheme of the error propagation in the SM calculations.

4. High precision SM calculations — the role of resummation

First order SM perturbative corrections split nicely into sum of three groups: "photonic" QED real and virtual (loop) corrections, pure EW loop corrections with heavy particles and QCD corrections. Beyond the 1$^{\rm st}$ order, this split gets fuzzy due to mandatory use of soft and collinear resummation of QED higher orders, renormalization group use in QCD and the presence of QCD insertions in the EW multiloops. Nevertheless, it is useful to maintain it in practice as far as it is possible.

Let us briefly characterize genuine EW loop corrections, omitting QCD component from our discussion, while on the QED class, we shall elaborate in a more detail in the following sections.

The EW loop corrections are relatively small, of the order of $\sim 1\%$, as compared to QED effects which are of the $\sim 10\%$ order. The $\mathcal{O}(\alpha^1)$ pure EW loop corrections to $e^+e^- \to f\bar{f}, W^+W^-$ processes were completed at the start of LEP experiments in 1989 by several groups, but only two calculations embedded in the codes DIZET [11] (ZFITTER [12]) and TOPAZ0 [13] were used in the LEP data analysis [14, 15]. It took another decade to complete most of $\mathcal{O}(\alpha^2)$ corrections to $e^+e^- \to f\bar{f}$ process [16, 17], but only recently missing bosonic 2-loop corrections to $e^+e^- \to f\bar{f}$ process were calculated [18, 19]. Generally, pure EW corrections are harder to calculate than QCD or QED corrections, because of their multi-scale character — with masses of gauge bosons, Higgs boson and all fermions spanning over the entire range from 0.5 MeV to 175 GeV, hence one cannot profit from smallness of some of them like in QCD or QED. Consequently, the phase space of loop integrals has to be calculated without any approximations. Spontaneous symmetry breaking and more complicated gauge group add to the problems. In most complicated cases, like bosonic 2-loops analytical calculations, analytical integrations are not feasible — only numerical integration methods are able to cope [18, 19].

In view of the 0.003% precision for some observables near Z peak in FCC-ee complete calculations of the $\mathcal{O}(\alpha^3)$ EW corrections, including 3-loop amplitudes will be needed. It looks that again only numerical integration methods may work for the integrations over 3-loop virtual phase space [7]. Such calculations will be rather slow and will have limited numerical precision, but it is argued that 2-digit precision is good enough.

As soon as the complete $\mathcal{O}(\alpha^1)$ EW corrections to $e^+e^- \to f\bar{f}$ process have been available, it was quite clear that their practical usefullness for the analysis of the LEP data near Z peak is severely limited due to numerically huge size of the pure QED component. Even the upgrade of QED part to $\mathcal{O}(\alpha^2)$ was not sufficient — only after the inclusion of soft photon resummation, the desired theoretical precision $\sim 0.1\%$ for this process near the Z resonance was attained [20] (similarly $\sim 0.5\%$ precision for the W-pair production process). In other words, the conservative order-by-order perturbative approach does not work in practice — one has to go to much higher perturbative orders for QED and QCD subclass of correction (even to infinite order for soft photons) than for the genuine pure EW parts. The immediate question is, therefore, how to disentangle in a systematic way the QED part and the so-called pure EW corrections, performing IR cancellations within the soft photon resummation. This question is especially intriguing beyond the $\mathcal{O}(\alpha^1)$, where in a single diagram, both photonic QED part and genuine EW parts may show up simultaneously.

The solution of this problem is described and implemented in the so-called CEEX scheme, see Refs. [21, 22], see also chapter C in Ref. [23]. In the KKMC program [21, 22], the pure EW (non-soft) corrections are complete to $\mathcal{O}(\alpha^1)$, QED corrections are complete to $\mathcal{O}(\alpha^2)$ and soft photon corrections are resummed to infinite order. The same scheme will work at higher orders, for instance, for genuine $\mathcal{O}(\alpha^2)$ or $\mathcal{O}(\alpha^3)$ EW corrections combined with sufficiently higher order complete QED non-soft corrections and soft-photon resummation. Moreover, this technique is implementable in the form of the Monte Carlo event generators which can provide SM predictions for arbitrary experimental cut-offs and/or detector efficiencies. It is formulated using spin amlitudes, so it works perfectly well for polarized initial and final particles. It can also accommodate resummation of the coherent initial–final state interferences [24] for narrow neutral/charged resonances, and can also deal with multiple photon emission from unstable charged particles before they decay [25].

