PRODUCTION OF PURELY GRAVITATIONAL VECTOR DARK MATTER*

Aqeel Ahmed

Theoretische Natuurkunde & IIHE/ELEM, Vrije Universiteit Brussel
Pleinlaan 2, 1050 Brussels, Belgium

Bohdan Grzadkowski, Anna Socha

Faculty of Physics, University of Warsaw
Pasteura 5, 02-093 Warszawa, Poland

(Received October 24, 2019)

We explore a possibility of dark matter (DM) interacting with the visible sector only through gravity. We consider the case of the vector DM and discuss both perturbative and non-perturbative mechanisms that can be relevant for DM production. In the first case, we investigate particle production during reheating phase via the freeze-in mechanism, while the latter refers to the particle creation in the time-varying background during inflation. In each case, we find a viable range in parameter space which reproduces the observed DM relic abundance.

DOI:10.5506/APhysPolB.50.1809

1. Introduction

The existence of dark matter (DM) is well-established by completely independent cosmological and astrophysical observations spanning over a wide range of cosmological scales. However, its nature remains one of the most puzzling and challenging problems in modern physics. The known properties of DM can be summarized in the following statement: it is invisible, extremely elusive, and electrically neutral. It should be emphasized that the existence of DM has been inferred only from gravitational effects. There is no direct evidence that DM interacts with forces other than gravity to the visible matter, however, it is commonly assumed that its interactions with the Standard Model (SM) are mediated via some non-gravitational forces

* Presented at the XLIII International Conference of Theoretical Physics “Matter to the Deepest”, Chorzów, Poland, September 1–6, 2019.

of weak strength. This is the well-known weakly interacting massive particle (WIMP) DM paradigm. However, decades of intensive searches of these WIMP DM remain fruitless with ever-strengthening constraints. These null results motivate to explore alternative paradigms for DM.

In this work, we consider an extreme possibility by assuming that DM interacts only gravitationally with the visible sector. We focus on the case where DM consists of spin-1 particles and investigate its perturbative and non-perturbative production mechanisms. In Sec. 2, we study the DM production via the freeze-in mechanism, where the SM particles annihilate to produce DM through the exchange of a graviton in s-channel [1, 2] and SM Higgs doublets annihilate into DM particles via a dim-6 effective operator [3]. By solving Boltzmann equations in Sec. 3, we find the region in parameter space that reproduces the observed DM abundance. In Sec. 4, we investigate DM production via the gravitational particle creation from vacuum fluctuation during inflation [4]. In particular, we focus on the production of longitudinal modes of the dark vector boson and find their number density today. We conclude in Sec. 5.

2. Vector DM production via freeze-in mechanism

To study the production of DM via the freeze-in mechanism, we consider the following action:

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left\{ \frac{R}{2\kappa^2} + \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\mathrm{DM}} \right\} , \tag{1}$$

where $\kappa \equiv \sqrt{8\pi G_{\mathrm{N}}} = M_{\mathrm{Pl}}^{-1}$ is the effective gravitational constant (G_{N} is Newton's constant and M_{Pl} is the Planck constant), R is the Ricci scalar, and $\mathcal{L}_{\mathrm{SM}}(\mathcal{L}_{\mathrm{DM}})$ denote the Lagrangian density of the SM (DM). In the weak-field approximation of general relativity, the graviton $h_{\mu\nu}$, *i.e.* the fluctuation of the metric tensor $g_{\mu\nu}$, is defined as follows

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu} ,$$

where $\eta_{\mu\nu}$ is the Minkowski metric diag$(+1, -1, -1, -1)$. We assume here that DM has no self-interactions and it does not interact directly with the SM particles. The only interaction between these two sectors is through gravity, which couples to their energy-momentum tensors. However, from the effective field theory perspective, one may expect interactions between the two sectors through the higher dimensional operators suppressed by the scale of a new physics Λ, which is assumed here to be of the order of the Planck mass M_{Pl}.

At the first order in the graviton perturbations, the effective action can be written as

$$S_{\text{eff}} = \int \mathrm{d}^4x\, \mathcal{L}_{\text{eff}} = \int \mathrm{d}^4x \left\{ \mathcal{L}^{(4)} + \mathcal{L}^{(5)}_{\text{int}} + \mathcal{L}^{(6)}_{\text{int}} + \mathcal{O}\left(\kappa^3, h^2\right) \right\}, \tag{2}$$

where $\mathcal{L}^{(4)} = \mathcal{L}^{(4)}_{\text{DM}} + \mathcal{L}^{(4)}_{\text{SM}}$ is the effective dim-4 renormalizable Lagrangian, whereas $\mathcal{L}^{(5)}_{\text{int}}$ and $\mathcal{L}^{(6)}_{\text{int}}$ contain operators of dim-5 and -6 suppressed by M_{Pl}^{-1} and M_{Pl}^{-2}, respectively. The dim-4 Lagrangian for an Abelian vector DM is

$$\mathcal{L}^{(4)}_{\text{DM}} = -\frac{1}{4} X_{\mu\nu} X^{\mu\nu} + \frac{1}{2} m_X^2 X_\mu X^\mu, \tag{3}$$

where the mass for the dark vector boson m_X is generated via the Abelian Higgs mechanism. We assume that the interaction part of the effective action is given by

$$\mathcal{L}^{(5)}_{\text{int}} = \frac{\kappa}{2} h^{\mu\nu} \left(T^{\text{DM}}_{\mu\nu} + T^{\text{SM}}_{\mu\nu}\right), \qquad \mathcal{L}^{(6)}_{\text{int}} = \frac{\kappa^2}{2} \mathcal{C}_X\, m_X^2 \mathcal{H}^\dagger \mathcal{H} X_\mu X^\mu, \tag{4}$$

where $T^{\text{SM}}_{\mu\nu}$ and $T^{\text{DM}}_{\mu\nu}$ denote the energy-momentum tensor for the SM and DM sectors, respectively, $\mathcal{H}$ is the SM Higgs doublet and $\mathcal{C}_X$ is the dimensionless Wilson coefficient.

