LIGHT THOMAS–FERMI DARK MATTER*

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We investigate the viability of a simple dark matter (DM) model consisting of a single fermion in the context of galactic dynamics; the model has a single free parameter, the DM mass. We follow an approach similar to the one used in the Thomas–Fermi model of the atom, and find that for the 76 galaxies considered, the model can explain most of the bulk galactic properties provided the DM mass is in the $\mathcal{O}(50\ \text{eV})$ range. More precise tests of the model require better modeling of the baryon profile, and a better control on the uncertainties in the data.

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1. Introduction

Particle dark matter (DM) provides the simplest hypothesis for explaining a variety of astrophysical and cosmological observations, such as the flat galactic rotation curves, the velocity profiles in galactic clusters, the non-uniformities in the cosmic microwave background, and the mass distribution in the bullet cluster (for a review see, *e.g.* [1]). Unfortunately as of yet, there is no clear indication of what constitutes DM, so that models have proliferated. These range from proposing DM as primordial black holes of a few solar masses, to ultralight particles with mass $\sim 10^{-22}$ eV, a span of some 90 orders of magnitude in mass. The simplest of these models, however, have encountered difficulties in explaining the distribution of DM in galaxies, an issue known as the cusp *versus* core problem [2].

In this paper, we address the cusp *versus* core problem assuming a simple model where the DM is composed of a single fermion ψ that carries a $\mathbb{Z}_2$ charge under which the Standard Model (SM) is even. In this case, there are no renormalizable couplings of the ψ to the SM, so the absence of a direct

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detection signal [3–6] or collider effects [7, 8] are easily accommodated. The observed relic abundance must follow from a mechanism other than the common freeze-out process [9], but we do not discuss this here; we instead concentrate on whether this DM model can address the cusp problem.

We assume that the DM fermions in the galactic halo are in thermodynamic, chemical and mechanical equilibrium. We include baryonic effects by choosing a mass profile consistent with observations, but we neglect baryonic contributions to the thermodynamic quantities. For simplicity, we neglect all non-gravitational interactions of the DM fermions (though these can be readily included). This approach is closely related to others that have appeared in the literature (*e.g.* [10, 11]), the difference is that the DM profile is derived and not postulated *a priori*, and that the gravitational baryon effects are included using a semi-realistic profile.

2. Fermionic DM in galaxies

As indicated above, we assume the DM gas is in local hydrostatic equilibrium, which implies

$$mn\nabla\Phi + \nabla P = 0\,, \tag{1}$$

where m is the DM mass, n the DM density, P its pressure, and Φ the gravitational potential. We also assume that the gas is in global thermodynamic equilibrium, so that its temperature T is a (position-independent) constant. Using then the standard thermodynamic relation $n\,\mathrm{d}\mu = \mathrm{d}P - s\,\mathrm{d}T$, where μ is the chemical potential, we find

$$m\Phi + \mu = \text{constant}. \tag{2}$$

that can also be obtained using elementary arguments.

Using Eq. (2) in the Poisson equation for Φ gives

$$\nabla^2\mu = -\frac{4\pi m}{M_{\mathrm{Pl}}^2}\left(\rho_B + mn\right)\,; \qquad n = -\frac{2}{\lambda^3}\mathrm{Li}_{3/2}\left(-\mathrm{e}^{\mu/T}\right)\,; \quad \lambda = \sqrt{\frac{2\pi}{mT}}\,, \tag{3}$$

where M_{Pl} denotes the Planck mass[1], ρ_B is the baryon mass density, λ is the thermal wavelength; the factor of 2 in n is due to spin; we used for n the standard expression for an ideal non-relativistic fermion gas. Except for the presence of ρ_B, these arguments are identical to the ones used in constructing the Thomas–Fermi model of the atom [12, 13]. The DM profile is obtained by solving this equation subject to the boundary conditions that ensure the observed flat rotation curves.

[1] We work in units where $k_{\mathrm{B}} = \hbar = c = 1$, where k_{B} is Boltzmann's constant.

