

MODIFICATION OF THE SYMMETRY ENERGY BY STRANGENESS*

I. BEDNAREK, J. ŚLADKOWSKI, J. SYSKA

Institute of Physics, University of Silesia
75 Pułku Piechoty 1, 41-500 Chorzów, Poland

(Received November 20, 2019)

Analysis of the density dependence of the symmetry energy in the case of strangeness-rich neutron star matter has been done. It has been shown that the equation of state which gives the maximum neutron star mass of the order of $2 M_{\odot}$ meets the experimental constraints obtained for the high density limit of the symmetry energy.

DOI:10.5506/APhysPolB.50.1849

1. Introduction

Correct modeling, *i.e.* reproduction of global parameters as well as the internal structure of a neutron star requires specification of an equation of state of dense asymmetric nuclear matter. Different models describing matter of a neutron star have been developed. They are becoming more and more complex mainly due to the increase in the diversity of the chemical composition of this matter. At densities relevant for neutron star interiors, the exotic form of matter such as hyperons is expected to emerge. To get a reliable neutron star model, every opportunity to confront a theoretical model with experimental constraints is extremely valuable. The most important astrophysical constraint requires that theoretical models must lead to an equation of state that gives a maximum mass at least equal to the largest mass determined observationally, which is of the order of $2 M_{\odot}$ [1, 2]. A very important aspect of theoretical modeling of a neutron star is the possibility of studying its internal structure, *i.e.* finding a solution for the radial distribution of particle numbers, isospin asymmetry or other quantities that characterize neutron star matter. This is currently beyond the experimental range. However, modification of the chemical composition of neutron star matter by including hyperons as additional degrees of freedom changes both

* Presented at the XLIII International Conference of Theoretical Physics “Matter to the Deepest”, Chorzów, Poland, September 1–6, 2019.

the properties of this matter as well as the structure and global parameters of the star. In this context, the reports that in the central zone of heavy-ion collision experiments the density of the order of $(2 \div 3) \times n_0$, where n_0 is the saturation density, can be achieved [3] take on special significance. The aforementioned density range is achieved in neutron star interiors, moreover, hyperons appear in this density range, specifically Λ hyperon. Thus, this gives the opportunity to analyze the properties of Λ -nucleon matter in terms of isospin asymmetry and symmetry energy.

2. Theoretical framework

Theoretical modeling of dense nuclear matter comprises baryons defined as Dirac spinors interacting through the exchange of the following mesons: σ (scalar), ω and ρ (vector). The attractive hyperon–hyperon interaction, seen in double- Λ hypernuclei, is simulated via the exchange of hidden strangeness mesons: scalar σ^* and vector φ . Here, this path is followed. The Lagrangian density function embodies contributions from baryons and mesons

$$\begin{aligned} \mathcal{L} = & \sum_B \bar{\psi}_B (i\gamma_\mu D^\mu - m_{\text{eff},B}) \psi_B + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} \partial_\mu \sigma^* \partial^\mu \sigma^* \\ & - \frac{1}{2} m_{\sigma^*}^2 \sigma^{*2} + \frac{1}{2} m_\omega^2 (\omega_\mu \omega^\mu) + \frac{1}{2} m_\rho^2 (\rho_\mu^a \rho^{a\mu}) + \frac{1}{2} m_\varphi^2 (\varphi_\mu \varphi^\mu) \\ & - \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} - \frac{1}{4} \mathbf{R}_{\mu\nu} \mathbf{R}^{\mu\nu} - \frac{1}{4} \Phi_{\mu\nu} \Phi^{\mu\nu} - U_S(\sigma) + U_{V,\text{ext}}(\omega, \rho, \varphi), \end{aligned} \quad (1)$$