A possible dim-4 kinetic mixing between $X_{\mu\nu}$ and the field tensor for the weak hypercharge is eliminated by a dark charge conjugation that ensures the stability of the vector. It is worthwhile to mention that $\mathcal{L}^{(5)}_{\text{int}}$ involves only interactions of the SM and DM with gravity, *i.e.* there is no dim-5 direct SM and DM interaction. The $\mathcal{L}^{(5)}_{\text{int}}$ will induce tree-level s-channel graviton exchange amplitudes $\mathcal{O}(\kappa^2)$. Note that the same suppression appears for the contact interactions (4) involving dim-6 operator, see Fig. 1.

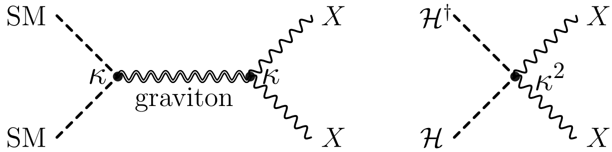


Fig. 1. Feynman diagrams that arise from (4). The left panel shows annihilation process for SM particles to vector DM X_μ via the s-channel graviton exchange while the right panel shows contact interaction induced by the dim-6 operator.

We briefly motivate the origin of dim-6 operator in (4), *i.e.* $\mathcal{H}^\dagger \mathcal{H} X_\mu X^\mu$, [5]. In our model, we assume that mass of the vector DM is generated by a heavy complex scalar Φ charged under the gauge $\mathrm{U}(1)_X$ symmetry. The covariant derivative of such a scalar field is

$$D_\mu \Phi = (\partial_\mu - i g_X X_\mu)\Phi \,,$$

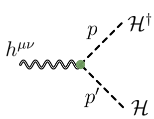
so that the kinetic term $D_\mu\Phi(D^\mu\Phi)^*$ contains $g_X^2 X_\mu X^\mu \Phi\Phi^*$. It is assumed that Φ acquires its vacuum expectation value (VEV), which leads to spontaneous symmetry breaking (SSB) of the extra $\mathrm{U}(1)_X$ gauge symmetry and generates a mass for the vector DM, $m_X^2 = 2g_X^2\langle\Phi\rangle^2$. The dim-6 interaction of Eq. (4) arises through the Higgs doublet coupling to the new scalar field Φ as

$$D_\mu\Phi(D^\mu\Phi)^*\mathcal{H}^\dagger\mathcal{H} \supset g_X^2\Phi\Phi^*\mathcal{H}^\dagger\mathcal{H}X_\mu X^\mu \to \frac{m_X^2}{2}\,\mathcal{H}^\dagger\mathcal{H}X_\mu X^\mu \,.$$

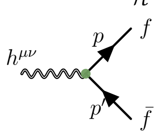
The first step toward computing DM production via the freeze-in from the SM is to calculate the annihilation amplitudes. To do this, we expand action (1) in the linearized level in the graviton field. By considering graviton interactions with spin-0, 1/2, 1 fields, we obtain Feynman rules that are collected in Table I. Using the Feynman rules we find cross sections for XX production in annihilation of SM spin 0,1/2 and 1 states. Note that the scattering amplitude for the SM scalars going into DM vectors is a sum of contributions from the SM annihilation via graviton exchange and contract interactions.

TABLE I

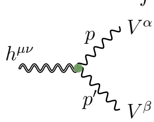
Feynman rules for the gravitational vector DM model, where h_i, f, $V(X)$ represent spin-0, 1/2, 1 fields, respectively.



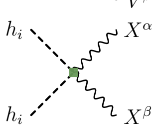
$$i\frac{\kappa}{2}\left[p_\mu p'_\nu + p_\nu p'_\mu - \eta_{\mu\nu}\left(p\cdot p' + m_H^2\right)\right]$$



$$i\frac{\kappa}{8}\left[\Big(\gamma_\mu\left(p-p'\right)_\nu + \gamma_\nu\left(p-p'\right)_\mu\Big) - 2\eta_{\mu\nu}\left(\left(\not{p}-\not{p}'\right) + 2m_f\right)\right]$$



$$-i\frac{\kappa}{2}\Big[C_{\mu\nu,\alpha\beta}\left(p\cdot p' + m_V^2\right) + \eta_{\mu\nu}p_\beta p'_\alpha + \eta_{\alpha\beta}\left(p_\mu p'_\nu + p_\nu p'_\mu\right) - p_\beta\left(\eta_{\mu\alpha}p'_\nu + \eta_{\nu\alpha}p'_\mu\right) - p'_\alpha\left(\eta_{\mu\beta}p_\nu + \eta_{\nu\beta}p_\mu\right)\Big]$$



$$i\kappa^2\mathcal{C}_X m_X^2\eta_{\mu\nu}\,, \qquad C_{\mu\nu,\alpha\beta} \equiv \eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta}$$