3. Spherical symmetry

Assuming all quantities depend only on $r = |\boldsymbol{r}|$, the above equations reduce to

$$\begin{aligned} \bar{u}'' &= x\mathrm{Li}_{3/2}\left(-\mathrm{e}^{\bar{u}/x}\right) - q\,xF(x\mathcal{R}/a)\,; \\ \frac{\bar{u}(x)}{x} &= \frac{\mu}{T}\,, \qquad q = \frac{3M_B\lambda^3}{8\pi m a^3}\,, \qquad x = \frac{r}{\mathcal{R}}\,, \qquad \mathcal{R} = \sqrt{\frac{TM_{\mathrm{Pl}}^2\lambda^3}{8\pi m^2}}\,, \end{aligned} \tag{4}$$

where $F(r/a) = (4\pi a^3/3)\rho_B/M_B$, and a is a typical scale of the galactic core. We choose boundary conditions that ensure that the circular velocity approach a constant at large r

$$v_{\mathrm{rot}}^2(r) = \frac{M_{\mathrm{tot}}(r)}{M_{\mathrm{Pl}}^2 r} \to \bar{v}_{\mathrm{rot}}^2 \quad \Rightarrow \quad \bar{u} \to x\ln\left(\frac{2}{x^2}\right)\,. \tag{5}$$

Such solutions have the following characteristics:

— $T = m\bar{v}_{\mathrm{rot}}^2/2$, which ensures that the fermions are non-relativistic as we have assumed (typically $\bar{v}_{\mathrm{rot}} \sim 10^{-3}$).

— In general, $\varPhi$ has a $1/r$ singularity near the origin

$$\varPhi \to -\frac{M_{\mathrm{BH}}}{M_{\mathrm{Pl}}^2}\frac{1}{r}\,, \qquad M_{\mathrm{BH}} = \left(\frac{\pi}{64}\right)^{1/4}\frac{\bar{v}_{\mathrm{rot}}^{3/2}M_{\mathrm{Pl}}^3}{m^2}\bar{u}(r=0)\,. \tag{6}$$

That is, such solutions require the presence of a central black hole. It is, however, possible to tune parameters so that $\bar{u}$ vanishes at the origin; the model can also accommodate galaxies without central black holes.

— Numerically, we find $\bar{u}(0) \lesssim 1.3$, so that

$$m^2 \lesssim (180\ \mathrm{eV})^2\frac{(10^3\bar{v}_{\mathrm{rot}})^{3/2}}{M_{\mathrm{BH}}/(10^9 M_\odot)}\,, \tag{7}$$

thus this model requires the DM fermions to be light.

— The size of the galactic halo (defined as the radius where the DM density equals 200 times the critical density of the Universe) is

$$R_{\mathrm{hal}} = \left(10^3\bar{v}_{\mathrm{rot}}\right) \times 240\ \mathrm{kpc}\,. \tag{8}$$

4. Results

To test the viability of this model, we collected a dataset of 68 galaxies [14–19] with the estimated values of the central black hole masses $M_{\rm BH}$, the total baryon masses M_B, the core radii a and the asymptotic rotational velocities $\bar{v}_{\rm rot}$. We also chose the baryon density function F from a set of commonly used profiles [20–22], an example is the so-called Hernquist profile

$$F(y) = \frac{2}{3y(1+y)^3}\,. \tag{9}$$

Combining all this, the above equations and boundary conditions give the DM mass m for each galaxy. Despite our simplifying assumptions, the results are gratifyingly consistent, giving masses in the 20–70 eV range (with a few outliers), as shown in Fig. 1.

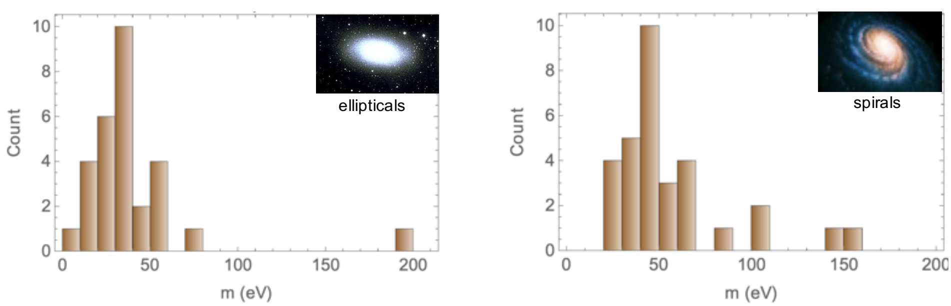
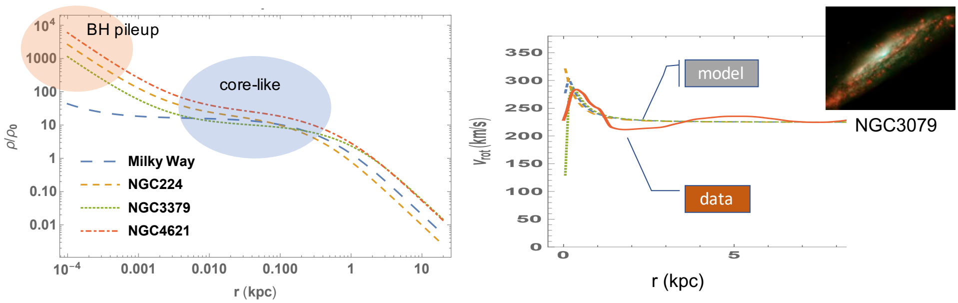