where $D^\mu = \partial^\mu + ig_{B\omega}\omega^\mu + ig_{B\varphi}\varphi^\mu + ig_{B\rho}\tau_B\rho^\mu$ is the covariant derivative and τ_B denotes the isospin operator. The baryon effective mass is defined as $m_{\text{eff},B} = m_B - g_{B\sigma}\sigma - g_{B\sigma^*}\sigma^*$ and $U_S = \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4$, while $\Omega_{\mu\nu}$, $\mathbf{R}_{\mu\nu}$, and $\Phi_{\mu\nu}$ are the field tensors of the ω , ρ , and ϕ mesons. Neutron star matter is considered electrically neutral and in β -equilibrium, so the presence of leptons is necessary. The Lagrangian density of leptons is given by

$$\mathcal{L}_l = \sum_{l=e,\mu} \bar{\psi}_l (i\gamma_\mu \partial^\mu - m_l) \psi_l. \quad (2)$$

The nonlinear vector meson mixed and self-coupling terms have been collected in the form of a vector potential

$$\begin{aligned} U_{V,\text{ext}} = & \frac{1}{4}c_3 (\omega_\mu \omega^\mu)^2 + \frac{1}{4}c_3 (\rho_{a\mu} \rho^{a\mu})^2 + \Lambda_V (g_\omega g_\rho)^2 (\omega_\mu \omega^\mu) (\rho_{a\mu} \rho^{a\mu}) \\ & + \frac{1}{2} \left(\frac{3}{2}sc_3 - \Lambda_V (g_\omega g_\rho)^2 \right) (\omega_\mu \omega^\mu + \rho_{a\mu} \rho^{a\mu}) (\varphi_\mu \varphi^\mu) \\ & + \frac{1}{4} \left(\frac{1}{2}c_3 + \Lambda_V (g_\omega g_\rho)^2 \right) (\varphi_\mu \varphi^\mu)^2. \end{aligned} \quad (3)$$

These terms were added in order to improve the high density limit of the equation of state [4, 5]. If the nonlinear vector meson coupling terms that involve the strange meson φ are neglected, then the vector potential $U_{V,\text{ext}}$ is reduced to a much simpler form

$$U_{V,\text{st}} = \frac{1}{4}c_3 (\omega_\mu \omega^\mu)^2 + \Lambda_V (g_\omega g_\rho)^2 (\omega_\mu \omega^\mu) (\rho_{a\mu} \rho^{a\mu}) . \quad (4)$$

The Lagrangian function (1) for the $U_{V,\text{ext}}$ defines the TM1_{ext} model whereas when $U_{V,\text{ext}}$ is replaced by $U_{V,\text{st}}$ the standard TM1 model (TM1_{st}) in which the strange meson are included in a minimal fashion is obtained. For all calculations the mean field approach has been adopted thus the quantum field operators are replaced by their classical expectation values: $\sigma = \bar{\sigma} + s_0$, $\sigma^* = \bar{\sigma}^* + s_0^*$, $\varphi_\mu = \bar{\varphi}_\mu + f_0 \delta_{\mu 0}$, $\omega_\mu = \bar{\omega}_\mu + w_0 \delta_{\mu 0}$, $\rho_\mu^a = \bar{\rho}_\mu^a + r_0 \delta_{\mu 0} \delta^{3a}$. The β -equilibrium for neutrino free matter and for only Λ hyperon relates chemical potentials of particles: $\mu_p = \mu_n - \mu_e$, $\mu_\Lambda = \mu_n$, where the chemical potential of baryon B takes the form of $\mu_B = \sqrt{k_{F,B}^2 + m_{\text{eff},B}^2} + g_{B\omega} w_0 + g_{B\rho} \tau_{3B} r_0 + g_{B\varphi} f_0$. Determination of the energy-momentum tensor allows one to calculate the energy and pressure of the strangeness-rich neutron star matter