3. Boltzmann equations

The thermal evolution in our model involves unstable massive particles ϕ (inflaton), a stable non-relativistic massive DM particle X_μ and ultra-relativistic SM species $\mathcal{R}$ (radiation). We assume that ϕ decays into the SM (radiation) and DM with rates $\Gamma_{\mathcal{R}}$ and Γ_X, respectively. The DM X are created by (and annihilated to) the SM with the cross section $\langle\sigma v\rangle_1$. To find the relic abundance of DM species, we generalize the system of Boltzmann equations [6]

$$\begin{aligned} \dot{\rho}_\phi + 3(1+w)H\rho_\phi &= -(\Gamma_{\mathcal{R}} + \Gamma_X)\rho_\phi \,, & (5)\\ \dot{\rho}_{\mathcal{R}} + 4H\rho_{\mathcal{R}} &= \Gamma_{\mathcal{R}}\rho_\phi + 2\langle E_X\rangle\langle\sigma v\rangle_1 \left[n_X^2 - \left(n_X^{\mathrm{eq}}\right)^2\right] , & (6)\\ \dot{n}_X + 3Hn_X &= \Gamma_X \frac{\rho_\phi}{m_\phi} - \langle\sigma v\rangle_1 \left[n_X^2 - \left(n_X^{\mathrm{eq}}\right)^2\right] , & (7) \end{aligned}$$

where $\rho_{\phi,\mathcal{R}}$ and n_X are corresponding energy and number densities, n_X^{eq} is the equilibrium number density of X, $\langle E_X\rangle \simeq m_X$ is an averaged energy of the DM field, w is the equation-of-state parameter for the inflaton field[1], and m_ϕ denotes its mass. For the sake of simplicity, we make here the simplifying assumption that $\Gamma_X \to 0$. Moreover, we also neglect the possibility of the inflaton annihilation into DM species. Time evolution of the Hubble rate is governed by the first Friedmann equation, *i.e.*

$$H^2 = \frac{\kappa^2}{3}\left(\rho_\phi + \rho_X + \rho_{\mathcal{R}}\right) .$$

To solve the above Boltzmann equations, it is instructive to introduce dimensionless variables

$$\tilde{\Phi} = \rho_\phi \frac{a^{3(1+w)}}{T_{\mathrm{RH}}^4} , \qquad \mathcal{R} = \rho_{\mathcal{R}} \frac{a^4}{T_{\mathrm{RH}}^4} , \qquad X = n_X \frac{a^3}{T_{\mathrm{RH}}^3} ,$$

where T_{RH} denotes the (end of) reheating temperature, when $\Gamma_{\mathrm{R}} = H$,

$$T_{\mathrm{RH}} = \left(\frac{90}{\kappa^2\pi^2 g_*(T_{\mathrm{RH}})}\right)^{\frac{1}{4}} \gamma H_{\mathrm{I}}^{\frac{1}{2}} .$$

In the above expression H_{I} denotes the Hubble rate at the end of inflation, $g_*(T_{\mathrm{RH}})$ counts effective number of relativistic degrees of freedom (d.o.f) at T_{RH}, and $\gamma^2 \equiv \Gamma_R/H_{\mathrm{I}} = H_{\mathrm{RH}}/H_{\mathrm{I}}$ parametrizes the efficiency of reheating.

[1] It is assumed here that during reheating, the total energy density ρ_{tot} is dominated by the inflaton component, *i.e.* $\rho_{\mathrm{tot}} \sim \rho_\phi$. In particular, at the end of inflation $\rho_{\mathcal{R}} = \rho_X = 0$.

To find approximate semi-analytical solutions to the above system of coupled equations, we assume that during reheating, *i.e.* the epoch between $H_{\rm I}^{-1}$ and $H_{\rm RH}^{-1}$, the total energy density ρ is dominated by the inflaton field, while after reheating, during the radiation-dominated period, the main contribution to the total energy density comes from the SM radiation. Those assumptions allow to solve the first two Boltzmann equations that provide relations between the Hubble rate and the scale factor and also between the temperature and the scale factor

$$T(a) \simeq \begin{cases} T_{\rm max} a^{-\frac{3}{8}(1+w)} & \text{for} \quad a_{\rm e} < a < a_{\rm RH} \\ T_{\rm RH} \frac{a_{\rm RH}}{a} & \text{for} \quad a \geq a_{\rm RH} \end{cases}$$

and

$$H(a) = \begin{cases} H_{\rm I} a^{-\frac{3}{2}(1+w)} & \text{for} \quad a < a_{\rm RH} \\ H_{\rm RH} \left(\frac{a_{\rm RH}}{a}\right)^2 & \text{for} \quad a > a_{\rm RH} \end{cases} ,$$

where $T_{\rm max}$ is the maximal temperature that was obtained during the reheating and $a_{\rm e(RH)}$ denotes the scale factor at the end of inflation (reheating). After obtaining $H(a)$ and $T(a)$, we can solve the last Boltzmann equation which, in terms of new variables, takes the form of

$$\frac{{\rm d}X}{{\rm d}a} = -\frac{\langle \sigma v \rangle_1}{H T_{\rm RH}^3} a^2 \left(n_X^2 - \left(n_X^{\rm eq}\right)^2 \right) . \tag{8}$$