However, in order to profit from the above technique, one has to calculate multi-loop SM corrections with QED component in a special way. For instance, in the $\mathcal{O}(\alpha^2)$ SM calculations for $e^+e^- \to l^+l^-$ process, *one should not use the Bloch–Nordsieck technique* of cancelling IR singularities by means of adding *(i)* IR-divergent 2-loop contribution with one virtual photon line and *(ii)* fully exclusive one-loop EW amplitudes for to $e^+e^- \to l^+l^-\gamma$ subprocesses (without IR singularity inside the virtual part)[2]. In the CEEX scheme [21, 22], the well-known IR component is *subtracted* from the both above corrections, because the IR cancellation is executed independently within the soft photon resummation part of the calculation.

5. QED challenges at FCC-ee precision

Trivial but numerically sizable pure QED effects were removed from all LEP data (observables) such as Z and W masses and widths, cross sections, asymmetries, decay widths. However, this was resulting in the induced QED uncertainty in the experimental errors of these observables. These QED uncertainties were usually at least factor three smaller than the genuine statistical and systematic experimental errors. This can be seen in the 3$^{\rm rd}$ column of Table I taken form Ref. [27]. Next columns in the table show the enormous progress, up to factor 100, in experimental errors to be attained in FCC-ee experiments. Obviously, the precision of the theoretical QED calculations has to progress at least to the level of the FCC-ee experiments. The corresponding minimal improvement factor of the QED calculations is shown in the last column of Table I. In fact, this factor has to be about three times better, if we want to be in the same comfortable situation as in LEP. The same information as in Table I is also visualised in Fig. 2.

Before we can discuss whether the desired improvement of the QED calculations shown in Table I and Fig. 2 is feasible, one has to answer even more basic and highly non-trivial question whether the methodology of removing QED effects from the listed observables, which was used at LEP data analysis, will still work at the tremendous precision of FCC-ee. This question will be briefly elaborated in the next section — for more detailed discussion, we refer the reader to Ref. [27].

The question whether improvement factors of QED calculations in Table I are achievable is discussed at length in Ref. [27]. Here, let us only summarize briefly on that in the following. The important point is that contrary to LEP data analysis, where semi-analytical programs like ZFITTER or TOAPZ0 have played major role, at FCC-ee only Monte Carlo calculations will be able to provide cut-off-dependent SM predictions with sufficient

[2] As it is done for instance in Ref. [26].

TABLE I

Table of electroweak observables most sensitive to QED effects from Ref. [27]. The LEP experimental errors (3rd column) are accompanied in the braces $\{\dots\}$ by the induced QED uncertainties. The FCC-ee experimental systematic errors in 4th column are from FCC-ee CDR [28] except τ polarisation [29]. The improvement factor in QED theoretical calculations needed to equalize with experimental precision of FCC-ee measurements is shown in the last column.

Observable	From	Present (LEP)	FCC stat.	FCC syst.	$\frac{\text{Now}}{\text{FCC}}$
M_Z [MeV]	Z linesh.	$91187.5\pm2.1\{0.3\}$	0.005	0.1	3
Γ_Z [MeV]	Z linesh.	$2495.2\pm2.1\{0.2\}$	0.008	0.1	2
$R_l^Z = \Gamma_h/\Gamma_l$	$\sigma(M_Z)$	$20.767\pm0.025\{0.012\}$	6×10^{-5}	1×10^{-3}	12
$\sigma^0_{\rm had}$ [nb]	$\sigma^0_{\rm had}$	$41.541\pm0.037\{0.025\}$	0.1×10^{-3}	4×10^{-3}	6
N_ν	$\sigma(M_Z)$	$2.984\pm0.008\{0.006\}$	5×10^{-6}	1×10^{-3}	6
N_ν	$Z\gamma$	$2.69\pm0.15\{0.06\}$	0.8×10^{-3}	$< 10^{-3}$	60
$\sin^2\theta_W^{\rm eff}\times10^5$	$A_{\rm FB}^{\rm lept}$	$23099\pm53\{28\}$	0.3	0.5	55
$\sin^2\theta_W^{\rm eff}\times10^5$	$\langle\mathcal{P}_\tau\rangle$, $A_{\rm FB}^{{\rm pol},\tau}$	$23159\pm41\{12\}$	0.6	< 0.6	20
M_W [MeV]	ADLO	$80376\pm33\{6\}$	0.5	0.3	12
$A_{{\rm FB},\mu}^{M_Z\pm3.5\,{\rm GeV}}$	$\frac{{\rm d}\sigma}{{\rm d}\cos\theta}$	$\pm0.020\{0.001\}$	1.0×10^{-5}	0.3×10^{-5}	100