$$\begin{aligned} \mathcal{E}_{\text{ext}} = & \frac{1}{2}m_\omega^2 w_0^2 + \frac{1}{2}m_\rho^2 r_0^2 + \frac{1}{2}m_\varphi^2 f_0^2 + \frac{1}{2}m_\sigma^2 s_0^2 + \frac{3}{4}c_3 (w_0^4 + r_0^4) + \frac{1}{2}m_{\sigma^*}^2 s_0^{*2} \\ & + \frac{3}{4} \left(\frac{1}{2}c_3 + \Lambda_V (g_\omega g_\rho)^2 \right) f_0^4 + \frac{3}{2} \left(\frac{3}{2}c_3 - \Lambda_V (g_\omega g_\rho)^2 \right) (w_0^2 + r_0^2) f_0^2 \\ & + U_S(s_0) + 3\Lambda_V (g_\omega g_\rho)^2 w_0^2 r_0^2 + \sum_B \frac{2}{\pi^2} \int_0^{k_{F,B}} k^2 dk \sqrt{k^2 + m_{\text{eff},B}^2} + \mathcal{E}_L , \quad (5) \end{aligned}$$

$$\begin{aligned} \mathcal{P}_{\text{ext}} = & \frac{1}{2}m_\omega^2 w_0^2 + \frac{1}{2}m_\rho^2 r_0^2 + \frac{1}{2}m_\varphi^2 f_0^2 - \frac{1}{2}m_\sigma^2 s_0^2 - U_S(s_0) + \Lambda_V (g_\omega g_\rho)^2 w_0^2 r_0^2 \\ & + \frac{1}{4}c_3 (w_0^4 + r_0^4) + \frac{1}{2} \left(\frac{3}{2}c_3 - \frac{1}{2}m_{\sigma^*}^2 s_0^{*2} \Lambda_V (g_\omega g_\rho)^2 \right) (w_0^2 + r_0^2) f_0^2 \\ & + \frac{1}{4} \left(\frac{1}{2}c_3 + \Lambda_V (g_\omega g_\rho)^2 \right) f_0^4 + \sum_B \frac{1}{3\pi^2} \int_0^{k_{F,B}} \frac{k^4 dk}{\sqrt{k^2 + m_{\text{eff},B}^2}} + \mathcal{P}_L , \quad (6) \end{aligned}$$

where \mathcal{E}_L and \mathcal{P}_L denote the contributions coming from leptons. The presence of nonlinear couplings between ω and ρ mesons and the strange meson φ modifies the strange sector of the equation of state leaving its nonstrange part almost unchanged.

3. Symmetry energy

Combining data from terrestrial experiments and using constraints from astrophysical observations, it is possible to estimate consistently the values of the symmetry energy coefficient $E_{\text{sym}}(n_0) = 31.6 \pm 2.66$ MeV and the slope $L(n_0) = 58.9 \pm 16$ MeV [6]. The density dependence of the symmetry energy can be expressed as a power law

$$E_{\text{sym}}(n_b) = E_{\text{sym}}^{\text{kin}}(n_b) + E_{\text{sym}}^{\text{pot}}(n_b) = \frac{1}{3}\epsilon_F \left(\frac{n_b}{n_0}\right)^{2/3} + C \left(\frac{n_b}{n_0}\right)^{\gamma}, \quad (7)$$

in which the kinetic and potential terms are included. The coefficient of the kinetic part equals $\epsilon_F/3$ with $\epsilon_F \approx 28$ MeV being the Fermi energy. The potential part of the symmetry energy is characterized by the exponent γ , the coefficient $C = 22$ MeV. Massive experimental efforts are devoted to find constraints on the value of γ . The early data from the FOPI-LAND experiment [7, 8] have been compared with calculations performed with the ultrarelativistic quantum molecular dynamics (UrQMD) transport model [9–11] and with the power law parametrization of the density dependence of the potential parts of the symmetry energy. This allows to put constraints on the density dependence of the symmetry energy and leads to the value of the coefficient $\gamma = 0.9 \pm 0.4$. To improve the accuracy of the experimental flow parameters, a new ASY-EOS experiment [3] was carried out. The analysis of the obtained data, with particular effort directed at the improvement of the statistical accuracy, leads to the value of $\gamma = 0.72 \pm 0.19$ [3].