The Gondolo–Gelmini formula for the thermally-averaged cross section is [7]:

$$\langle \sigma v \rangle = \frac{1}{8 m_X^4 T K_2(m_X/T)^2} \int\limits_{4m_X^2}^{\infty} {\rm d}s \sqrt{s} \left(s - 4m_X^2\right) \sigma(s) K_1\left(\frac{\sqrt{s}}{T}\right) ,$$

with

$$\sigma(s) = \frac{-1}{16\pi s \left(s - 4m_X^2\right)} \int\limits_{t_+}^{t_-} {\rm d}t |\mathcal{M}|^2 ,$$

where $t_\pm = -(\sqrt{s/4 - m_X^2} \mp \sqrt{s/4}\,)^2$ and we have assumed that masses of the SM d.o.f. are negligible.

The total cross section for the SM annihilations into spin-1 DM can be written as a sum of the following three contributions:

$$\langle \sigma v \rangle_1 = N_0 \langle \sigma v \rangle_{0\to1} + N_{1/2} \langle \sigma v \rangle_{1/2\to1} + N_1 \langle \sigma v \rangle_{1\to1} ,$$

where $N_0 = 4, N_{1/2} = 45, N_1 = 12$ denote the number of d.o.f. of the SM scalars, fermions and vectors, respectively (before the EW SSB), whereas,

$\langle \sigma v \rangle_{0,1/2,1\to 1}$ refers to the thermally averaged cross sections for the SM scalars, fermions and vectors to the vector DM, respectively. Assuming that $n_X \ll n_X^{\mathrm{eq}}$, we rewrite Eq. (8) as

$$X(T_0) = \int_{a_{\mathrm{e}}}^{a_{\mathrm{RH}}} \mathrm{d}a \frac{\langle \sigma |v| \rangle_1}{HT_{\mathrm{RH}}^3} a^2 \left(n_X^{\mathrm{eq}}\right)^2 + \int_{a_{\mathrm{RH}}}^{a_0} \mathrm{d}a \frac{\langle \sigma |v| \rangle_1}{HT_{\mathrm{RH}}^3} a^2 \left(n_X^{\mathrm{eq}}\right)^2 \,,$$

where a_0 denotes the present-day scale factor and we can adopt $a_0 \to \infty$ since in this limit, the production rate is exponentially suppressed.

Finally, the relic abundance of the vector DM is defined as

$$\Omega_X h^2 = \frac{\rho_X}{\rho_c} h^2 = \frac{m_X n_X(T_0)}{\rho_c} h^2 \,, \tag{9}$$

where the present-day number density is given by

$$n_X(T_0) = X(T_0) T_{\mathrm{RH}}^3 \gamma^{\frac{4}{1+w}} \frac{s_0}{s_{\mathrm{RH}}} \,, \tag{10}$$

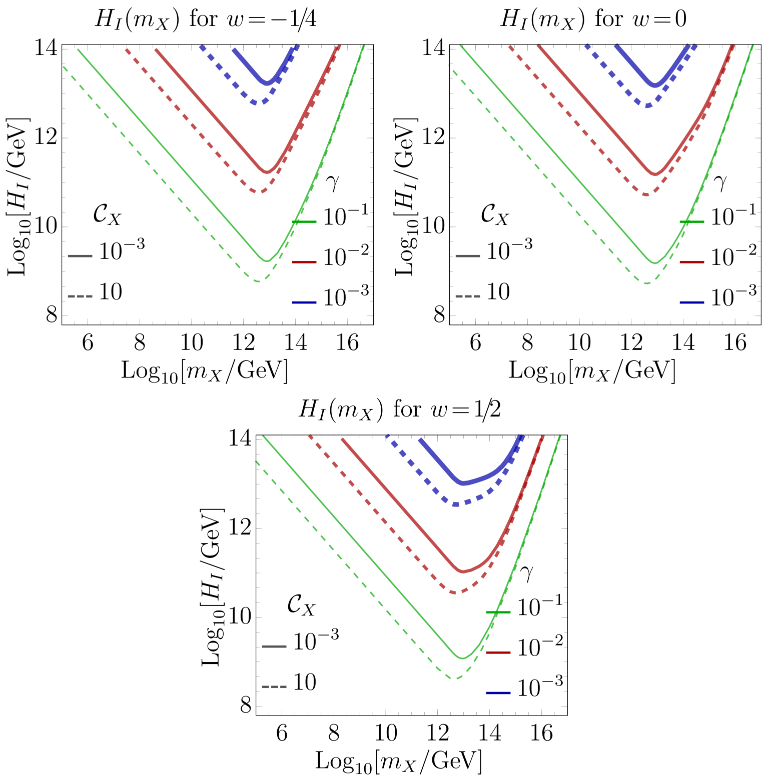


Fig. 2. Relations between the Hubble rate at the end of inflation H_{I} and vector DM mass m_X that reproduces the observed relic abundance $\Omega_{\mathrm{DM}}^{\mathrm{obs}} h^2$.