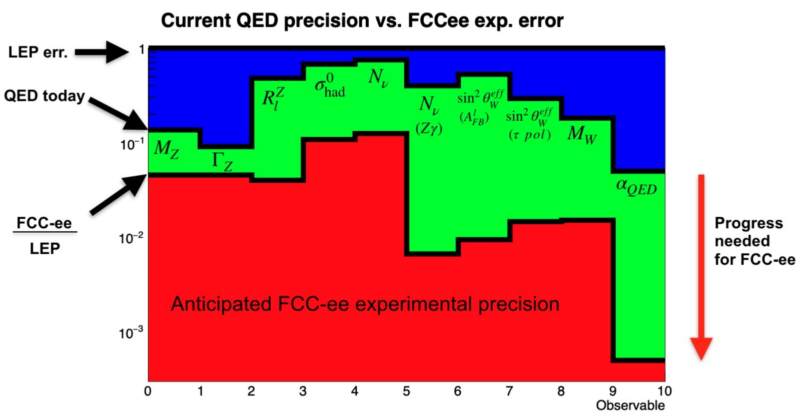

Fig. 2. QED challenges at FCC-ee of Table I in a graphical form.

precision[3]. Moreover, fitting data to full SM prediction or to "effective SM parametrization" of data in the form of EW pseudo-observables (see next section) will be also done using MC programs.

[3] Perhaps with the only exception of total hadronic cross section.

Just to give a few examples, near the Z peak improvements in precise measurement of the hadronic total cross section providing experimental $\delta M_Z, \delta\Gamma_Z \leq 0.1$ MeV will require QED to be reduced to ≤ 0.03 MeV *i.e.* by a factor 10. Better modelling of light fermion production and the inclusion of $\mathcal{O}(\alpha^2 L_e^0, \alpha^3 L_e^2, \alpha^4 L_e^4)$, $L_e = \ln(s/m_e^2)$ initial state QED corrections will be mandatory. Data analysis for final leptonic states near Z resonance will be more demanding. In the MC programs of the KKMC class with CEEX matrix element, at least the inclusion of $\mathcal{O}(\alpha^2 L_e)$ penta-boxes and of $\mathcal{O}(\alpha^3 L_e^3)$ photonic corrections will be necessary. Provisions for SM parameter fitting and extracting EWPOs from data will have to be included in the MC programs. Measurement of charge asymmetry with the experimental error $\delta A_{\rm FB}^{\mu}(M_Z) \simeq 1 \times 10^{-5}$, leading to $\delta \sin^2\theta_W^{\rm eff} \simeq 0.5 \times 10^{-5}$ will require factor 50–150 improvement in the control of QED effects. Such improvements are particularly urgent for the Bhabha process. Similarly, the anticipated experimental error $\delta \sin^2\theta_W^{\rm eff} \simeq 0.6 \times 10^{-5}$ from spin asymmetry measurements in tau pair production and decay at FCC-ee will require factor 20–60 better understanding of QED effects. As it is seen in Table 3 in Ref. [9], the precision of the QED coupling constant $\alpha_{\rm QED}(M_Z)$, as an input in the SM calculations, is critical for precision of all SM predictions [9]. In Ref. [30], it was proposed to extract $\alpha_{\rm QED}(M_Z)$ from the measurment of $A_{\rm FB}(M_Z \pm 3.5$ GeV) with precision of 3×10^{-5}, that is factor 200 more precisely than at LEP. However, the QED initial–final state interference IFI is here the main obstacle! While IFI cancels partly in the difference of $A_{\rm FB}(M_Z \pm 3.5$ GeV), the 1% effect remains in $A_{\rm FB}(M_Z \pm 3.5$ GeV). Can one control IFI in the charge asymmetry near Z resonance with the precision 3×10^{-5}? In Ref. [24], it was shown, using KKMC and new KKfoam programs, that one may reach precision of 10^{-4}. More effort is needed to get another improvement factor of 10.