4. Parameters of the model

TM1 parametrization gives large symmetry energy slope $L = 118$ MeV which requires extension of the isovector sector. This has been done by the inclusion of a nonlinear ω – ρ coupling and enables modification of the high density limit of the symmetry energy. The strength of this coupling is characterized by the parameter Λ_V . For each value of Λ_V , the parameter g_ρ has to be adjusted to reproduce the symmetry energy $E_{\text{sym}} = 25.68$ MeV at $k_F = 1.15 \text{ fm}^{-1}$ [12, 13]. The description of dense, hyperon-rich nuclear matter given by Lagrangian (1) in the case of nonstrange matter it is reduced to the standard TM1 model with an extended isovector sector. In the case when hyperons are taken into account, uncertainties associated with the description of nuclear matter are intensified due to the incompleteness of the available experimental data. In general, the experimental knowledge of the hyperon–nucleon (YN) and hyperon–hyperon (YY) interactions is quite limited. Hyperon–vector meson coupling constants are taken from the SU(6) quark model. In the scalar sector, the scalar couplings $g_{A\sigma}$ of A hyperons

require constraining in order to reproduce the estimated values of the potentials felt by Λ in the saturated nuclear matter. In the case of Λ hypernuclei, there is a considerable amount of data on binding energies and single-particle levels allowing the identification of the potential felt by a single Λ in nuclear matter in the range of $-30 \text{ MeV} \leq U_{\Lambda}^{(N)} \leq -27 \text{ MeV}$ [14]. As far as YY interactions are considered, the only sources of information are the double-strange hypernuclear systems. Several events have been identified that suggest an attractive $\Lambda\Lambda$ interaction. The most promising results, known as the NAGARA event [15] with the ${}^6_{\Lambda\Lambda}\text{He}$ hypernucleus, indicate that the $\Lambda\Lambda$ interaction is weakly attractive and its estimated value is $U_{\Lambda}^{(\Lambda)} = -5 \text{ MeV}$. The potential that describes hyperon–nucleon and hyperon–hyperon interaction can be written in a form that involves both the scalar and vector coupling constants

$$\begin{aligned} U_Y^{(B)} &= g_{Y\sigma}s_0 - g_{Y\omega}w_0 + g_{Y\sigma^*}s_0^* - g_{Y\varphi}f_0 \\ &= m_Y - m_{\text{eff},Y}(s_0, s_0^*) - (g_{Y\omega}w_0 + g_{Y\varphi}f_0) , \end{aligned} \quad (8)$$

where $m_{\text{eff},Y}(s_0, s_0^*)$ is the effective mass and Y stands for the Λ hyperon. The value of the potential $U_{\Lambda}^{(N)} = -30 \text{ MeV}$ has been used for the determination of the $g_{\Lambda\sigma}$ coupling constant. The coupling constants used in this model are given in Tables I and II.

TABLE I

The standard TM1 parameter set [16].

$m_{\sigma} = 511.2 \text{ MeV}$	$g_{\sigma} = 10.029$	$g_2 = 7.2325 \text{ fm}^{-1}$
$m_{\omega} = 783 \text{ MeV}$	$g_{\omega} = 12.614$	$g_3 = 0.6183$
$m_{\rho} = 770 \text{ MeV}$	$g_{\rho} = 9.264$	$c_3 = 71.0375$

TABLE II

Vector and scalar meson coupling constants.