Fig. 1. Number of spiral and elliptical galaxies in our dataset for which the predicted DM mass m falls in a given mass bin.

In addition to the consistency of the DM masses, the obtained DM profile has also other desirable properties. For example, the model generates a core-like behavior at small r (with a pileup very close to the origin caused by the central black hole), as shown in Fig. 2 (a); for some choices of F, it also exhibits some of the observed features in $v_{\rm rot}$ at small r, as shown in Fig. 2 (b).

The model can also be applied to galaxies without a central black hole by imposing the additional requirement $\bar{u}(r=0)=0$, which (within this model) imposes an additional constraint, so that not only the DM mass is predicted, but a relation between the galactic parameters is needed. It is notable that the observational data meets this added constraint (within observational errors) and that the predicted DM masses fall also in the range obtained previously, as shown in Table I. For these galaxies, the circular velocity profile is also in good agreement with the observations (Fig. 3).



(a) (b)

Fig. 2. (a) DM density as a function of r for a set of 4 galaxies. The decrease at large r ensures the observed flat rotation curves. The behavior at small r is constant (emphasized by the shaded ellipse marked 'core-like'); as $r \to 0$, there is an increase due to the black hole (emphasized by the ellipse marked 'BH pileup'). Note that the r scale is logarithmic. (b) circular velocity profiles for the Milky Way at small r comparing the data (solid line — without error bars, for clarity), with the results obtained using 3 baryon density profiles (dotted and dashed lines).

TABLE I

Dwarf galaxies.

Galaxy	m [eV]
DDO 154	92.8
DDO 168	117.6
NGC 2915	89.2
NGC 3741	114.6
UGC 7603	159.5
UGC 5721	85.5
UGC 7690	65.5
UGC 8550	189.2

Summarizing: the spherically-symmetric Ansatz gives a reasonable fit to the observations for a DM mass in the $\mathcal{O}(100 \text{ eV})$ range. Yet there are discrepancies. These may be due to *(i)* the fact that many of the galaxies studied are far from being spherical, *(ii)* errors in the observational data we used, *(iii)* corrections in the baryon density profile. It is, of course, also possible that this model is not realized in Nature.

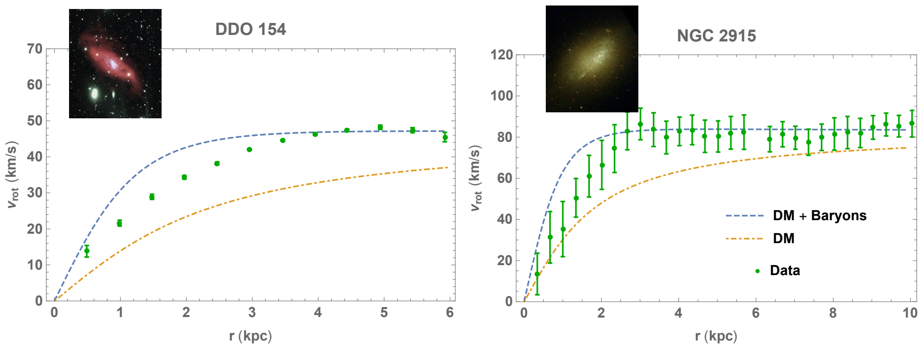


Fig. 3. Circular velocity profiles for two sample dwarf galaxies. The dot-dashed curve gives the prediction in the absence of DM, the dashed curve gives the result for DM+baryons.