with $s_0(s_{\mathrm{RH}})$ being the entropy density at the temperature $T_0(T_{\mathrm{RH}})$. We use relations (9)–(10) and require the parameters to be such that the present-day abundance $\Omega_X h^2$ reproduces the value measured by Planck $\Omega_{\mathrm{DM}}^{\mathrm{obs}} h^2 = 0.1198 \pm 0.0015$ [8]. The results are shown in Fig. 2 in H_{I}–m_X plane for several values of the equation-of-state parameter $w \in \{-\frac{1}{4}, 0, \frac{1}{2}\}$, reheat efficiency $\gamma \in \{10^{-3}, 10^{-2}, 10^{-1}\}$, and $\mathcal{C}_X \in \{10^{-3}, 10\}$. We note that the difference between the case with Wilson coefficient $\mathcal{C}_X = 10^{-3}$ and $\mathcal{C}_X = 10$ is rather small and is important only in the small mass limit. This means that the contribution from the term $N_0\langle\sigma|v|\rangle_{0\to 1}$ to the vector DM production is small compared to the $N_{1/2,1}\langle\sigma|v|\rangle_{1/2,1\to 1}$. On the other hand, this mechanism of DM production is very sensitive to reheating efficiency. The allowed range in parameter space is quite large in the limit of efficient reheating and shrinks significantly as γ declines.

4. Gravitational production of vector DM

In this section, we study DM matter production in the rapidly expanding early Universe. We consider the following action for a massive vector DM:

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left(-\frac{1}{4} g^{\mu\alpha} g^{\nu\beta} X_{\mu\nu} X_{\alpha\beta} + \frac{1}{2} m_X^2 g^{\mu\nu} X_\mu X_\nu \right) .$$

The background metric is the Friedmann–Lemaître–Robertson–Walker (FLRW) form, *i.e.* $\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\mathrm{d}\vec{x}^{\,2}$. To derive equation of motion (e.o.m.) for the X_μ field, it is convenient to go to the Fourier space

$$X_\mu(t, \vec{x}\,) = \int \frac{\mathrm{d}^3 k}{(2\pi)^{3/2}} \mathcal{X}_\mu(t, \vec{k}\,) \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} ,$$

where the reality of X_μ implies $\mathcal{X}_\mu(t, \vec{k}\,) = \mathcal{X}_\mu^*(t, -\vec{k}\,)$. Three physical components of the vector field satisfy, $\vec{k} \cdot \vec{\mathcal{X}} = k\mathcal{X}_{\mathrm{L}}, \vec{k} \times \vec{\mathcal{X}} = \pm k|\vec{\mathcal{X}}_\pm|$. It should be emphasized that the $\mathcal{X}_0$ mode is non-dynamical and can be express as

$$\mathcal{X}_0 = \frac{-\mathrm{i}k\dot{\mathcal{X}}_{\mathrm{L}}}{k^2 + a^2 m_X^2} . \tag{11}$$

The e.o.m. for the transverse modes is

$$\mathcal{X}_\pm'' + \left(k^2 + a^2 m_X^2\right) \mathcal{X}_\pm = 0 ,$$

and for the longitudinal mode, it is

$$\mathcal{X}_{\mathrm{L}}'' + \frac{2k^2}{k^2 + a^2 m_X^2} \frac{a'}{a} \mathcal{X}_{\mathrm{L}}' + \left(k^2 + a^2 m_X^2\right) \mathcal{X}_{\mathrm{L}} = 0 , \tag{12}$$

where the prime denotes the derivative with respect to conformal time τ which is related to the physical time t by $\mathrm{d}t = a(\tau)\mathrm{d}\tau$.

For the two transverse modes $\mathcal{X}_{\pm}$, we get e.o.m in a form of the oscillator equation with frequency $\omega_{\mathrm{T}}^2 = k^2 + a^2 m_X^2$, whereas in the case of longitudinal mode $\mathcal{X}_{\mathrm{L}}$, we redefine $\mathcal{X}_{\mathrm{L}}$ as follows:

$$\mathcal{X}_{\mathrm{L}} = \frac{\sqrt{k^2 + a^2 m_X^2}}{a m_X} \tilde{\mathcal{X}}_{\mathrm{L}} \tag{13}$$

in order to rewrite Eq. (12) in a form of the oscillator equation, *i.e.*

$$\tilde{\mathcal{X}}_{\mathrm{L}}'' + \omega_{\mathrm{L}}^2(\tau) \tilde{\mathcal{X}}_{\mathrm{L}} = 0\,, \tag{14}$$

where

$$\omega_{\mathrm{L}}^2(\tau) \equiv k^2 + a^2 m_X^2 - \frac{k^2}{k^2 + a^2 m_X^2} \left(\frac{a''}{a} - \frac{3a^2 m_X^2}{\left(k^2 + a^2 m_X^2\right)} \frac{a'^2}{a^2} \right) . \tag{15}$$

Note that $\omega_{\pm}^2$ is always positive, while $\omega_{\mathrm{L}}^2(\tau)$ can be negative. It can be shown that this happens for $m_X \ll H_{\mathrm{I}}$. This phenomenon leads to the so-called *tachyonic enhancement* and results in the growth of the super-horizon modes ($k < aH$) during inflation. Therefore, in what follows, we focus on the longitudinal modes because it turns out that in this case, the gravitational production is much more efficient than in a case of transverse modes.