Baryon (B)	$g_{B\omega}$	$g_{B\phi}$	$g_{B\rho}$	$g_{B\sigma}$	$g_{B\sigma^*}$
n	g_{ω}	0	g_{ρ}	g_{σ}	0
p	g_{ω}	0	g_{ρ}	g_{σ}	0
Λ	$\frac{2}{3}g_{\omega}$	$-\frac{\sqrt{2}}{3}g_{\omega}$	0	6.227	5.568

5. Results

For matter with nonzero strangeness, calculations were carried out using two methods. In the first case, the two-component model describing matter of a neutron star was supplemented with the full baryon octet and with strange mesons σ^* and φ . However, with parameters given in Tables I and II, this approach leads to the maximum mass of a neutron star much lower than the value determined observationally. An alternative method consists in taking into account various types of couplings between mesons with particular emphasis on couplings with the strange φ meson. This leads to the stiffening of the EoS and results in an appropriate value of the maximum mass. The EoSs calculated for these two models are presented in Fig. 1. The composi-

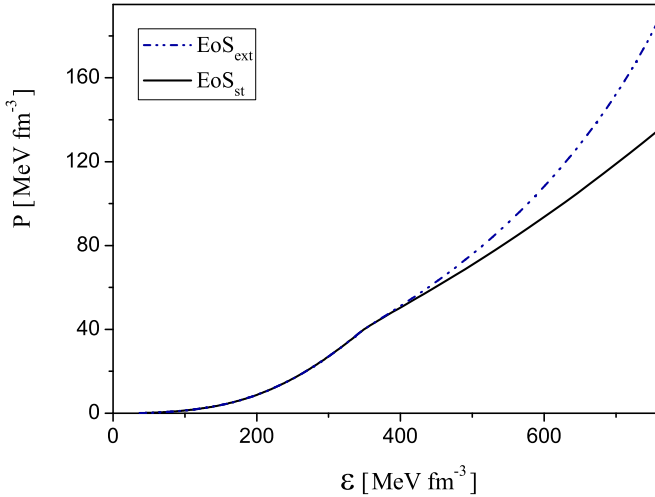


Fig. 1. The EoSs calculated for TM1 parameterization. The stiff EoS — dash-dotted line — has been obtained in the case of the extended nonlinear TM1 model (EoS_{ext}) for the value of $\Lambda_V = 0.0165$. The solid line represents the case of the standard TM1 model extended by the inclusion of strange mesons which have been introduced in a minimal fashion (EoS_{st}). Nonlinear vector meson couplings stiffen the EoS which results in a higher value of the maximum mass.

tion of strangeness-rich neutron star matter is controlled by the equilibrium conditions (β -equilibrium and charge neutrality) and the values of potentials $U_A^{(N)}$ and $U_A^{(\Lambda)}$ [17]. Relative concentrations of particular components are defined as the ratio $Y_i = n_i/n_b$, where n_i denotes the number density of the component i and n_b is the total baryon number density. In general, the appearance of Λ hyperons reduces the concentrations of nucleons and leptons. In addition, the chemical composition of neutron star matter is altered by the modification of the vector meson sector. Relative concentrations of

nucleons and Λ hyperons as a function of the baryon number density n_b for the fixed value of parameter $\Lambda_V = 0.0165$ are presented in the left panel of Fig. 2. From this figure, it can be seen that concentration of Λ hyperons is significantly reduced in the case of TM1_{ext} model. The concentration of leptons, which is higher in the case of TM1_{ext} model is depicted in the right panel of Fig. 2. The relative baryon–lepton composition of neutron

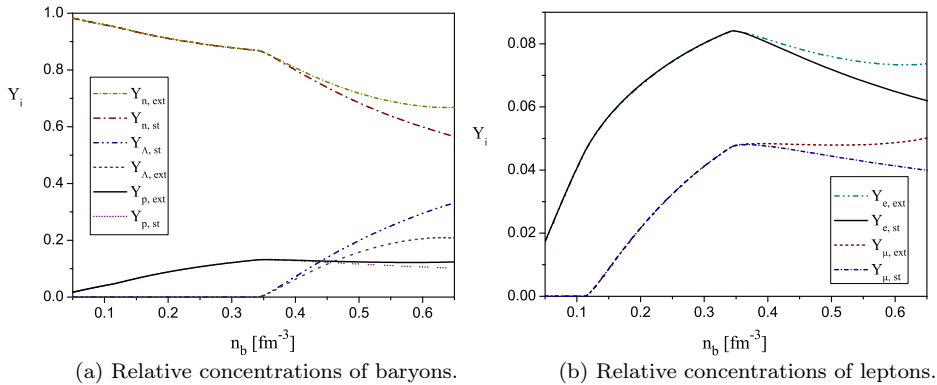


Fig. 2. Relative concentrations of baryons calculated for TM1_{ext} and for TM1_{st} models, for the fixed value of parameter $\Lambda_V = 0.0165$. The extended nonlinear model leads to the reduced number density of Λ hyperons. The right panel shows the relative concentrations of leptons.