The precision determination of the luminosity using low angle Bhabha process at FCC-ee will be again limited by the knowledge of higher order QED effects and hadronic contributions to vacuum polarization (VP) correction. In Ref. [31] and Chapter B of Ref. [7], it was shown that 10^{-4} precision of theoretical calculation of the low angle Bhabha for FCC-ee luminometer is feasible. This will allow to reduce error of the invisible Z decay rate measured in terms of the "number of neutrinos" N_ν from present $\delta N_\nu = \pm 0.006$ down to $\delta N_\nu = \pm 0.001$. Similar precision of N_ν, also limited by the QED effects, will be achievable using the process $e^+e^- \to X\gamma$, $X \to$ invis. [32].

New more precise calculation of the $e^+e^- \to W^+W^-$ is needed for the FCC-ee measurements of W mass and couplings. The 0.5 MeV precision of W mass from the threshold cross section and the mass distributions in the final state will require clever resummation of the QED effects using QED resummation techniques [25], Effective Field Theory [7, 33] and new higher order EW calculations beyond the $\mathcal{O}(\alpha^1)$ [34]. Precise measurement of the WW cross sections (distributions) and W mass (~ 0.5 MeV) will require: *(i)* $\mathcal{O}(\alpha^2)$ calculation of EW corrections for double-resonant (on-shell) — non-trivial but feasible, to be done, *(ii)* $\mathcal{O}(\alpha^1)$ calculation for single-resonant component (partly done in Ref. [34]), *(iii)* tree-level for non-resonant part (available), and *(iv)* the consistent scheme of combining all that within the Monte Carlo event generator! QED component will be again most sizeable[4] and equally important as pure EW corrections.

6. The need of new ideas for EW pseudo-observables

In the LEP era data analysis based on Ref. [37] and summarized in Refs. [14, 15], there were two types of observables, realistic observables (ROs), *i.e.* cross sections and distributions for well-defined experimental cut-offs (after removing detector inefficiencies using Monte Carlo) and EW pseudo-observables (EWPOs), in which QED effects were removed (deconvoluted). The simplest example of EWPO is hadronic (or total) cross section exactly at the mass of Z, $\sigma^0_{\rm had}$. It was obtained in LEP in such a way that experimental cross section at seven energy points was fitted with the following formula:

$$\sigma_{\rm had}(s) = \int_0^1 {\rm d}z\; \sigma^{\rm Born}_{\rm had}(zs)\; \rho_{\rm QED}(z)\,, \tag{6.1}$$

where $\sigma^{\rm Born}(s)$ comes from analytical formula in Eq. (3.8) in Ref. [14]. Mass and width of Z and couplings of Z to electron and final quarks are also obtained from the same fit. The effective radiator function $\rho_{\rm QED}(z)$ represents perturbative QED result for the initial-state multiphoton radiation. Finally, hadronic cross section, $\sigma^0_{\rm had}$, is obtained from analytical formula $\sigma^0_{\rm had} \equiv \sigma^{\rm Born}_{\rm had}(M_Z^2)$ inserting into it all parameters from the fit to data. Leptonic cross section σ^0_l is obtained in the same way.

Similarly, the charge asymmetry for lepton pair production process $e^+e^- \to l^+l^-$ is obtained using another convolution formula

$$\frac{{\rm d}\sigma^\mu}{{\rm d}\cos\theta^*}(s,\theta^*) = {\rm CONV}\left\{\frac{{\rm d}\sigma^{\rm Born}_\mu(s)}{{\rm d}\cos\theta}, \rho_{\rm QED}\right\}, \tag{6.2}$$

where θ^* is some experimentally well defined effective angle of outgoing leptons (they are not back to back due to photon emission). The meaning of the convolution CONV and the definition of the analytical formula for the effective Born distribution can be found in Ref. [14]. The value of the

[4] As in the LEP era calculations of RACOONWW [35] and YFSWW+KORALW [36].

pseudo-observable charge asymmetry $A^l_{\rm FB}$ does not correspond directly to the asymmetry of some well-defined experimental angular distribution, but results from the following analytical formula:

$$A^l_{\rm FB} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_l\,, \qquad \mathcal{A}_f = \frac{g^2_{{\rm L}f} - g^2_{{\rm R}f}}{g^2_{{\rm L}f} + g^2_{{\rm R}f}}\,, \tag{6.3}$$

where again the values of Z couplings $g_{{\rm L}f}$ and $g_{{\rm R}f}$ to fermion f are determined in the fit of $\frac{{\rm d}\sigma^\mu}{{\rm d}\cos\theta^*}(s,\theta^*)$ to data. The effective EW mixing angle is obtained also from the simple analytical formula

$$\frac{g_{{\rm V}f}}{g_{{\rm A}f}} = 1 - \frac{2Q_f}{T^3_f}\sin^2\theta^f_{\rm eff}$$

using fitted values of Z couplings.