Our ultimate goal is to find the present-day energy density of DM particles produced by the vacuum fluctuations during inflation. For this purpose, we start with quantizing the longitudinal mode as

$$\left[\hat{\tilde{X}}_{\mathrm{L}}(\tau, \vec{x}\,), \hat{\tilde{\Pi}}_{\mathrm{L}}(\tau, \vec{y}\,)\right] = i\delta^{(3)}(\vec{x} - \vec{y}\,)\,, \tag{16}$$

where

$$\hat{\tilde{X}}_{\mathrm{L}}(\tau, \vec{x}\,) = \int \frac{\mathrm{d}^3 k}{(2\pi)^{3/2}} \left[\epsilon_{\mathrm{L}}(\vec{k}\,)\hat{a}(\vec{k}\,)\mathcal{X}_{\mathrm{L}}(\tau, \vec{k}\,)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} + \epsilon_{\mathrm{L}}^*(\vec{k}\,)\hat{a}^\dagger(\vec{k}\,)\mathcal{X}_{\mathrm{L}}^*(\tau, \vec{k}\,)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}}\right] \tag{17}$$

and $\hat{\tilde{\Pi}}_{\mathrm{L}}(\tau, \vec{x}) = \hat{\tilde{X}}_{\mathrm{L}}'(\tau, \vec{x})$ denotes the canonical momenta. It is easy to check that (16) implies

$$\left[\hat{a}(\vec{k}\,), \hat{a}^\dagger(\vec{k'}\,)\right] = \delta^{(3)}(\vec{k} - \vec{k'}\,)\,, \qquad \left[\hat{a}(\vec{k}\,), \hat{a}(\vec{k'}\,)\right] = 0 = \left[\hat{a}^\dagger(\vec{k}\,), \hat{a}^\dagger(\vec{k'}\,)\right] , \tag{18}$$

with the time-independent Wronskian normalized as

$$W\left[\mathcal{X}_{\mathrm{L}}, \mathcal{X}_{\mathrm{L}}^*\right] \equiv \mathcal{X}_{\mathrm{L}}' \mathcal{X}_{\mathrm{L}}^* - \mathcal{X}_{\mathrm{L}}'^* \mathcal{X}_{\mathrm{L}} = -\mathrm{i}\,.$$

Using the energy-momentum tensor we get the energy density as

$$\rho_X = T_{00} = \frac{1}{2a^2}\left(\left|\dot{\vec{X}} - \nabla X_0\right|^2 + \frac{1}{a^2}\left|\vec{\nabla}\times\vec{X}\right|^2 + m_X^2 a^2 X_0^2 + m_X^2 \vec{X}^2\right).$$

In the following computations, we replace the above expression with its vacuum expectation value. We define the vacuum state by the following Bunch–Davies initial condition, *i.e.*,

$$\lim_{\tau\to-\infty} \tilde{\mathcal{X}}_{\mathrm{L}} = \frac{1}{\sqrt{2k}} \mathrm{e}^{-\mathrm{i}k\tau}\,.$$

Using Eqs. (11), (13), (17) and (18), we find

$$\begin{aligned}\frac{\mathrm{d}\langle\rho_{\mathrm{L}}\rangle}{\mathrm{d}\ln k} = \frac{k^3}{2\pi^2 a^4}\Bigg\{&\left|\mathcal{X}_{\mathrm{L}}'\right|^2 - \left(\mathcal{X}_{\mathrm{L}}'\mathcal{X}_{\mathrm{L}}^* + \mathcal{X}_{\mathrm{L}}'^*\mathcal{X}_{\mathrm{L}}\right)\frac{k^2}{k^2 + a^2 m_X^2}\frac{a'}{a} \\ &+ \left[\frac{k^4}{\left(k^2 + a^2 m_X^2\right)^2}\left(\frac{a'}{a}\right)^2 + k^2 + a^2 m_X^2\right]\left|\mathcal{X}_{\mathrm{L}}\right|^2\Bigg\}\,. \end{aligned} \tag{19}$$

Therefore, in order to find $\langle\rho_{\mathrm{L}}(\tau)\rangle$, one should first determine solutions to Eq. (14) at some time τ.

Assuming that during inflation the Universe undergoes almost purely the de Sitter expansion, we find that $\mathrm{d}\langle\rho_{\mathrm{L}}\rangle/\mathrm{d}\ln k$ at the end of inflation is proportional to

$$\frac{\mathrm{d}\langle\rho_{\mathrm{L}}(\tau_{\mathrm{e}})\rangle}{\mathrm{d}\ln k} \propto \begin{cases} \left(\frac{k}{a_{\mathrm{e}}}\right)^4 & \text{for sub-horizon modes } a_{\mathrm{e}}H_{\mathrm{I}} \ll k \\ \left(\frac{kH_{\mathrm{I}}}{a_{\mathrm{e}}}\right)^2 & \text{for super-horizon modes } k \ll a_{\mathrm{e}}H_{\mathrm{I}} \end{cases},$$

where a_{e} denotes the scale factor at the end of inflation. Then we obtain the evolution of $\langle\rho_{\mathrm{L}}(\tau)\rangle$ after inflation as follows

$$\frac{\mathrm{d}\langle\rho_{\mathrm{L}}(\tau)\rangle}{\mathrm{d}\ln k} \propto \begin{cases} a^{-4}\,, & \max[a(\tau)m_X, a(\tau)H(\tau)] \ll k \\ a^{-2}\,, & \max[a(\tau)m_X, k] \ll a(\tau)H(\tau) \\ a^{-3}\,, & \max[a(\tau)H(\tau), k] \ll a(\tau)m_X \end{cases}.$$