star matter can be also analysed through the parameters that specify the isospin asymmetry and strangeness content of the system. In the case of two-component nucleonic matter, the isospin asymmetry is defined as

$$\delta_a^N = \frac{n_n - n_p}{n_n + n_p}. \quad (9)$$

The same definition can be used when neutron star matter contains also Λ hyperons. Then, the nucleon concentrations are changed due to equilibrium conditions. In general, the isospin asymmetry of neutron star matter containing hyperons can be interpreted in accordance with definition (9) or further modified by application of the following relation:

$$\delta_a^\Lambda = \frac{n_n - n_p}{n_b} = 1 - 2Y_p - Y_\Lambda. \quad (10)$$

The strangeness content given by $\delta_S = \sum_B n_B S_B / n_b$, where S_B denotes the strangeness of baryon B in the case of Λ -nucleon matter is simply the relative Λ concentration Y_Λ . The β -equilibrium condition can be used to establish the relationship between chemical potentials of neutron matter constituents

and the symmetry energy. In the case of Λ -nucleon matter, it takes the following form:

$$E_{\text{sym}}(n_b) = \frac{1}{4\delta_a^A} \frac{n_N}{n_b} (\mu_{\text{asym}} - m_n + m_p), \quad (11)$$

where $\mu_{\text{asym}} = \mu_n - \mu_p$ and $n_N = n_n + n_p$ is the density of nucleons. When the difference between neutron and proton masses is neglected and $n_N = n_b$, Eq. (11) reduces to $E_{\text{sym}}(n_b) = \mu_{\text{asym}}/(4\delta_a^N)$. The density dependences of the asymmetry parameters δ_a^N and δ_a^A are depicted in the left panel of Fig. 3. Application of Eq. (10) results in a significantly reduced value of the isospin

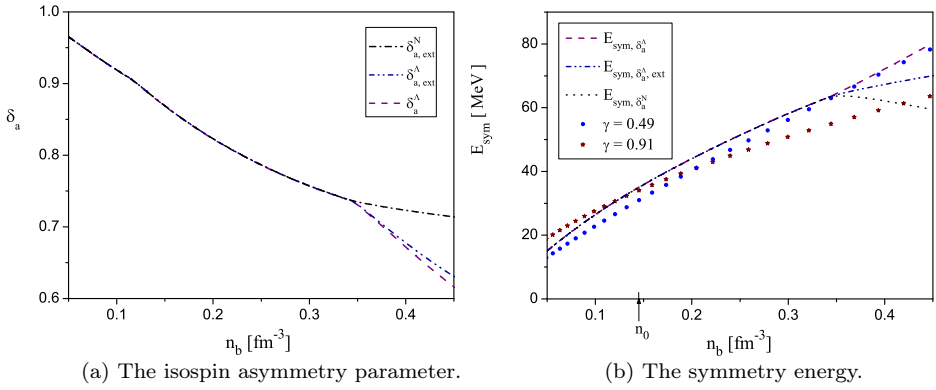


Fig. 3. The asymmetry parameters δ_a^N and δ_a^A as functions of baryon density n_b calculated for the extended TM1_{ext} and the standard TM1_{st} models, for the value of parameter $\Lambda_V = 0.0165$. The isospin asymmetry δ_a^N reaches the highest value whereas δ_a^A for both models leads to less asymmetric neutron star matter. The right panel: the density dependence of the symmetry energy. This figure contains results obtained based on both models (TM1_{ext} and TM1_{st}) for δ_a^N and δ_a^A . Experimental constraints are marked by stars and dots.