In a similar way, that is using simple analytical formula with fitted Z couplings, mass and width inside, all nine EWPOs listed in Table 2.4 in Ref. [14] $m_Z, \mathit{\Gamma}_Z, \sigma^0_{\rm had}, R^0_e, R^0_\mu, R^0_\tau, A^{0,e}_{\rm FB}, A^{0,\mu}_{\rm FB}, A^{0,\tau}_{\rm FB}$ were obtained. The fundamental role of these EWPOs was to encapsulate in a compact way experimental data, such that SM predictions including $\mathcal{O}(\alpha)$ EW corrections were confronted with cut-off-independent EWPOs with the removed QED effects, instead of cut-off-dependent realistic data including QED effects.

Of course, the use of EWPOs at LEP was dangerous, because the convolution formulas were including simplified version of QED calculation (without initial–final state interference and with fully inclusive treatment of the final-state radiation). The use of the effective Born with effective Z couplings could also be incompatible, at a certain precision level, with the presence of the $\mathcal{O}(\alpha^1)$ EW corrections in the data (additional angular dependence from EW boxes). In Refs. [14, 37], it was proven that at the LEP data precision such dangers were avoided, by means of comparing realistic data with the predictions of the SM, in which internal parameters were previously fit to EWPOs. Such a "circular cross-check" is illustrated in Fig. 3.

It is quite likely that the above LEP construction of EWPOs will not pass the above circular cross-check test due to much smaller errors of FCC-ee experimental data. How to upgrade the definitions of EWPOs, such that they work at the FCC-ee precision level? Answering this question requires dedicated study. Most likely, two elements will have to be modified. In the transition from realistic data in step (B) to new EWPOs in step (C) in Fig. 3, semianalytical codes such as ZFITTER or TOPAZ0 will have to be replaced by the Monte Carlo programs of the KKMC class, or even more sophisticated ones. Most likely, the effective Born-like formula for spin amplitudes used to parametrize data in the (B)→(C) transition will have to include more of genuine $\mathcal{O}(\alpha^1)$ EW corrections, removing them from the data in the form of new EWPOs, in the same way as trivial pure QED effects.

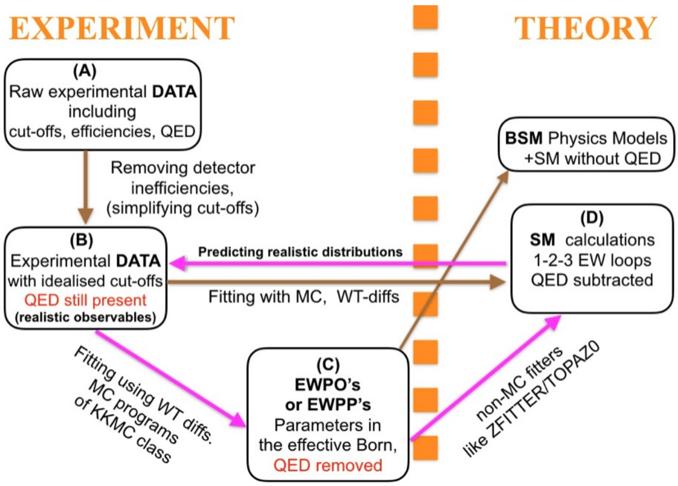

Fig. 3. Scheme of construction of EWPOs at FCC-ee. Main difference with LEP is Monte Carlo use in steps (B)→(C) and (B)→(D) instead of programs like ZFITTER/TOPAZ0.

7. Summary and outlook

We cannot get to better understanding of fundamental laws on Nature without answering a lot of big intriguing questions! Unfortunately, there is no clear hint from the theory where to look for the answers. Hence, one should explore all possible experimental fronts: highest possible energies, very weak and rare processes (neutrinos), astrophysics.

High precision measurements in the future electron–positron colliders will require major effort in order to improve SM/QED predictions for FCC-ee observables by a factor of 10–200. In particular, precision of QED calculations of asymmetries near the Z resonance has to be improved by a factor up to 200. New algorithms of extracting EW pseudo-observables from experimental data has to be worked out and cross-checked. The increased role of MC event generators at all levels of data analysis and in comparisons with the theory is anticipated.

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