Evolution of $\mathrm{d}\langle\rho_{\mathrm{L}}\rangle/\mathrm{d}\ln k$ with time is shown in Fig. 3. As the Universe evolves, the redshift of the energy density varies for different modes. We

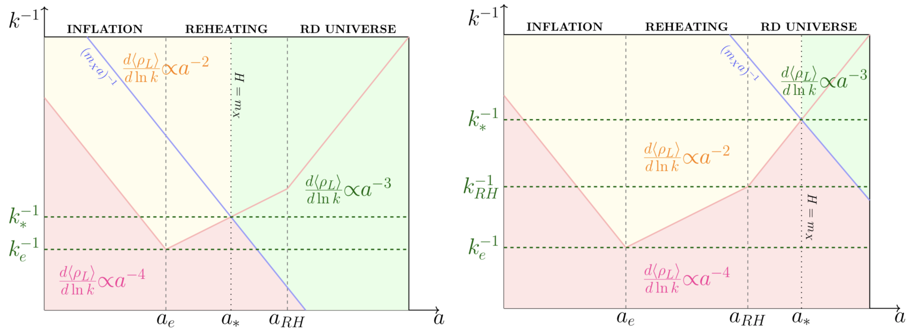


Fig. 3. Scaling of the energy density as a function of the scale factor for heavy *i.e.* $H_{\rm RH} < m_X$ (left diagram) and light $H_{\rm RH} > m_X$ (right diagram) vector DM. The main contribution to the total energy density comes from the mode $k_* \equiv a(\tau : H = m_X)m_X$. Here, $k_{\rm e} \equiv a_{\rm e}H_{\rm e}$ and $k_{\rm RH} \equiv a_{\rm RH}H_{\rm RH}$.

see that at the end of inflation ($a = a_{\rm e}$) the main contribution to the total energy density comes from modes with the shortest wavelength. However, after inflation, those modes receive the strongest suppression proportional to a^{-4}. Note that the energy density of modes with $k = k_*$ is the least redshifted. It turns out that the present-day number density of DM particles could be expressed in terms of number density $\mathrm{d}\langle n_*\rangle/\mathrm{d}\ln k$ at $H(T_*) = m_X$. After some tedious calculations, we have:

— For DM vector bosons with mass $H_{\rm RH} < m_X < H_{\rm I}$

$$\frac{\mathrm{d}\langle n_*\rangle}{\mathrm{d}\ln k} = \begin{cases} \frac{1}{8\pi^2} H_{\rm I}^{\frac{2(1+3w)}{3(1+w)}} m_X^{\frac{1-3w}{3(1+w)}} \left(\frac{k}{a_{\rm e}}\right)^2 , & k < k_* \\ \frac{1}{8\pi^2} H_{\rm I}^{\frac{2\left(3w^2+3w+2\right)}{(w+1)(3w+1)}} m_X^{\frac{2}{1+w}} \left(\frac{a_{\rm e}}{k}\right)^{\frac{3(1-w)}{(1+3w)}} , & k_{\rm e} > k > k_* \end{cases} .$$

— For DM vector bosons with mass $m_X < H_{\rm RH}$

$$\frac{\mathrm{d}\langle n_*\rangle}{\mathrm{d}\ln k} = \begin{cases} \frac{1}{8\pi^2} H_{\rm I} \gamma^{\frac{2(1-3w)}{3(1+w)}} \left(\frac{k}{a_{\rm e}}\right)^2 , & k < k_* \\ \frac{1}{8\pi^2} m_X^{3/2} H_{\rm I}^{5/2} \gamma^{\frac{-1+3w}{3(1+w)}} \left(\frac{a_{\rm e}}{k}\right) , & k_* < k < k_{\rm RH} \\ \frac{1}{8\pi^2} m_X^{3/2} \gamma^{\frac{1-3w}{1+w}} H_{\rm I}^{\frac{3(w+3)}{2(3w+1)}} \left(\frac{a_{\rm e}}{k}\right)^{\frac{3(1-w)}{1+3w}} , & k_{\rm e} > k > k_{\rm RH} \end{cases} .$$

Above, the equation-of-state parameter w corresponds to the total energy density, *i.e.* $p = w\rho_{\rm tot}$. The role of w is to allow for an extra freedom while describing gravitational evolution during reheating. More microscopically,

w would be an effective parameter that takes into account the presence of an inflaton, radiation and perhaps another components of the Universe at that time. We note that the number density per log frequency has a peak structure if and only if $w \in (-\frac{1}{3}, 1)$. Then, $\mathrm{d}\langle n_* \rangle / \mathrm{d} \ln k$ is dominated by modes with $k = k_* \equiv a(\tau : H = m_X) m_X$. The observed value of DM number density, $n_X(T_0)$, is related to the number density $n_* \equiv \langle n_* \rangle$ at $T_* : H(T_*) = m_X$ as

$$n_X(T_0) = n_* \left(\frac{a_*}{a_0} \right)^3 = n_* \frac{s_0}{s_*} \, ,$$

where $s_0(s_*)$ refers to the entropy density at $T = T_0 (T = T_*)$, respectively. The entropy density is

$$s_* = \frac{p + \rho}{T_*} = \frac{4}{3} \frac{\rho_{\mathrm{R}}}{T_*} = \frac{4 \kappa^{-2} m_X^2}{T_*} \, ,$$