asymmetry compared to the value calculated on the basis of Eq. (9). Considering the results obtained for δ_a^A , it can be seen that TM1_{ext} model leads to a system with an enhanced asymmetry. The density dependence of the symmetry energy is depicted in the right panel of Fig. 3. Experimental limitations determined based on the power law dependence given by Eq. (7), for two boundary values of γ are represented by stars and dots. The dashed and dotted lines represent results obtained for the standard TM1 parametrization with the symmetry energy calculated according to Eq. (11). They differ in the form of the function determining the isospin asymmetry of neutron star matter. The symmetry energy considered in terms of δ_a^N asymmetry is a decreasing function of density when hyperons are present. This can lead to

rather soft form of the symmetry energy. Instead of using δ_a^N , the solutions may be discussed in terms of δ_a^A . In this case, a more satisfactory results have been obtained. For the standard TM1 model (TM1_{st}), the symmetry energy is close to the upper limit for experimental constraints. However, the most promising result has been got for the model TM1_{ext} which is even more important as this model meets astrophysical constraints.

6. Conclusions

Analysis of neutron matter properties in the density range of $(2 \div 3) \times n_0$ is of particular importance because this density range is achievable in HIC experiments thus the obtained theoretical predictions can be related to the experimental limitations. In this paper, the symmetry energy was analyzed for matter of a neutron star with nonzero strangeness. The couplings of Λ to scalar mesons have been determined basing on the existing hypernuclei data, whereas Λ -vector mesons couplings were taken from the SU(6) quark model. Calculations were made for TM1 parameterization using two different approaches to matter with non-zero strangeness. This gave the opportunity to compare the results obtained for a model that complies with astrophysical limitations and gives a correspondingly high value of the maximum mass with the model that does not meet this condition. The density dependence of the symmetry energy is affected by the presence of hyperons. Both the isospin asymmetry and symmetry energy density dependence are modified by the presence of the Λ hyperon in the matter of a neutron star. The best fit to the experimental constraints was obtained for the TM1 model and for the isospin asymmetry δ_a^A .

REFERENCES

- [1] P. Demorest *et al.*, *Nature* **467**, 1081 (2010).
- [2] J. Antoniadis *et al.*, *Science* **340**, 6131 (2013).
- [3] P. Russotto *et al.*, *Phys. Rev. C* **94**, 034608 (2016).
- [4] I. Bednarek, R. Manka, *J. Phys. G* **36**, 095201 (2009).
- [5] I. Bednarek *et al.*, *Astron. Astrophys.* **543**, A157 (2012).
- [6] B.A. Li, *Nucl. Phys. News* **27**, 7 (2017).
- [7] Y. Leifels *et al.* [FOPI Collaboration], *Phys. Rev. Lett.* **71**, 963 (1993).
- [8] D. Lambrecht *et al.*, *Z. Phys. A* **350**, 115 (1994).
- [9] Q. Li *et al.*, *J. Phys. G* **31**, 1359 (2005).
- [10] Q. Li *et al.*, *J. Phys. G* **32**, 151 (2006).
- [11] Q. Li *et al.*, *J. Phys. G* **32**, 407 (2006).
- [12] C.J. Horowitz, J. Piekarewicz, *Phys. Rev. Lett.* **86**, 5647 (2001).

- [13] B.K. Sharma, S. Pal, *Phys. Lett. B* **682**, 23 (2009).
- [14] A. Gal, E.V. Hungerford, D.J. Millener, *Rev. Mod. Phys.* **88**, 035004 (2016).
- [15] H. Takahashi *et al.*, *Phys. Rev. Lett.* **87**, 212502 (2001).
- [16] Y. Sugahara, H. Toki, *Progr. Theor. Phys.* **92**, 803 (1994).
- [17] I. Bednarek, R. Manka, M. Pienkos, *PLoS ONE* **9**, e106368 (2014).