5. Beyond spherical symmetry

For slight deviations from spherical symmetry, we can assume $\rho_B \to \rho_B(r) + \delta\rho_B(\boldsymbol{r})$ and $\mu \to \mu(r) + \delta\mu(\boldsymbol{r})$ and work to first order in the corrections. Equation (3) then gives

$$\left[-\nabla^2 + \frac{1}{A^2}\mathrm{Li}_{3/2}\left(-\mathrm{e}^{-\bar{u}/x}\right)\right]\delta\mu = \frac{4\pi m}{M_{\mathrm{Pl}}^2}\delta\rho_B \tag{10}$$

with solution

$$\delta\mu(\boldsymbol{r}) = \int \mathrm{d}^3\boldsymbol{r}'\, G\left(\boldsymbol{r},\, \boldsymbol{r}'\right)\, \delta\rho_B\left(\boldsymbol{r}'\right)\,, \tag{11}$$

where

$$G\left(\boldsymbol{r}, \boldsymbol{r}'\right) = \frac{1}{r\,r'}\sum_{l>0,m}\gamma_l^+(r_<)\gamma_l^-(r_>)Y_{lm}(\Omega)Y_{lm}^*\left(\Omega'\right)\,,$$

$$\gamma_l^\pm(r) \simeq \begin{cases} r^{\frac{1}{2}\pm\left(l+\frac{1}{2}\right)} & r \lesssim \mathcal{R} \\ r^{\frac{1}{2}\pm\nu_l} & r \gtrsim \mathcal{R} \end{cases} \quad ; \qquad \nu_l = \sqrt{\left(l+\tfrac{1}{2}\right)^2 - 2}$$

and $r_> = \max\{r, r'\}$, $r_< = \min\{r, r'\}$. This describes small shifts in the shape of the DM density, but does not affect the value of m. For large deviations from spherical symmetry, a full azimuthally-symmetric calculation must be performed.

6. Stability

It is of interest to determine whether the DM+baryon configurations we obtained are stable under small perturbations. In studying this, we assume that the temperature and baryon density remain constant, and that local chemical equilibrium holds ($\mathrm{d}P = n\,\mathrm{d}\mu$). Denoting by $\boldsymbol{v}$ the DM velocity, the hydrodynamic equations become

$$\partial_t \boldsymbol{v} + \left(\boldsymbol{v}\dot{\nabla}\right)\boldsymbol{v} = -\frac{1}{mn}\nabla P - \nabla\Phi\,, \qquad \partial_t n + \nabla\left(n\boldsymbol{v}\right) = 0\,. \tag{12}$$

Assuming the presence of harmonic perturbations δn, $\delta\mu$, $\delta\Phi$ proportional to $\exp(i\omega t)$, and noting that $\boldsymbol{v}$ will be of the same order, we find

$$\begin{aligned} \mathrm{i}\omega m\boldsymbol{v} &= \nabla\left(\delta\mu + m\delta\Phi\right)\,, & \mathrm{i}s\omega\delta n = \nabla\left(n\boldsymbol{v}\right)\,, \\ \delta n &= s\frac{2}{\lambda^3}\mathrm{Li}_{1/2}\left(-\mathrm{e}^{-\mu/T}\right)\frac{\delta\mu}{T}\,, & \nabla^2\delta\Phi = \frac{4\pi m}{M_{\mathrm{Pl}}^2}\delta n\,. \end{aligned} \tag{13}$$

One can verify that these expressions imply that the DM flux is irrotational. Writing then $\boldsymbol{v} = \nabla\chi$, we find

$$-\omega^2\frac{m}{T}\nabla^2\chi = \left(\nabla^2 + \kappa^2\,\mathrm{Li}_{1/2}\right)\left[\frac{\nabla\left(\mathrm{Li}_{3/2}\nabla\chi\right)}{\mathrm{Li}_{1/2}}\right]\,, \qquad \kappa^2 = \frac{2m^4\bar{v}_{\mathrm{rot}}}{\sqrt{\pi}\,M_{\mathrm{Pl}}^2}\,, \tag{14}$$

where the polylogarithmic functions have argument $-\exp(\mu/T)$. The above equation must be solved numerically; the unperturbed configuration is stable if all such solutions require $\omega^2 \geq 0$.

7. Conclusions

The proposed simple model fits some of the bulk aspects of galactic dynamics provided the DM mass in the $\sim$ 20–70 eV range; the observed discrepancies can be explained by data uncertainties and non-spherical symmetric effects. The model closely resembles the Thomas–Fermi model of the atom, though in the present case, exchange effects are very small, but this can change dramatically in case DM self-interactions are included. The validity model can be tested in a more refined way by including non-spherical effects and, of course, by more accurate data.

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