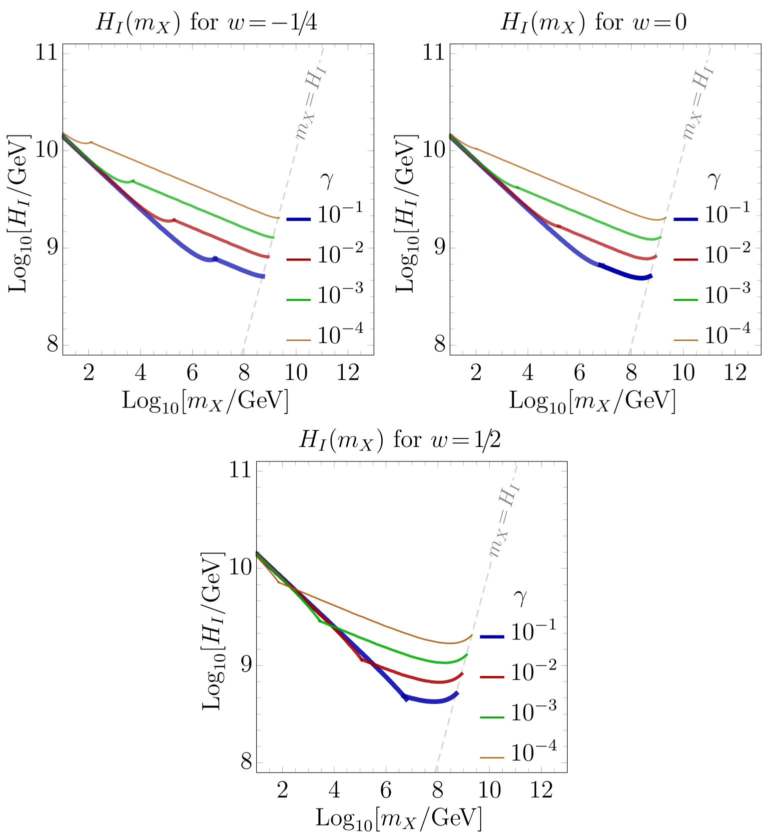


Fig. 4. Relations between the Hubble rate at the end of inflation H_{I} and vector DM mass m_X that reproduces the observed relic abundance $\Omega_{\mathrm{DM}}^{\mathrm{obs}} h^2$.

where T_* is determined by the following condition:

$$T_* = T_{\rm RH} \times \begin{cases} \left(\frac{m_X}{\gamma^2 H_{\rm I}}\right)^{1/4}, & H_{\rm RH} < m_X \\ \left(\frac{m_X}{\gamma^2 H_{\rm I}}\right)^{1/2}, & m_X < H_{\rm RH} \end{cases} .$$

Relations between $H_{\rm I}$ and m_X for several values of w and γ that reproduce the observed value of DM density $\Omega_{\rm DM}^{\rm obs} h^2$ are shown in Fig. 4. We limit ourselves to mass $m_X \ll H_{\rm I}$, since in this case, we expect the tachyonic enhancement in longitudinal mode production. We see that if the reheating was extended in time (*i.e.* for $\gamma = 10^{-4}$), then the Hubble rate at the end of inflation has to be larger than in the case of very efficient reheating ($\gamma = 10^{-1}$) to produce particle with the same mass. Moreover, we notice that gravitational particle production is most relevant for relatively light DM vector bosons.

5. Conclusion

In this work, we have demonstrated that massive vector particles that communicate with the SM sector only through gravity can serve as a viable DM candidate. It was shown that even in the minimal scenario, there exist mechanisms, *i.e.* freeze-in from SM and pure gravitational production from vacuum fluctuations during inflation that can reproduce the observed relic abundance of the vector DM. In each case, we showed the viable range in parameter space $H_{\rm I}$–m_X that produces the correct relic abundance. We have found out that the thermal mechanism of particle production prefers rather effective scenario for reheating and could produce very heavy ($\mathcal{O}(10^{17})$ GeV) vector DM, while the pure gravitational production is most relevant for particles with masses smaller than the Hubble rate during reheating.

REFERENCES

[1] M. Garny, A. Palessandro, M. Sandora, M.S. Sloth, *J. Cosmol. Astropart. Phys.* **1802**, 027 (2018).

[2] Y. Tang, Y.-L. Wu, *Phys. Lett. B* **774**, 676 (2017).

[3] E.W. Kolb, A.J. Long, *Phys. Rev. D* **96**, 103540 (2017).

[4] Y. Ema, K. Nakayama, Y. Tang, *J. High Energy Phys.* **1907**, 060 (2019).

[5] M. Duch, B. Grzadkowski, J. Wudka, *J. High Energy Phys.* **1505**, 116 (2015).

[6] G.F. Giudice, E.W. Kolb, A. Riotto, *Phys. Rev. D* **64**, 023508 (2001).

[7] P. Gondolo, G. Gelmini, *Nucl. Phys. B* **360**, 145 (1991).

[8] P.A.R. Ade *et al.*, *Astron. Astrophys.* **594**, A13 (2016).